On the dynamic contact angle in spontaneous capillary flow

J. Berthier, D. Gosselin, G. Delapierre

University Grenoble Alpes, F-38000, Grenoble
Department of Biotechnology, CEA, LETI, Campus MINATEC, 38054, Grenoble, France
Corresponding author: jean.berthier@cea.fr

ABSTRACT

Capillary spontaneous flow is an interesting solution to move fluids either in microgravity conditions, i.e. in space, in microfluidic systems. In both cases, gravity is negligible and capillarity is the dominant force.

Spontaneous capillary flow onset as well as the dynamics of capillary flows have been recently documented in the literature for channels of different shapes, confined or open. However, the role of the dynamic contact angle is still unclear. This work aims at bringing a new light on the effect of the dynamic contact angle in the dynamics of capillary flows.

Keywords: spontaneous capillary flow (SCF), capillarics, capillary velocity, dynamic and static contact angles.

1 INTRODUCTION

Capillary actuation of fluids is an interesting solution when the gravitational forces are small: this is the case in space , or in microfluidic systems. In both cases, the ratio between gravity and capillary forces characterized by the Bond number is very small.

The onset of spontaneous capillary flow (SCF) and its dynamics have been widely investigated, first in the years 1920s [1-3], then more recently with the development of microfluidic systems for biotechnology [4-9].

Most of the time, the interpretation of the dynamics of the flow is performed using a constant contact angle. Although the study of the dynamic contact angle on the dynamics of wetting has been the subject of many investigations [10-16], the effect of the dynamic contact angle during a sponaneous capillary flow is still unclear.

In this work, we propose a correction to the capillary velocity that takes into account the dynamic contact angle, based on correlations reported in the literature. Comparison between static and dynamic velocities are presented.. It is concluded that the dynamic contact angle has an importance only on the few first millimeters of the channel. The length scale of the device is then of utmost importance. A non-dimensional number characterizing the effect of the dynamic contact angle is proposed

2 THEORETICAL APPROACH

Let us consider a closed channel of uniform cross section and arbitrary shape (figure 1).

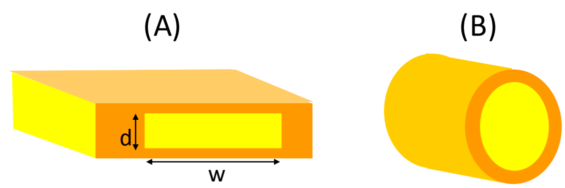

Fig.1. Sketch of a rectangular (A) and cylindrical channel (B).

It has been shown that the capillary force writes [6,7]

$$F_{cap} = \gamma \, p_W \cos\theta \, , \tag{1}$$

where γ is the surface tension between the liquid and air, and p_W and θ are respectively the wetted perimeter and the dynamic contact angles in a cross-section of the channel

From a dynamic standpoint, the velocity of the capillary flow can be determined using a balance between capillary forces and friction with the wall [1-3]. The friction force is

$$F_{drag} = \int_S \tau \, ds = \left(\int_\Gamma \tau \, dl \right) z(t) = \bar{\tau} \, p_W \, z(t) \tag{2}$$

where τ is the local wall friction, S the wetted surface between the origin and the front end of the liquid flow, Γ the wetted contour of the cross-section, $\bar{\tau}$ the averaged wall friction in a cross-section, i.e. $\bar{\tau} = (1/p_W)\int_\Gamma \tau\, dl$, and z the distance of the interface from inlet which depends on the time t.

The force balance on the fluid flow is then

$$m\frac{dV}{dt} = F_{cap} - F_{drag} \, , \tag{3}$$

where m is the mass of the fluid in the channel and V the average velocity. The mass of fluid being proportional to the penetration distance, (3) can be written under the form

$$\rho\, z(t) S_c \frac{dV}{dt} = \gamma\, p_W \cos\theta - \bar{\tau}\, p_W\, z(t) \, , \tag{4}$$

where S_c is the cross-section area and ρ the volumic mass of the fluid. The Reynolds number of the fluid being small, the flow is laminar. For a Newtonian fluid, the friction τ then depends on the geometry of the channel and on the average velocity V. Locally, the wall fiction is

$$\tau = \mu\frac{\partial v}{\partial n} = \frac{\mu V}{\lambda} \tag{5}$$

where λ is a local friction length and n the coordinates perpendicular to the wall (figure 2).

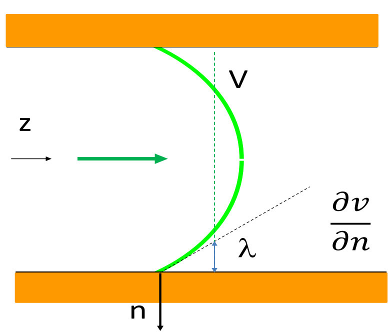

Fig 2. Definition of the friction factor from the velocity profile.

Conceptually, the friction can be averaged in a whole cross-section

$$\bar{\tau} = \frac{1}{p_w}\int_\Gamma \tau\, dl = \frac{1}{p_w}\int_\Gamma \frac{\mu V}{\lambda} dl = \frac{\mu V}{\bar{\lambda}} \quad , \tag{6}$$

where $\frac{1}{\bar{\lambda}} = \frac{1}{p_W}\int_\Gamma \frac{1}{\lambda} dl$ is by definition the average friction length. Note that the derivation of (6) assumes a constant value of the viscosity, which is the case of Newtonian fluids. The case of non-Newtonian fluids is more complex and will be the subject of further work. Using the instantaneous relation

$$V = \frac{dz}{dt} \quad , \tag{7}$$

Substitution of (6) and (7) in (4) yields the differential equation

$$\rho\, z\, S_c \frac{d^2 z}{dt^2} = \gamma\, p_W \cos\theta - \mu\frac{p_W}{2\bar{\lambda}}\frac{d\, z^2}{d\, t} \, . \tag{8}$$

It can be shown that, most of the time in capillary microsystems, inertia can be neglected because the Reynolds number is small, i.e.

$$\mathrm{Re} = \frac{V\, w}{\nu} \leq \mathrm{O}(1) \, , \tag{9}$$

where w is a characteristic dimension of the channel, ν the kinematic viscosity, and O(1) means "order of 1". Instead of numerically integrating (10), a closed form expression for the travel distance z can be obtained by neglecting the inertial term on the left hand side of (8). Hence, we can write

$$\frac{dz^2}{dt} = \frac{2\gamma\bar{\lambda}}{\mu}\cos\theta \, . \tag{10}$$

The solution of (10) is

$$z = \sqrt{\frac{\gamma}{\mu}}\sqrt{\cos\theta}\ \sqrt{2\,\bar{\lambda}\, t} \, . \tag{11}$$

The travel distance varies as the square root of the time, in agrement with the Lucas-Washburn-Rideal (LWR) law for capillary flows inside cylinders [1-3]. The liquid velocity can be readily derived from (11)

$$V = \sqrt{\frac{\gamma}{\mu}}\sqrt{\cos\theta}\sqrt{\frac{\bar{\lambda}}{2t}} \, . \tag{12}$$

Under this form, the fluid velocity is the product of the square root of a "physical" velocity $\sqrt{\gamma/\mu}$ —related to the physical properties of the materials—by the square root of a "geometrical" velocity $\sqrt{\bar{\lambda}/2t}$, and by the cosine of the dynamic contact angle. Note that the friction length $\bar{\lambda}$ is purely a geometrical data (in the case of Newtonian fluids). The value of $\bar{\lambda}$ can found in numerous published tables [8].

2

On the other hand, because $dz^2/dt = 2zV$ relation (10) immediately produces a relation between the velocity and the travel distance

$$V = \frac{\gamma\bar{\lambda}}{\mu}\cos\theta\frac{1}{z}. \qquad (13)$$

3 DYNAMIC CONTACT ANGLE

It has been shown that the dynamic contact angle depends on the capillary number [13-16]. In our case, the capillary number for the flow can be derived from relation (13). Let us recall that the capillary number is the ratio of the viscous forces to the surface tension forces

$$Ca = \frac{\mu V}{\gamma}\ . \qquad (14)$$

Substitution of (14) in (13) yields

$$Ca = \cos\theta\frac{\bar{\lambda}}{z}\ . \qquad (15)$$

Relation (15) determines the capillary number—in the case of SCF—as the product of the ratio $\bar{\lambda}/z$ by the cosine of the wetting angle. Relation (15) shows that the capillary number is proportional to the non-dimensional ratio $\bar{\lambda}/z$.

Let us now recall some empirical models for the estimation of the dynamic contact angle [12-16]. Noting θ_0 the static contact angle and θ the dynamic contact angle, Bracke and colleagues [13] proposed the formula

$$\cos\theta = \cos\theta_0 - 2(1+\cos\theta_0)Ca^{\frac{1}{2}} \qquad (16)$$

Relation (16) shows that the dynamic contact angle θ is always larger than the static contact (Young) angle θ_0. The empirical correlation proposed by Seebergh and coworkers [14] is only slightly different.

If we substitute the expression (15) of the capillary number in relation (16), we find the expression of the dynamic contact angle

$$\cos\theta = \cos\theta_0 - 2(1+\cos\theta_0)\sqrt{\cos\theta}\sqrt{\frac{\bar{\lambda}}{z}}\,. \qquad (17)$$

The cosine of the dynamic contact angle appears on both sides of (17) so that (17) is an implicit relation. Far from the channel inlet ($\bar{\lambda}/z$ <<1), we can substitute to (17) the explicit formula

$$\cos\theta = \cos\theta_0 - 2(1+\cos\theta_0)\sqrt{\cos\theta_0}\sqrt{\frac{\bar{\lambda}}{z}}\,. \qquad (18)$$

Relations (17) and (18) are plotted in figure 3, for a flat rectangular channel (aspect ratio 7): the explicit formulation is legitimate except near the channel inlet (z=0). Note that, in the inlet region ($\bar{\lambda}/z$ >1), the expression of the contact angle is derived from (17) is

$$\cos\theta = \cos\theta_0 + 2(1+\cos\theta_0)^2\frac{\bar{\lambda}}{z}\left[1 - \sqrt{\frac{\cos\theta_0}{(1+\cos\theta_0)^2}\frac{z}{\bar{\lambda}} + 1}\right] \qquad (19)$$

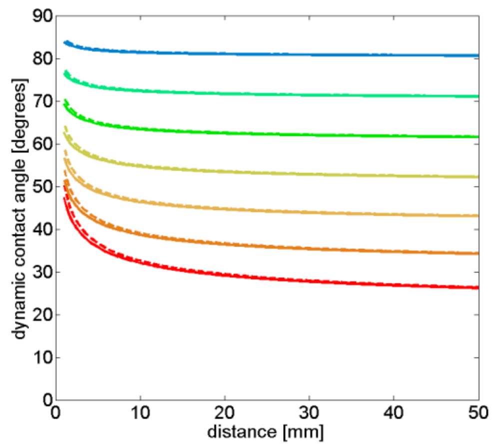

Fig.3. Variation of the dynamic contact angle as a function of the travel distance z, for seven values of the static contact angle {20, 30,40, 50, 60, 70, 80} degrees. The dotted lines correspond to the implicit solution (17) while the continuous lines correspond to the explicit relation (18).

Substituting back (18) in the velocity expression (13) we find the expression for the capillary velocity

$$V = \frac{\gamma}{\mu}\frac{\bar{\lambda}}{z}\left[\cos\theta_0 - 2(1+\cos\theta_0)\sqrt{\cos\theta_0}\sqrt{\frac{\bar{\lambda}}{z}}\right]. \qquad (20)$$

Or, noting $\varepsilon = (V_{stat} - V)/V_{stat}$ the relative error due to the use of the static contact angle instead of the dynamic contact angle, where $V_{stat} = (\gamma/\mu)(\bar{\lambda}/z)\cos\theta_0$, we find

$$\varepsilon = \frac{V_{stat} - V}{V_{stat}} = 2\frac{(1+\cos\theta_0)}{\sqrt{\cos\theta_0}}\left(\frac{\bar{\lambda}}{z}\right)^{\frac{1}{2}}. \qquad (21)$$

4 MODELING AND EXPERIMENTS

Relation (20) is plotted in figure 4 for $\bar{\lambda} = d/6$—which corresponds to a flattened rectangle cross section with $w >> d$ —and a static contact angle of 45°. The relative error is small after a few millimeters in the channel. It is however not negligible in the first few millimeters.

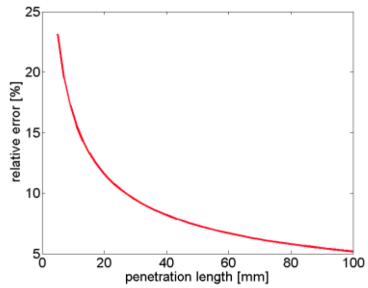

Fig.4. Relative error on the velocity ($\bar{\lambda} = d/6$, θ=45°)

Let us now re-analyze the case of water flowing in a rectangular channel composed of COC walls and a thin plastic cover (aspect ratio 0.3), described in [8]. We find that the best fit with a static contact angle is 49°, while it is 47° using the dynamic formulation (figure 5).

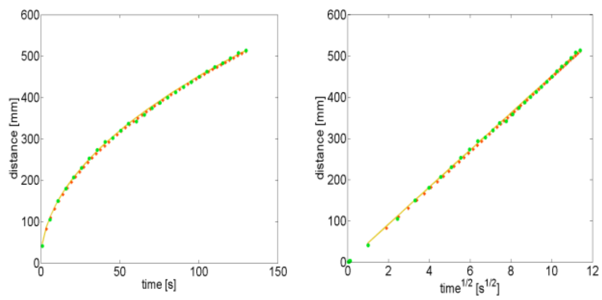

Fig.5. Comparison between experiments and model for the travel distance in a rectangular channel: left, travel distance vs. time; right, travel distance vs. square root of time. The green dots correspond to the experimental results, the yellow continuous line to the model with a static contact angle and the red crosses to the model with the dynamic contact angle.

5 CONCLUSION

The effect of dynamic contact angle is still controversial in the literature. Some papers report that the dynamic contact is not very different from the static contact angle [17,18], on the opposite some papers point out its importance [16]. In fact, all depends on the value of the capillary number.

In this work, we showed that the usual definition of the capillary number can be replaced by another one, based on the friction length, which we have shown to be $Ca = (\bar{\lambda}/z)\cos\theta$. This "channel capillary number" points out the importance of the ratio between the friction length and the channel length.

It is concluded that the effect of the dynamic contract angle strongly depends on the characteristic length of the channel.

REFERENCES

[1] R. Lucas, "Ueber das Zeitgesetz des Kapillaren Aufstiegs von Flussigkeiten, Kolloid Z., **23**, 15, 1918.

[2] E.W. Washburn, The dynamics of capillary flow. Phys. Rev., **17**, 273-283, 1921.

[3] E.K. Rideal. On the flow of liquids under capillary pressure. Philos. Mag. Ser. **6**(44):1152— 1159, 1922.

[4] R.R.Rye, F.G. Yost, Wetting Kinetics in Surface Capillary Grooves Langmuir, **12**, 4625-2627, 1996.

[5] M. Kitron-Belinkov, A. Marmur, T. Trabold, G.V. Dadheech, Groovy-drops: effect of groove curvature on spontaneous capillary flow, Langmuir, **23** (16), 8406-8410, 2007.

[6] F. F. Ouali, G. McHale, H. Javed, C. Trabi, N. J. Shirtcliffe, M. I. Newton, Wetting considerations in capillary rise and imbibition in closed square tubes and open rectangular cross-section channels, Microfluid Nanofluid., **15**, 309-326, 2013.

[7] J. Berthier, K. Brakke, E. Berthier, A general condition for spontaneous capillary flow in uniform cross-section microchannels, Microfluid Nanofluid, **16**:779–785, 2014.

[8] J. Berthier, D. Gosselin, E. Berthier, A generalization of the Lucas-Washburn-Rideal law to composite microchannels of arbitrary cross-section, Microfluid. Nanofluid., accepted, in print, 2015.

[9] M.M. Weislogel, Compound capillary rise, J. Fluid Mech., **709**, 622-64, 2012.

[10] V. Dussan, On the spreading of liquids on solid surfaces: static and dynamic contact lines, Ann. Rev. Fluid Mech., **11**, 371, 1979.

[11] Y.D. Shikhmurzaev, The moving contact line on a smooth solid surface, Int. J. Multiphase Flow, **19**, 584, 1993.

[12] T.D. Blake, Dynamics of contact angles and wetting; in J.C. Berg (Ed.), Wettability, Marcel Dekker, New-York, p. 251, 1989.

[13] M. Bracke, E De Voeght, and P. Joos, The kinetics of wetting: the dynamic contact angle, Progr Colloid Polym Sci **79**:142-149, 1989.

[14] J.E. Seebergh, J.C. Berg, Dynamic wetting in the low capillary number regime, Chem. Eng. Sc., **47** (17), 4455, 1992.

[15] T.S. Jiang, S.G. Oh, J.C. Slattery, Corrections for dynamic contact angle, J. Colloid Interface Sc., **69**, 74, 1979.

[16] S. van Mourik, A.E.P. Veldman, M.E. Dreter, Simulation of capillary flow with a dynamic contact angle, Microgravity sci. technol. XVII-3, 2005.

[17] A. Han, G. Mondin, N. G. Hegelbach, N. F. de Rooij, U. Staufer, Filling kinetics of liquids in nanochannels as narrow as 27 nm by capillary force, Journal of Colloid and Interface Science **293**, 151–157, 2006.

[18] V. D. Sobolev, N. V. Churaev, M. G. Velarde, and Z. M. Zorin, Surface tension and dynamic contact angle of water in thin quartz capillaries, Journal of Colloid and Interface Science **222**, 51–54, 2000.