

# A Mechanical Metamaterial with Extreme Stiffness and Strength

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## ABSTRACT

A material geometry has been identified that achieves the theoretical upper bound for stiffness and strain energy storage. Extreme lightweighting and systems with unprecedented performance can be achieved with accessible fabrication methods and materials. The symmetries and alignments of material in this design form a relatively simple geometry that is amenable to fabrication using a variety of techniques. Unique and highly anisotropic subgeometries facilitate functional gradation, simple parameterized optimization, and even smart structures.

**Keywords:** metamaterials, cellular materials, mechanical metamaterials, lightweighting

## 1 INTRODUCTION

High-density materials that possess high specific stiffness and strength—such as boron carbide, beryllium, and carbon fiber composites—are essential in the design of high-performance engineered systems. These advanced materials are key enabling technologies in modern space, air and naval vehicles, sports equipment, and fuel-efficient transportation, to name but a few examples. They provide structural shape stability and high strength with minimum mass penalty. As solid materials, however, they occupy only a portion of the available specific-stiffness and specific-strength materials design space. There is a large region of high specific stiffness and strength materials that can only be occupied by mechanical metamaterials<sup>1</sup>. Mechanical metamaterials can achieve extraordinary properties via combinations of constituent and geometric mechanical properties. Some examples of these materials include composite honeycomb sandwich panels and ceramic microlattices<sup>2,3</sup>. At its core, this approach utilizes design at an intermediate length scale—between the small scale of the constituent’s microstructure, and the large scale of parts and loads—as is often found in natural systems such as bone and wood. The population of this property space is fundamentally a design problem, with previously known solutions falling far short of theoretical limits.

Recent developments of key technologies in the early 21<sup>st</sup> century have facilitated a revolution in materials development, and the growing commercial relevance of mechanical metamaterials. Modern automated fabrication techniques—including the wide variety of 3D printing processes available and in development—allow for historically unprecedented part complexity, with little to no associated cost. These designs can be fabricated using industry relevant materials, such as aluminum and titanium alloys, ceramics, and even polymer matrix carbon fiber

composites. This is cotemporaneous with the decline in the cost of computing resources to the point where the problem size, which can be large with problems of this type, is now tractable on a lab scale, allowing researchers to explore this design space<sup>1,4,5</sup>.

In an effort to expand the property space of high specific stiffness materials, we have identified the first and only known material geometry to achieve the theoretical upper bounds for isotropic stiffness and strain energy storage<sup>1</sup>. This geometry stores strain energy more uniformly, independent of loading direction, than any other known geometry. The symmetries and alignments of material, and its unique subgeometries (**Fig. 1**), likely make it, and its properties, unique in three-dimensional space. The relatively simple geometry is amenable to origami-like sheet folding<sup>6</sup>, providing access to a wide range of constituent materials. In this review, we will explore the design space and capabilities of this unique material architecture.

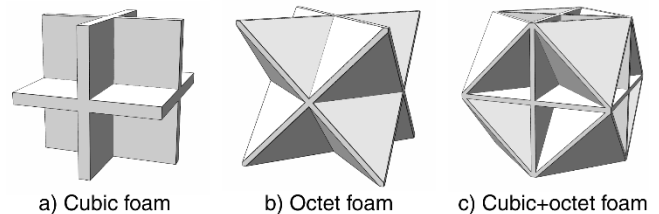


Figure 1: Unit cells of the cubic foam (a), octet foam (b), and the combined cubic+octet foam (c). The unique subgeometries (a & b) possess unique symmetries and associated mechanical properties. [Adapted from Berger, 2017]

## 2 METHODS

### 2.1 Theoretical upper bounds

The upper limit on the modulus of mechanical metamaterials is governed by a suite of theoretical bounds that limit the strain energy density<sup>7</sup>; for a detailed derivation of the bounds see Hashin (1963) [Ref. 7]. These bounds govern, in part, the Young’s, shear, and bulk moduli of isotropic cellular materials. They can be simplified to a single value

$$\Omega = \frac{\bar{E} + 2\bar{G}(1-\bar{\nu})}{E_{HSU} + 2G_{HSU}(1-\nu_{HSU})} \quad (1)$$

which uses the Zener anisotropy to quantify the overall performance<sup>1</sup>, where  $E$ ,  $G$ , and  $\nu$  are the Young’s and shear moduli, and the Poisson ratio respectively, the bar notation is used to denote the property of the metamaterial, and the subscript  $HSU$  to denote the respective Hashin-Shtrikman upper bound.

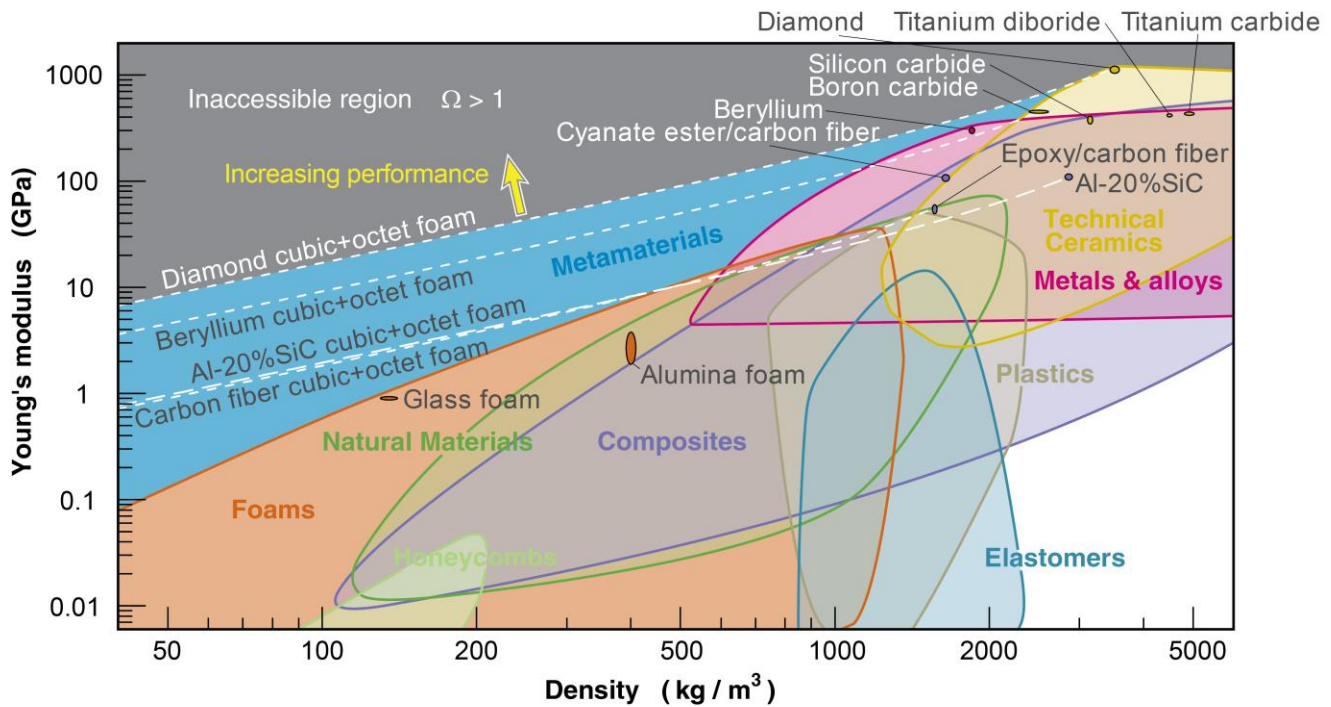


Figure 2: Property space of isotropic and nearly isotropic materials. Materials with the highest specific stiffness lie towards the upper left quadrant. Mechanical metamaterials achieve properties that exceed that of natural and more conventional materials through combinations of high-performance constituents and design (the properties of non-optimal material geometries are not shown). The single crystal diamond system bounds the property space. [Adapted from Berger, 2017]

## 2.2 Finite element homogenization

The effective moduli of material geometries are calculated using a finite element homogenization technique. Unit cells are subject to periodic boundary conditions consistent with uniform macroscopic strains—behaving as if part of a larger body undergoing deformation. To assess the efficiency of designs, distributions of strain energy are used to identify the geometric features responsible for high-

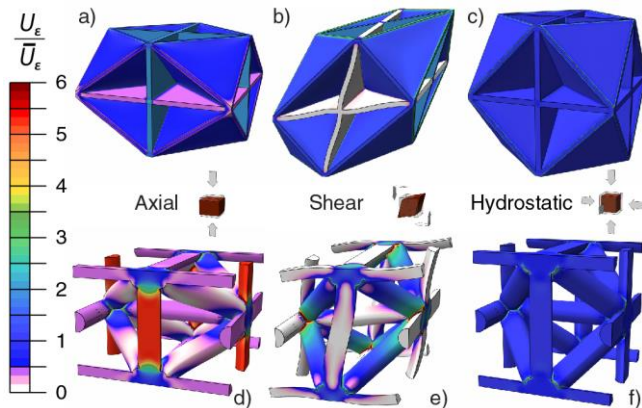


Figure 3: Normalized strain energy distributions in two material geometries that achieve theoretical upper bounds for isotropic stiffness. Closed cell material geometries fundamentally outperform open cell geometries, evident in the relatively homogeneous strain energy distributions (a-c). [Reprinted from Berger, 2017]

stiffness performance. Local strain energy densities are normalized by the average solid fraction strain energy density. Calculated moduli are normalized by the properties of a density of equivalent continuum, in which stresses and strains are uniform.

## 2.3 Fabrication and mechanical testing

Mechanical tests have been performed on 3D printed ABS plastic specimens<sup>6</sup>. Four single unit cell specimens were tested to failure using a MTS servohydraulic press and the resulting load displacement curves recorded.

## 3 RESULTS AND DISCUSSION

A combination of experimental, computer, and analytical models are all found to agree, and show that the cubic+octet material geometry is capable of achieving the theoretical upper bound for isotropic stiffness and strain energy storage, over a wide range of relative densities<sup>1</sup>.

### 3.1 Specific stiffness and property space

A majority of the upper region of high specific stiffness materials property space can only be populated by mechanical metamaterials (Fig. 2). When the cubic+octet geometry is composed limiting materials, it bounds the upper limits of property space. Available engineering materials can be used to produce structures with unparalleled performance.

When compared to an optimal truss geometry<sup>1,8</sup>, strain energy distributions in the closed cell geometry are far

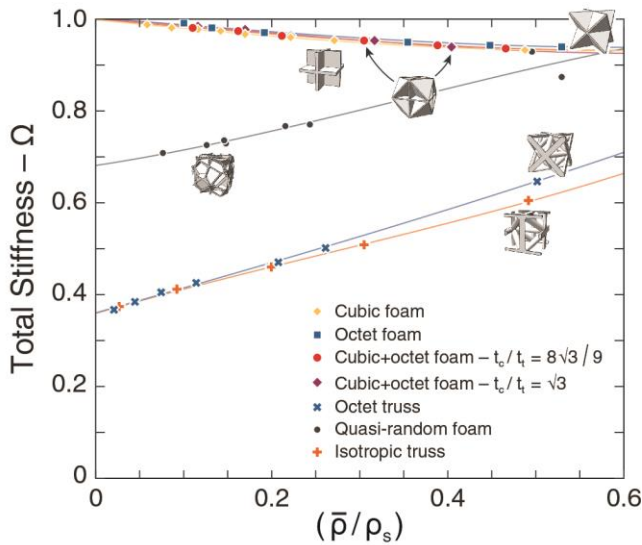


Figure 4: The total stiffness (Eqn. 1) of closed cell geometries can approach the theoretical limit at low densities, outperforming optimal lattice geometries by nearly a factor of three. The anisotropic subgeometries of the cubic+octet foam also have essentially maximal structural efficiency. [Reprinted from Berger, 2017]

more homogeneous (Fig. 3). Lattices have historically been investigated for their lightweight and high specific stiffness and strengths<sup>9</sup>, yet underperform closed cell geometries by a large margin in terms of total structural efficiency (Fig. 4).

### 3.2 Strength

Experiments performed on 3D printed ABS indicate very high strengths in the cubic+octet geometry (Fig. 5), where roughly 75% of the material has yielded at peak load in structures with only ~25% relative density<sup>6</sup>.

The geometry is insensitive to edge effects and imperfections. Finite element models of single unit cells perform only slightly worse, in terms of stiffness and strength, than periodic structures. Eigenmode imperfections, whose magnitude is on the order of the wall thickness, reduce performance only slightly.

### 3.3 Functional Gradation

The unique subgeometries of the cubic+octet foam allow for simple parameterized functional gradation and the creation of smart structures. Cubic and octet wall thicknesses can be varied independently to modify Young's and shear modulus locally (Fig. 6). Large optimized structures can be devised with only a small number of parameters.

### 3.3 Fabrication by sheet folding

The cubic+octet geometry is composed of continuous sheets of material that can be assembled by origami like sheet folding<sup>6</sup>. A pattern can be cut and folded from a sheet of material (Fig. 7) to form a half unit cell (Fig. 8) which can be bonded and assembled to form structures

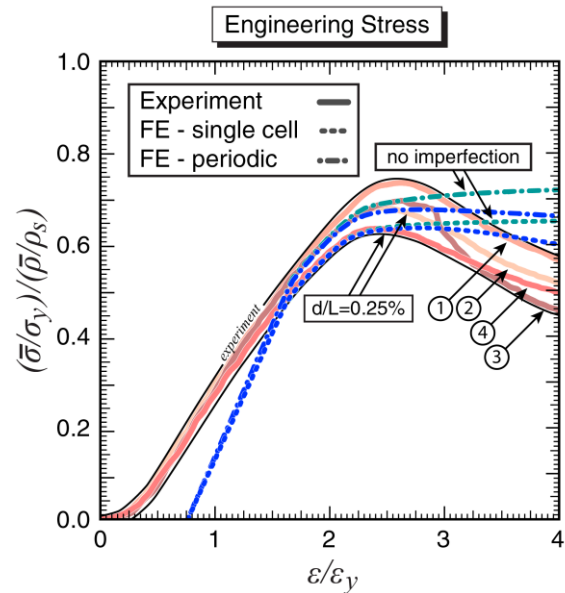


Figure 5: Experimental stress strain curves for 3D printed ABS plastic specimens and corresponding finite element predictions. Stress is normalized by the yield stress and relative density, strain is normalized by the yield strain of the constituent. Despite unpredicted delamination failures, the performance is well predicted, with strengths that achieve more than 75% of the loosest theoretical upper bound. Significant imperfections have little effect on peak strength. [Adapted from Berger, 2015]

(Fig. 9) of arbitrary size and periodicity. Using this method structures with extremely low relative density can be made.

## 4. CONCLUSIONS

A geometry has been identified that achieves the theoretical bounds for stiffness performance, over a wide

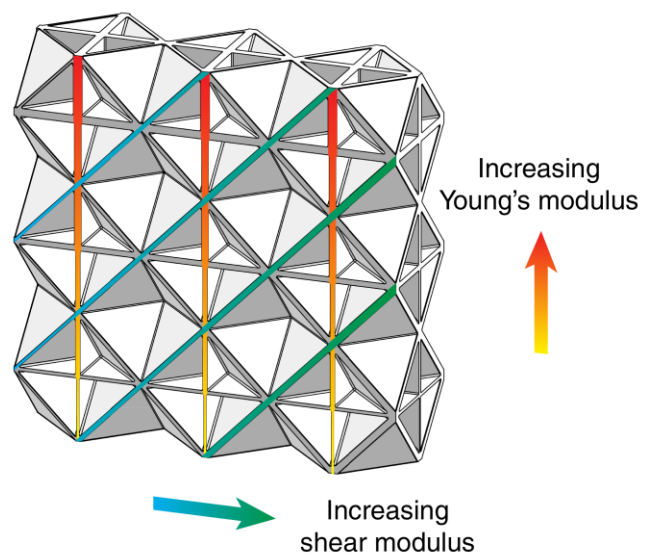


Figure 6: Example of a functionally graded structure. [Adapted from Berger, 2015].



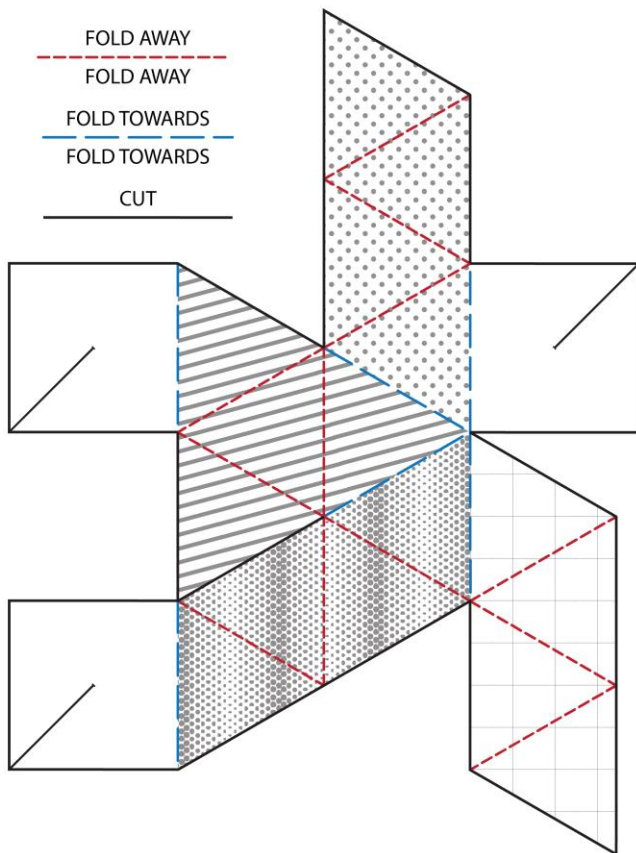


Figure 7: Two-dimensional pattern that can be cut and folded to form a half unit cell of the cubic+octet geometry, in an example of fabrication by sheet folding. Each pattern corresponds to a single regular tetrahedron in the folded half unit cell. [Adapted from Berger, 2015]

range of relative densities. This closed cell geometry outperforms optimal lattice designs by nearly a factor of three. The design lends itself to optimization, functional gradation, and can be manufactured with origami like sheet folding to produce very low-density structures. Experiments indicate very high strengths and defect insensitivity. With such extraordinary properties, flexibility, robustness, and with the continued development of automated assembly

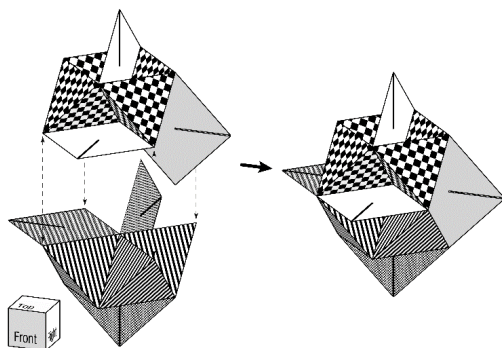


Figure 8: Two half unit cells can be bonded together and then interlocked with neighboring unit cells to form a structure of many unit cells. [Reprinted from Berger, 2015]

techniques, there is a vast potential to produce systems with unprecedented performance for a wide range of applications.

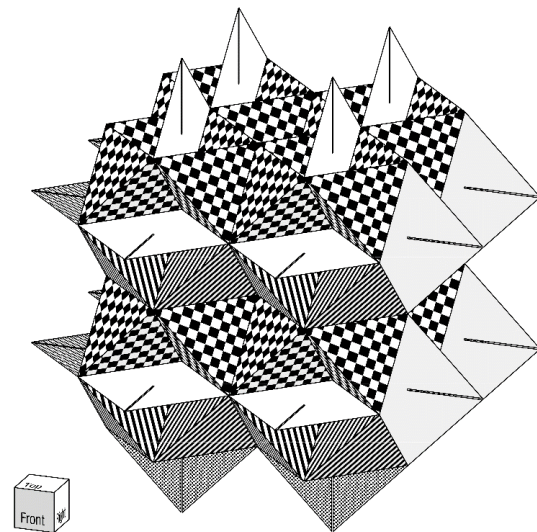


Figure 9: An assembly of interlocking unit cells, each composed of folded sheets of material. [Reprinted from Berger, 2015]

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