A Novel Co-Resonantly Coupled Cantilever Sensor Platform

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ABSTRACT

Co-resonance is a novel approach to strongly enhance the sensitivity of dynamic mode cantilever sensors while maintaining the ease of detection with state-of-the-art laser-based methods. The concept makes use of the coupling and eigenfrequency matching of a micro- and a nanocantilever. While the highly sensitive nanocantilever is in interaction with the environment, the detection of the co-resonantly coupled system’s oscillatory state takes place at the microcantilever.

Here we give an overview over the sensor concept, its consequences on sensor parameters and first applications of the co-resonant approach in the characterization of novel (nano)materials.

Keywords: cantilever sensor, co-resonance, sensitivity enhancement, nanocantilever, materials research

1 INTRODUCTION

Dynamic mode cantilever sensors are used for many different applications, for example in materials research to study magnetic properties of small particles and thin films [1, 2, 3], in biology for real-time observation of cell processes and as balances [4, 5, 6] or as gas sensors to detect trace analytes in gases (“artificial nose”) [7].

In contrast to static mode cantilever sensors where the static bending is used as a measurement signal, amplitude, oscillation frequency and phase shift are observed in the dynamic mode. For this case, the cantilever’s sensitivity is mainly determined by the beam’s stiffness, i.e. its spring constant, and effective mass. Both quantities need to be decreased in order to increase sensitivity. This can be achieved by reducing the cantilever’s dimensions, especially its thickness and width, leading to the use of nanocantilevers (at least in two out of three dimensions). However, this approach is limited as the commonly employed laser-based detection methods require a certain thickness and width of the cantilever for reliable oscillation detection.

Hence, methods need to be devised which allow the use of highly sensitive nanocantilevers but at the same time maintain the ease of oscillation detection.

Figure 1: Principle of the co-resonant sensor concept with optical read out at the microcantilever. The external interaction can have various forms, such as a mass/pressure change on the nanocantilever, magnetic field or force gradient. Furthermore, a sample can be placed onto the nanocantilever and in combination with a known interaction, the sample properties can be derived (e.g. cantilever magnetometry).

2 CO-RESONANT SENSOR CONCEPT

Our recently developed co-resonant sensor concept addresses this challenge by mechanical coupling of a micro- and a nanocantilever and the matching of their eigenfrequencies [8]. This introduces a strong interplay between the two cantilevers. Consequently, any interaction applied at the highly sensitive nanocantilever influences the oscillatory state of the coupled system as a whole and the change of the oscillation can be detected with standard laser-based methods at the microcantilever (see figure 1). Please note that, depending on the application, the nanocantilever is either subject to an unknown external interaction (e.g. mass change, force gradient) which is to be quantified (e.g. in scanning probe methods) or a sample is placed on the nanocantilever and a known external interaction applied, allowing for a characterization of the sample (e.g. in cantilever magnetometry).
Another consequence of the co-resonance is that the parameters of both resonance peaks of the coupled system lie in between the properties of the individual beams. Hence, a large fraction of the high sensitivity of the nanocantilever is accessible with this concept without requiring advanced methods to determine the nanocantilever’s oscillatory state. Consequently, the nanocantilever has the potential for further miniaturization without any restrictions applied by a possible detection method.

2.1 Modeling

A cantilever can be modeled as a coupled harmonic oscillator consisting of lumped elements (spring \(k\), damping element \(d\) and effective mass \(m\)) for each of its eigenfrequencies [9]. Consequently, the coupled system can be represented by a coupled harmonic oscillator model as depicted in figure 2a. The numerical values for micro- (index 1) and nanocantilever (index 2) have to be chosen depending on the eigenfrequencies of the beams that are matched [8].

According to the model, the coupled system’s behavior is described by a set of differential equations. An approach which facilitates evaluation is to transform the mechanical model into an electric circuit model (see figure 2b). This gives the opportunity of studying the system with circuit analysis tools. Details thereof can be found in [10].

2.2 Coupled system’s parameters

In the following, we will consider an exemplary system with numerical values summarized in table 1. Furthermore, it will be necessary to distinguish between the parameters of the individual beams and those of the two resonance peaks of the coupled system which is indicated by the indexes 1,2 (individual systems) and a,b (coupled system), respectively. In addition, we will focus our discussion on the amplitude response curve of the microcantilever as that is the one measured experimentally. Please note that similar results are obtained by evaluating the nanocantilever [10].

Figure 3 shows the coupled system’s amplitude response curve taken at the microcantilever for the unmatched and closely matched (2% eigenfrequency deviation) state. Please note that the amplitudes have been normalized to the highest amplitude of the unmatched state.

It is evident that for the unmatched state the amplitude response curve only exhibits one clear resonance peak at a frequency \(f_a\) very close to the eigenfrequency of the microcantilever \(f_1\). Although one would expect two resonance peaks due to the coupling, the second peak’s amplitude is so small that it cannot be seen in the coupled system’s amplitude response curve measured at the microcantilever. This second peak \(f_b\) would be very close to the eigenfrequency of the nanocantilever \(f_2\) but due to invisibility of the peak, the relevant frequency range is not shown in figure 3.

For a 2% frequency deviation of micro- and nanocantilever, i.e. \(f_2 = 1.02 \cdot f_1 = 204\) kHz, two clear resonance peaks occur at the resonance frequencies \(f_a = 198.3\) kHz and \(f_b = 205.8\) kHz of the coupled system. This is a deviation from the individual beam’s eigenfrequencies by -2.8% for \(f_a\) with respect to \(f_1\) and +1.9% for \(f_b\) with respect to \(f_2\). Furthermore, the oscillation amplitude is changed depending on the degree of frequency matching.

The most important effect in terms of a sensor ap-
Table 1: Numerical values for micro-(1) and nanocantilever(2) derived from the experimental implementation of co-
resonantly coupled sensors. Please note that the values are given as the initial values before frequency matching for the
individual subsystems (columns 2 and 3). The two right-hand side columns summarize the values for both resonance
peaks of the coupled system in case of 2% eigenfrequency deviation $\Delta f_e$ between micro- and nanocantilever ($f_1 = 200$
kHz, $f_2 = 204$ kHz).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Micro (1)</th>
<th>Nano (2)</th>
<th>Left peak (a)</th>
<th>Right peak (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency $f$</td>
<td>200.0 kHz</td>
<td>400.0 kHz</td>
<td>198.3 kHz</td>
<td>205.8 kHz</td>
</tr>
<tr>
<td>Spring constant $k$</td>
<td>1 N/m</td>
<td>0.001 N/m</td>
<td>0.0044 N/m</td>
<td>0.0013 N/m</td>
</tr>
<tr>
<td>Quality factor $Q$</td>
<td>10000</td>
<td>800</td>
<td>2670</td>
<td>1008</td>
</tr>
</tbody>
</table>

Table 2: Frequency shift values for the individual sub-
systems and for both resonance peaks of the coupled sys-
tem for the unmatched and matched (2% eigenfrequency
deviation) state. The calculation was done based on the
model in figure 2 and for an interaction $k_3 = 5 \cdot 10^{-6}$
N/m.

<table>
<thead>
<tr>
<th>Frequency shift $\Delta f$</th>
<th>$\frac{\Delta f}{f_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual subsystems</td>
<td></td>
</tr>
<tr>
<td>Microcantilever (1)</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Nanocantilever (2)</td>
<td>999.5 Hz</td>
</tr>
<tr>
<td>Unmatched</td>
<td></td>
</tr>
<tr>
<td>Left peak (a)</td>
<td>0.6 Hz</td>
</tr>
<tr>
<td>Right peak (b)</td>
<td>not visible</td>
</tr>
<tr>
<td>Matched (2%)</td>
<td></td>
</tr>
<tr>
<td>Left peak (a)</td>
<td>120.2 Hz</td>
</tr>
<tr>
<td>Right peak (b)</td>
<td>390.0 Hz</td>
</tr>
</tbody>
</table>

2.3 Sensitivity enhancement

The enhanced sensitivity due to co-resonant coupling
is illustrated when considering an external interaction
(modeled as a spring) $k_3 = 5 \cdot 10^{-6}$ N/m on the system.
The model given in figure 2 was used to calculate the
frequency shift $\Delta f$ induced by the interaction according to:

$$\frac{\Delta f}{f_0} = \frac{k_3}{2k}.$$  (1)

Equation 1 was employed for each individual subsystem, hence $f_0 = f_{1,2}$ and $k = k_{1,2}$ for micro- and na-
ocantilever, respectively, and for each resonance peak
of the coupled system, i.e. $f_0 = f_{a,b}$ and $k = k_{a,b}^{ef}$ for
the unmatched and the matched state (2% deviation).
The resulting frequency shifts and the percentage values
in relation to the original frequency are summarized in
table 2. These values illustrate how the co-resonance
gives access to part of the nanocantilever’s high sensi-
tivity while still allowing to use the detection on the
microcantilever.

3 APPLICATIONS

The immense potential of this concept was demon-
strated in first proof-of-principle experiments in can-
tilever magnetometry and magnetic force microscopy for
the study of magnetic properties of samples [3, 11].
In case of cantilever magnetometry, the sample was a
carbon nanotube filled with few individual Co$_2$FeGa Heu-
sler nanoparticles (diameter $\approx$35 nm) and we were, to
our knowledge for the first time, able to directly observe
magnetic switching of these individual nanoparticles at
room temperature and with simple laser-deflection de-
tection [3]. These experiments strongly indicate the po-
tential for highly sensitive investigations of novel nano-
materials without the need for low temperatures or ad-
vanced setups.
In magnetic force microscopy, a likewise increase in sen-
sitivity was reported for the study of a magnetic multi-
layer sample [12, 13].
4 CONCLUSION

The sensor concept is not limited to materials research but may instead be employed as a platform concept which can be tailored for other applications, for example in gas sensors, magnetic field sensors or mass detection. First numerical considerations indicate a likewise sensitivity increase as in the study of novel nanomaterials.

ACKNOWLEDGEMENT

The author acknowledges Christopher F. Reiche and Thomas Mühl for helpful discussions. Furthermore, funding by Deutsche Forschungsgesellschaft (DFG grant no. KO5508/1-1) is gratefully acknowledged.

REFERENCES


