# Spin-dependent scattering of electron wave on multi-barrier non collinear system

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## ABSTRACT

The elastic scattering of a non-spin-polarized electron wave on a film containing three ferromagnetic nanolayers separated by nonmagnetic layers is considered. Materials of the layers can be chosen so that the potential relief of the structure contains three identical barriers separated by nonmagnetic quantum wells. We assume that the induction vectors  $\vec{B}_i$  (*i* = 1, 2, 3) of the internal magnetic fields of the barriers are parallel to the surfaces of the nanolayers and form, respectively, the angles  $0, \theta_1, \theta_2$  with a certain direction (z axis,  $\vec{B}_1 \Box z$ ), that is, are noncollinear in general. It is shown that the transport characteristics of the structure, such as: the transmission coefficient and the degree of spin polarization of the transmitted wave are functions with double periodicity over the angles  $\theta_1$  and  $\theta_2$ . In addition, the dependence of these characteristics on one of the angular variables and the asymptotic momentum electron for a fixed value of the second angular variable was investigated.

Keywords: spin-dependent transport, double noncollinearity, transfer-matrix, partial amplitudes, internal magnetic field, potential relief

## **1 INTRODUCTION**

Quite a lot of papers [1-5] are devoted to solving the main problems of spintronics - the creation and control of a spinpolarized current. In this regard, we have proposed the use of layered systems (films) containing ferromagnetic nanolayers, separated for example by non-magnetic metal layers. The potential relief of these structures is a series (a chain) of identical magnetized barriers separated by nonmagnetic quantum wells. Each of the barriers is characterized by its own internal magnetic field induction.

The magnetizations of the barriers are autonomous, that is, we will assume that the exchange interaction between the two nearest barriers is missing. The set of vectors of internal fields, in general, are non-collinear or non-coplanar, and their spatial orientations are determined by a certain number of angular degrees of freedom. This leads to the fact that the elastic scattering of the electron wave on such a system occurs (happens) through two channels - without spin flip and with its flip. The corresponding probabilities of forward and backward scattering depend on the degrees of freedom mentioned above. The manipulation of the degrees of freedom leads to the variation of these scattering probabilities and, ultimately, to the variation of the transport characteristics of the system on the transmission coefficient of the electron wave and its degree of spin polarization. Thus, in papers [6], the scattering of an electron wave on a system consisting of two barriers with noncollinear internal fields was considered. Interesting features are revealed in the behavior of the transmission coefficient and the degree of polarization of the transmitted wave, which are functions of the same angular degree of freedom. Certain analogies with the behavior of optical systems were found in [6]. The scattering on a system of collinear barriers containing a single "non-coplanar defect" are considered [7] and it is shown that the presence of this "defect" significantly affects the behavior of transport quantities. Also it is assumed that the incident electron wave is not spin polarized. We are actually interested in the spin-polarizing properties of the systems under consideration.

### 2. STATEMENT OF THE PROBLEM

Thus, we consider the scattering of a non-spin-polarized electron wave on a system consisting of three identical magnetic barriers arranged equidistant and separated by non-magnetic quantum wells. The three barriers are characterized by induction vectors of internal magnetic fields which are parallel to the planes of the layers. This means that the magnetic layers possess an "easy-plane" anisotropy. Besides, we will assume that the magnetic layers are isotropic with respect to the rotations of the internal field vector both clockwise and counterclockwise. We also make



Fig.1 Diagram of a heterostructure containing three magnetic nanolayers. The axes *x* and *z* are located in the plane parallel to the planes of the layers, and *y* is perpendicular to these layers. The vectors of internal fields are equal in magnitude  $(B = B_1 = B_2)$  and differ only in orientations.

the following assumption, which greatly simplifies the solution of the scattering problem: the magnetic field in the film plane can be considered homogeneous.

From Fig. 1, it can be seen that the vectors  $\vec{B}_1$  and  $\vec{B}_2$  are obtained by rotating the vector  $\vec{B}$  around an axis parallel to y. In general, the vectors  $\vec{B}$ ,  $\vec{B}_1$ ,  $\vec{B}_2$ , can be arbitrarily distributed across the nodes *A*, *B*, *C*. However, it is this configuration that will be discussed below.

The heterostructure described above corresponds to the potential relief shown in Fig 2:



Fig.2 Potential relief of a heterostructure with three magnetic nanolayers. We assume the barriers are identical, that is, their geometry is the same, and the characteristic "dimensions" match up.

#### **3. TRANSFER MATRIX**

Electron interaction with the barrier occurs along the y-axis, that is the interaction potential depends only on the variable y. Therefore, the interaction Hamiltonian commutes with the operators of momentum components  $\hat{k}_x$  and  $\hat{k}_z$ . This leads to the separation of variables in the Schrödinger equation. Thus, the scattering of the electron wave on a single barrier or a system of multiple barriers, has a one-dimensional nature.

This fact makes it possible to apply the well-known transfer-matrix method to construct the amplitudes of transmission and reflection. For the scalar scattering problem, this method is described in [8]. In the case of spin-dependent scattering, it is summarized in the papers [6,7].

In the case of spin-dependent scattering on a threebarrier non-collinear system, the transfer- matrix is as follows:

$$S = S_0 \left( \hat{U}_1 S_0 \hat{U}_1^+ \right) \left( \hat{U}_2 S_0 \hat{U}_2^+ \right) A^{-3}$$

where  $S_0$  describes the scattering on the first barrier, and

 $(\hat{U}_{\ell}S_{0}\hat{U}_{\ell}^{+})(\ell=1,2)$  – on the second and third barriers respectively.

The transfer matrix  $S_0$  is as follows:

$$S_0 = \begin{pmatrix} \alpha, \beta^* \\ \beta, \alpha^* \end{pmatrix}, \alpha = \begin{pmatrix} 1/t_1 & 0 \\ 0 & 1/t_2 \end{pmatrix}, \beta = \begin{pmatrix} r_1/t_1 & 0 \\ 0 & r_2/t_2 \end{pmatrix},$$

 $r_{\ell}, t_{\ell}$  – are partial amplitudes of reflection and transmission, i.e. amplitudes on the barrier with a "Zeeman-split-ceiling":

$$r_{\ell} = r(V_0 \mp \gamma B, E), t_{\ell} = t(V_0 \mp \gamma B, E),$$

E – the transverse part of scattering energy; each of the  $\hat{U}_{\ell}$  matrix is a block-diagonal matrix, the blocks of which diagonalize the operator of electron interaction with the second and third barriers:

Below we present in brief the results we obtained in the study of the transport characteristics of the three-barrier structure described above.

$$\hat{U}_{\ell} = \begin{pmatrix} U_{\ell} & 0 \\ 0 & U_{\ell} \end{pmatrix}, \ U_{\ell} = \begin{pmatrix} \cos\frac{\theta_{\ell}}{2}, -\sin\frac{\theta_{\ell}}{2} \\ \sin\frac{\theta_{\ell}}{2}, \ \cos\frac{\theta_{\ell}}{2} \end{pmatrix}, \ (\ell = 1, 2)$$

A is a block-diagonal matrix of translation of the barrier on a distance a.

## 4. DISCUSSION

For numerical estimates and plotting of the transmission coefficient and the degree of spin polarization of the transmitted electron wave, we need to select a particular type of interaction Hamiltonian. We represent it as follows:

$$H_{\rm int} = (\omega - \gamma B \sigma_z) \delta(y) + (\omega - \gamma \vec{B}_1 \vec{\sigma}) \delta(y - a) + (\omega - \gamma \vec{B}_2 \vec{\sigma}) \delta(y - 2a),$$

where as the finite interaction function, the Dirac delta function is chosen, which is different from zero at the nodes with coordinates  $y = 0, a, 2a, \ \omega = \Omega/E_0a$ ,  $\Omega$  – is the amplitude of  $\delta$  –potentials,  $\gamma = \mu/2E_0$ ,  $\mu$  – is the electron magnetic momentum,  $E_0 = \hbar^2/2ma^2$  – the energy value, which matches in order of magnitude with the energy of "zero oscillations" of an electron in a region of width a. It is introduced in order to make  $H_{\rm int}$  dimensionless. The amplitudes of the  $\delta$  – potentials include vector quantity  $\vec{\sigma}$  –representing three Pauli matrices.

The above mentioned physical quantities (transport characteristics) are functions of three variables: the angular degrees of noncollinearity  $\theta_1$  and  $\theta_2$ , and the electron wave



Fig.3 The dependence of the degree of polarization of the transmitted wave on the angular degrees of freedom  $\theta_1$  and  $\theta_2$ .

momentum. When building 3D graphs, one of these variables is fixed.

Both physical quantities are periodic functions of  $\theta_1$  and  $\theta_2$ . . The maximum values of these quantities are  $P_{\text{max}} \approx 0.6$ ,  $D_{\text{max}} = 0.5$ . These values are achieved at k = 1. The oscillation amplitude of the transmission



Fig. 4 The dependence of the transmission coefficient on the degrees of freedom  $\theta_1$  and  $\theta_2$ .

coefficient is small; it does not exceed  $10^{-2}$ .



Fig. 5 Dependence of the degree of polarization of the transmitted wave on the variables k and  $\theta_1$ , and at a fixed value of  $\theta_2 = \pi/2$ .

 $P(k, \theta_1)$  is a non-monotonic function. With an increase in the momentum k – the amplitude of oscillations in the variable  $\theta_1$  – decreases. The dependence of P(k) depends essentially on the value of a fixed variable  $\theta_1$ .



coefficient on the variables k and  $\theta_1$  at a fixed value  $\theta_2 = \pi/2$ .

It is clear from the graph that the oscillations in the angular variable  $\theta_1$  are weak, and with the growth of k they practically disappear. Non-monotonic behavior depending on the momentum k - is due to the following reasons: the emergence of the maximum is due to the resonance of the transmission of the corresponding collinear system (partial resonance of transmission). Since noncollinearity leads to the two-channel scattering — without and with spin-flip, the transmission coefficient does not reach a maximum value

equal to unity. In this case  $P_{\max} \approx 0.8$ . The same fact leads to an increase in the resonance width compared with the case of a collinear system. By virtue of the choice of the interaction potential in the form of  $\delta$ -function, there is no separation by scattering energies into sub-barrier and overbarrier regions. That is why the oscillations in momentum variable k are absent.

The manipulation of degrees  $\theta_1$  and  $\theta_2$  means the rotation of the vectors of the internal fields around the *y* - axis, and besides two options of rotation are possible: clockwise and counterclockwise.



From Fig. 7, it can be seen that the difference in the degrees of polarization of the transmitted wave  $\Delta P$  in certain ranges of variables k and  $\theta_1$  significantly differs from zero.



From Fig.8 it can be seen that the difference  $\Delta P_z$  is practically zero. This means that when the rotation of the vector is conversed  $\vec{B}_1(\theta_1 \rightarrow -\theta_1)$ , in the transmitted wave, in certain regions of the variables k and  $\theta_1$  there arise non-zero polarization components  $P_x$  and  $P_y$ .

## 5. CONCLUSION

The scattering of a non- spin- polarized electron wave on a three-barrier magnetic noncollinear system is considered. The behavior of the degree of spin polarization of the transmitted wave and the transmission coefficient as functions of the electron momentum and noncollinear degrees of freedom  $\theta_1$  and  $\theta_2$  are investigated. The asymmetry in these characteristics is shown when one of the vectors  $\vec{B}_1$ ,  $\vec{B}_2$ , rotates clockwise and counterclockwise. This asymmetry is clearly manifested in the behavior of the degree of polarization of the transmitted wave.

## 6. ACKNOWLEDGEMENT

The work at California State University was supported by the National Science Foundation-Partnerships for Research and Education in Materials under Grant DMR-1523588.

### REFERENCES

- [1] A. Fert Nobel Lecture: Origin, development, and future of spintronics. Rev. Mod. Phys., 80, 1517 (2008).
- [2] T. Dietl A ten year perspective on dilute magnetic semiconductors and oxides. Nature Mat. 9 965-974 (2010).
- [3] I. Zutic, J. Fabian and S. Das Sarma 2004. Spintronics: Fundamentals and applications. Rev. Mod. Phys. 76 323-410 (2004).
- [4] A.Brataas, G.E. Bauer and P.J. Kelly Noncollinear magnetoelectronics. Phys. Rep. 427, 157 (2006).
- [5] H.L.He, W.Zhang, Z.P.Wang, B.Dai, Y.Ren. Effects of magnetic barriers on transport and magnetoresistance in a 2D-electronic device. AIP ADVANCES,6,055122(2016).
- [6] A.N. Kocharian, A.S. Sahakyan and R.M. Movsesyan, Spin-polarizing properties of heterostructures with magnetic nano elements. JMMM 322 19 (2010).
- [7] A. Sahakyan, A. Poghosyan, R. Movsesyan, A.Kocharian, Transport properties through multibarrier magnetic system containing a "noncoplanar defect" IEEE TRANSACTIONS ON MAGNETICS, 1-5, (2019).

[8] P. Erdos, R.C. Herndon. Theories of electrons in one dimensional disordered system. Advances in Physics, 31,2 (1982).