Numerical modeling of the formation of dynamically configurable L^2 lens in a microchannel

Adwaith Gupta*, Justin Kitting** and Ioannis H. Karampelas*

*Flow Science Inc.,
683 Harkle Rd Suite A, Santa Fe, NM, adwaith@flow3d.com
**Department of Mechanical Engineering, The University of New Mexico, Albuquerque, NM

ABSTRACT

This paper presents the numerical modeling of a dynamically reconfigurable liquid-core liquid-cladding (L^2) lens in a microfluidic channel [1], followed by a quantitative validation against experimental results. L^2 lenses are dynamically configurable, which means that their focal lengths can be changed in real time by adjusting the flow rate of the core and the cladding streams. The lens is formed in a microchannel by three laminar co-flowing streams of fluids with different refractive indices. The core stream, which is sandwiched between the cladding streams, has a higher refractive index, causing the light to bend while passing through the layers of microfluidic streams. Based on the relative flow magnitudes of the core flow rates and the cladding flow rates, different lens shapes (defined by the curvatures) are formed. Each curvature leads to a different focal length, thus governing the path of light rays passing through the microchannel.

Keywords: optofluidics, L^2 lens, dynamically configurable lens, refractive index, focal length

1 L^2 LENS

As stated in [1] , "focusing of light in a microfluidic device is important for lab-on-a-chip applications that require optical characterization." Proper illumination of the area of interest can be accomplished through insertion of optical fibers, however, the light transmitted from an optical fiber is divergent and therefore requires focusing [1]. There is also a need to focus light in the plane of a microchannel while avoiding cumbersome optical instruments that focus light in the perpendicular direction. There are some advantages to a lens design based on the laminar flow of multiple streams of fluids that focuses light in the plane of the microchannels. Compared to alternate illumination methods, the proposed L^2 lens [1] has a few additional advantages.:

1. Dynamically configurable: The rates of fluid flows change the curvature and focal length in real time
2. Laminar flow: The interface between the core and the cladding streams is smooth despite the roughness of the wall channel
3. Co-fabrication: Integration and pre-alignment of the lens is simple [1]

The schematic representation of the experimental setup for focusing light exiting an optical fiber through the L^2 lens is shown in Figure 1.

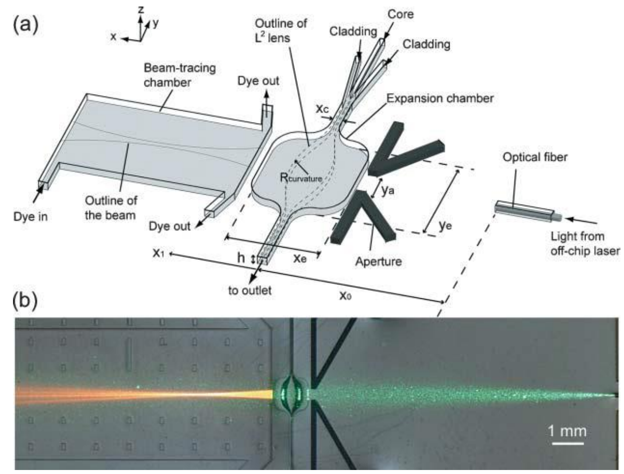

Figure 1: (a) The solid lines show the walls of the channel, and the dashed lines show the interfaces between the core and the cladding streams. $R_{curvature}$ is the radius of curvature of this interface. (b) Bright field image of the L^2 lens. Refer to [1] for further details about the experimental setup. (Image courtesy: Tang, Stan & Whitesides.)

2 NUMERICAL MODELING

Accurate numerical modeling allows researchers to test and analyze designs while minimizing the need for time- and resource-consuming physical experimentation. The objective of this work is to first numerically model the formation of the L^2 lens using the commercial flow solver ***FLOW-3D*** (www.flow3d.com) and then validate the numerical model against the experimental results presented in [1].

The accuracy of the modeled results is calculated by comparing the numerically estimated curvatures of the various lenses with the known experimental curvatures. Accurate curvature calculation of the modeled lenses is central to establishing the accuracy of ***FLOW-3D*** as a modeling tool for L^2 lens formation. Therefore, a rigorous curvature estimation technique, described in section 3, was used.

Total flow rate going through the microchannel is fixed at 10 mL/hr to keep the Reynold's number fixed for all the experiments [1]. The individual cladding flow rates and core flow rates, however, can be varied within the constraint,

$$Q_{core} + Q_{left} + Q_{right} = 10 \text{ ml/hr} \quad (1)$$

where Q_{left} and Q_{right} are the cladding flow rates and Q_{core} is the core flow rate as shown in the schematic below.

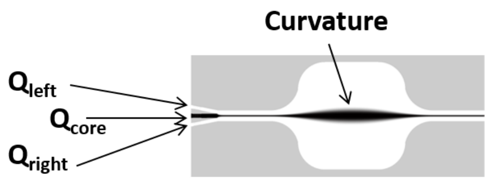

Figure 2: Schematic showing the cladding flows (left and right) and central core flow forming a lens inside the microchannel

Two case studies arise working within the constraint (1): Case Study A and Case Study B, which are explained in the following sections.

2.1 Case study A: Variable core flow rate

In this case, Q_{core} can vary anywhere in the range (0,10) ml/hr. Q_{left} is always set equal to Q_{right}, leading to the formation of only bi-convex lenses (Figure 3). Due to symmetry, only single curvature calculation is required.

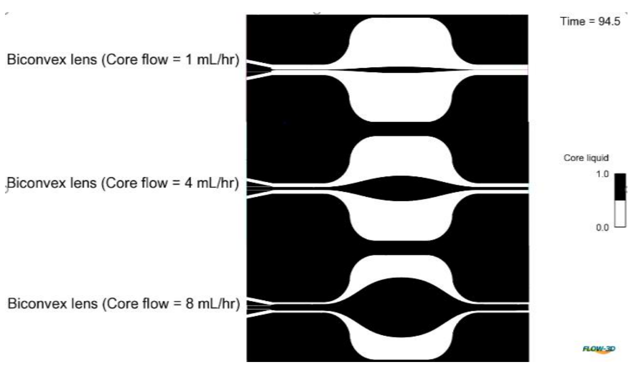

Figure 3: Bi-convex lens formed with different curvatures corresponding to different core flow rates

2.2 Case Study B: Fixed core flow rate

In this case, Q_{core} is fixed at 3 ml/hr which means Q_{left} and Q_{right} can vary in the range (0.0, 7.0) ml/hr [1]. Due to the asymmetric nature of flow rates, lens types (Figure 4) are not restricted to bi-convexity anymore. For asymmetrical lens shapes, double curvature calculation is required.

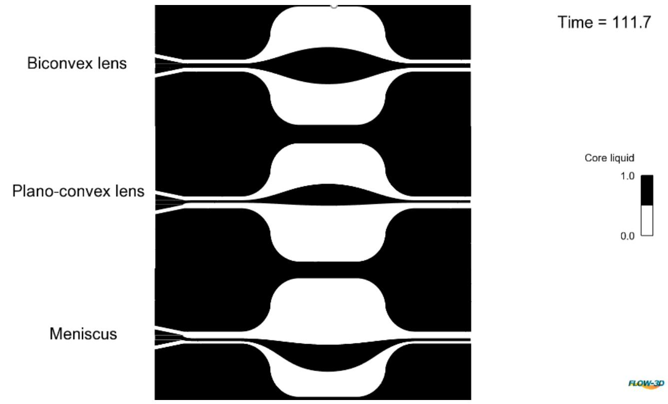

Figure 4: Varying lens formations corresponding to different combinations of Q_{left} and Q_{right} cladding flow rates (Q_{core} fixed to 3 ml/hr in all cases, i.e. bi-convex, plano-convex and meniscus).

3 CURVATURE CALCULATION

The curvature of each lens is calculated using the curvature equation for a parabola (3) derived from the general curvature equation (2),

$$\kappa_{curve} = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} \quad (2)$$

$$\kappa_{parabola} = \frac{2a}{[1 + \{2a(x-h)\}^2]^{3/2}} \quad (3)$$

where a can be calculated from the standard equation of a parabola (4)

$$y = a(x-h)^2 + k \quad (4)$$

Replacing (x, y) in (4) with a point on the parabola (x_1, y_1), fixing h to zero and measuring k using FlowSight (an advanced post-processor by Flow Science, Inc.) allows the calculation of a in (4). a can then be substituted in (3) to calculate the curvature of the lens.

4 VALIDATIONS AND DISCUSSION

Calculated curvature values for cases A and B are compared with the experimental curvature values reported in [1]. Figures 5 and 6 below show the comparison of curvature values between ***FLOW-3D*** results (top) and experimental results (bottom).

Upon visual comparison, computational results and the experimental results are in excellent agreement in case study A (Figure 5). The interface curvature increases up to a core flow rate of 8 ml/hr and then starts declining as the core fluid starts taking the shape of the walls (flat) at high

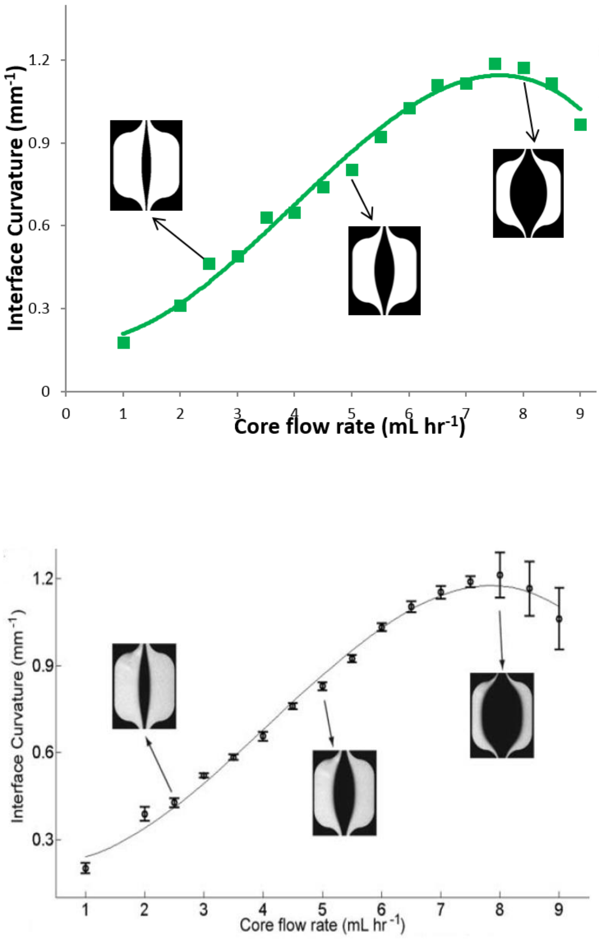

Figure 5: Comparison of ***FLOW-3D*** results (top) with the experimental results (bottom)[1] for case study A (variable core flow rate)

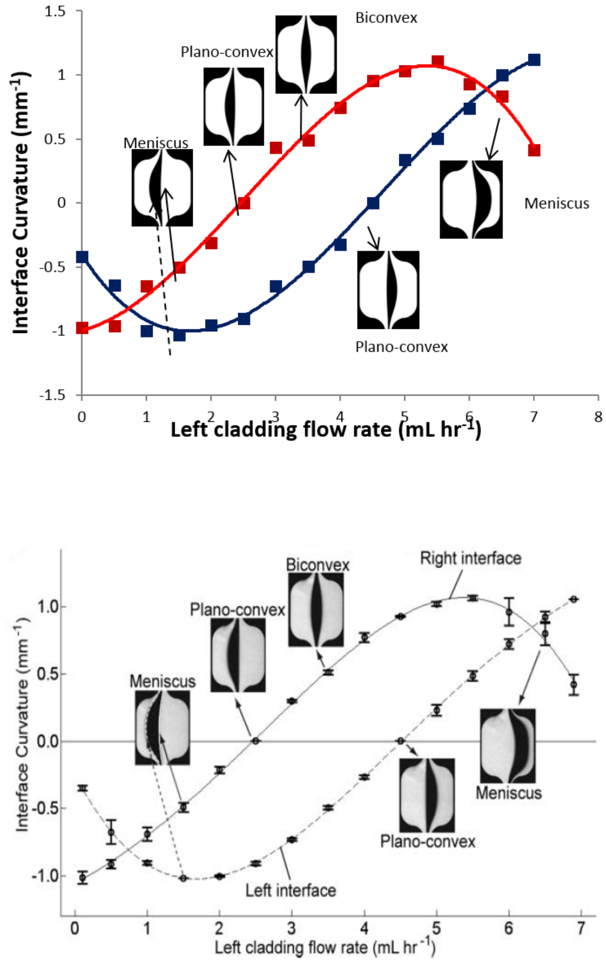

Figure 6: Comparison of ***FLOW-3D*** results (top) with the experimental results (bottom) [1] for case study B (fixed core flow rate)

core flow rates, forcing a reduction in curvature. The snapshots of the lenses at 3, 5 and 8 ml/hr flow rates also match very well with the ***FLOW-3D*** simulation snapshots at the same flow rates.

Case study B has two curvature values per lens. As shown in Figure 6, both left and right interfaces' curvature values produced by ***FLOW-3D*** are in excellent agreement with the experimental results.

In addition to the visual comparison, regression analysis (Figure 7 & 8) is performed and the R-square value calculated for both cases to quantify the accuracy of the predicted results. An R-square value of 0.987 is found for case study A and an R-square value of 0.999 is found for both the curvatures of case study B.

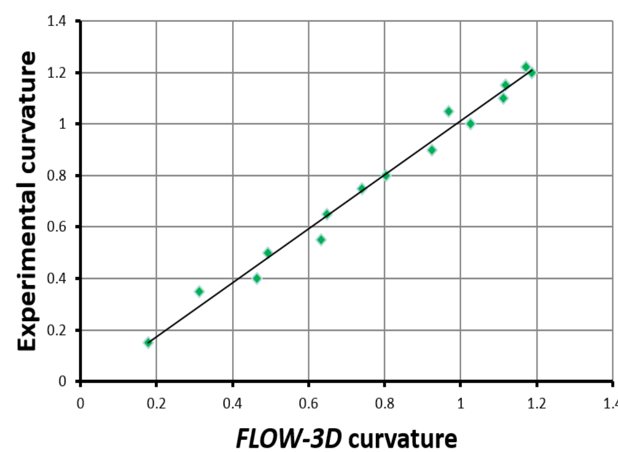

Figure 7: Regression analysis for case study A

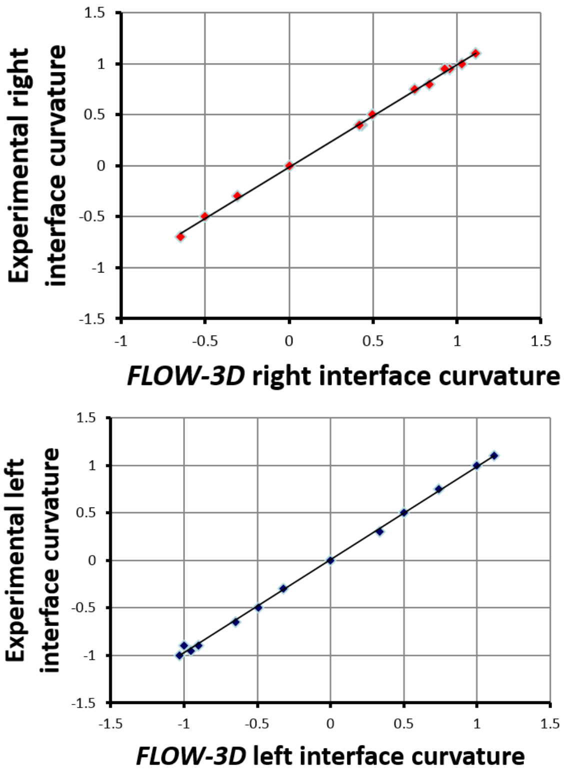

Figure 8: Regression analysis for right and left interface curvatures of case study B

These are excellent values, indicating a very high accuracy of the commercial software ***FLOW-3D*** model for the L^2 lens. These results suggest a benefit to researchers in related fields since compuational modeling can accurately guide the rational design of the experimental setup and can be readily extended to various similar processes.

REFERENCES

[1] Sindy K. Y. Tang, Claudiu A. Stan, George M. Whitesides, "Dynamically reconfigurable liquid-core liquid-cladding lens in a microfluidic channel" ***Lab Chip***, 2008,**8**, 395-401, DOI: 10.1039/b717037h, 2008.