Information-Theoretic Estimates of Classical and Quantum Communication and Processing at Nanoscale

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ABSTRACT

This paper examines fundamentals of classical, *hybrid* and quantum information technologies which are the bases of computer science and computer engineering. We define, analyze and evaluate classical and quantum-mechanical information measures pertained to communication and processing at nanoscale. The *macroscopic* and *microscopic* physical communication and processing platforms and hardware are considered. *Macroscopic* and *microscopic* systems operate on the electron- and photon-induced transductions. However, conventional electrostatic and quantum processing are fundamentally distinct. There are major differences in device physics, utilized phenomena, device- and system-level solutions, arithmetics, software and hardware solutions, etc. This paper contributes to design and analysis of multi-physics quantum↔classical communication and processing. We advance a knowledge base in cognizant areas across disciplines. Our findings enable existing and future communication, sensing, information fusion, computing and processing platforms. These enable technical readiness, technological capabilities and commercialization capacities by developing relevant tools, methods and practical schemes.

Keywords**:** electronics; information theory; processing; nanotechnology; quantum informatics

1. INTRODUCTION

Photonics and optoelectronics resulted in new communication and processing concepts. While conventional communication, computing and processing are supported by nanoscaled digital systems, quantum-centric computing, communication and processing are emerging. Quantum information technologies imply new paradigms. Fundamental, applied and experimental premises of quantum-classical and quantum information theories are under developments in order to advance technology-centric and compliant schemes. Practical solutions lead to new computing paradigms, communication platforms, data fusion schemes and processing premises.

Communication, data fusion, processing, data exchange and computing are achieved by using different premises. The majority of solid-state nanoscaled microelectronic devices operate utilizing electrostatic phenomena, while quantum effects degrade device and system performance [1-4]. Current microelectronics guarantees sensing, interfacing, communication and high-performance computing to meet practical needs. Optoelectronic, photonic and other processing devices and systems emerged [1, 5]. In life and physical sciences, advancements in knowledge base and transformative discoveries are of a great importance. There are a number of open problems, such as, quantum information theory, quantum processing, design of *microscopic* systems, etc. Quantum communication, computing and processing are under intensive developments [1]. Preliminary results in quantum-probabilistic and quantum-deterministic communication and computing are reported in [1-4, 6-8].

A quantum↔classical processing is exhibited by nervous systems [6-10]. This paper studies quantum and *hybrid* (classical→quantum and quantum→classical) communication and processing schemes. We depart from a theoretical computer science which attempts to justify a possibility of "quantum computing" by means of narrative mathematical models and a hypothetical use of not observable mathematical operators such as the wave function Ψ, probability amplitudes, etc. We develop and evaluate device- and physics-consistent quantum communication and processing concepts using real-valued, detectable and measurable physical variables. The quantum transitions of these physical quantities and device transductions ensure sensing, communication and processing in electronic, optoelectronic and photonic systems. This solution typifies asynchronous spatially- and time-distributed quantum sensing, communication and information processing in living organisms. Illustrative examples are reported.

2. QUANTUM OPTOELECTRONIC SCHEMES

There are quantum transductions in *microscopic* systems due to photon emission, photon propagation and photon absorption with the photon-electron interactions. Optical photon absorption and emission, as well as electronic transductions, are utilized by living organisms and *engineered* systems. Quantum phenomena are utilized by single-photon biomolecular sensors, waveguides and *processors* in visual, photosynthesis and other systems. *Engineered* optoelectronic sensors, receivers, waveguides and processors operate on many-photon mainly utilizing photoelectric effect, electrostatics, etc. Classical, *hybrid* and quantum communication and processing on photons and a single photon are examined in [1-4, 6-8]. The prototypes are designed integrating laser – nanowaveguide – sensor (detector) – post-processing optoelectronics.

Communication, interfacing and computing by optical photons have been extensively examined by IBM, Intel, HP and other leading companies [1-4]. Very promising results were achieved, reaching robust ~100 Gbit/sec interfacing and communication [2-4]. Communication and processing schemes should be examined in order to assess and approach the Shannon and quantum-mechanical performance limits in classical and quantum domains. These developments are emphasized in [11, 12].

3. INFORMATION-THEORETIC ESTIMATES

The quantitative information-centric performance estimates and capability measures should be derived to evaluate communication and processing. We analyze to assess communication and processing schemes to approach the Shannon and quantum-mechanical performance limits in classical and quantum domains. For a random variable X_i with n possible outcomes $\{x_i: i=1,2,\ldots,n-1,n\}$, which has the probability density functions $p(x_i)$, the entropy is [13]

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i)\cdot \quad (1)$$

The mutual information between the stimulus X and the output Y quantitatively defines the amount of information received on average. The mutual information $I(X;Y)$ and quantum mutual information $\mathcal{I}(\mathcal{X};\mathcal{Y})$ are defined by the classical and quantum entropies $H(X)$ and $\mathcal{H}(\mathcal{X})$ [6-8, 13-15]. These $I(X;Y)$ and $\mathcal{I}(\mathcal{X};\mathcal{Y})$ provide a quantitative estimate on the dependence of X and Y. We have [6-8]

$$I(X;Y)=I(Y;X)=H(X)-H(X|Y)=H(Y)-H(Y|X)=H(X)+H(Y)-H(X,Y),$$

$$I(X;Y)=\sum_{x\in X}\sum_{y\in Y} p_{X,Y}(x,y)\log_2\frac{p_{X,Y}(x,y)}{p_X(x)p_Y(y)} \text{ [bits/symbol]},$$

$$\mathcal{I}(\mathcal{X};\mathcal{Y})=\mathcal{H}(\mathcal{X})-\mathcal{H}(\mathcal{X}|\mathcal{Y}), \quad (2)$$

The aforementioned positively-definite mutual information $I(X;Y)\geq 0$ provides the average amount of information received per symbol transmitted or processed. The conditional entropy $H(X|Y=y)$ of a random variable X, conditional on a particular realization y of Y, is the expected conditional information content. We have $H(X|Y=y)=-\sum_x p_{X|Y}(x|y)\log_2 p_{X|Y}(x|y)$. The expected conditional information content with respect to both X and Y is defined by the conditional entropy

$$H(X|Y)=-\sum_{x\in X}\sum_{y\in Y} p_{X,Y}(x,y)\log_2 p_{X|Y}(x|y),\ H(X|Y)\geq 0. \quad (3)$$

The conditional entropy $H(X|Y)$ corresponds to the average loss of information, and, $H(X)\geq H(X|Y)$. In particular, $H(X|Y)=H(X)$ if X and Y are independent, yielding $I(X;Y)=0$.

The joint entropy is the entropy of the joint random variable (X,Y). One has

$$H(X,Y) = H(X) + H(Y|X) = H(Y) + H(X|Y),$$

$$H(X,Y)=-\sum_{x,y} p_{X,Y}(x,y)\log_2 p_{X,Y}(x,y)\cdot \quad (4)$$

Analog and digital deterministic computing and processing guarantee exceptional performance. These paradigms are utilized in all existing data processing platforms. The deterministic quantum processing on real-valued physical variables is studied. Quantum communication and processing on a single photon, or, on few photons and electrons result in significant uncertainties, distortions and errors [1]. We apply an inherent quantum determinism to ensure quantum-deterministic communication and processing on the *utilizable* initial (I) and final (F) *state* transductions $\mathcal{S}=[\mathcal{S}_I,\mathcal{S}_F]^T$ $\mathcal{S}_I{:}\,\boldsymbol{\nu}_I\Rightarrow\mathcal{S}_F{:}\,\boldsymbol{\nu}_F$ performed on real-valued, directly *detectable*, *measurable* and *processable* physical variables $\boldsymbol{\nu}$ [7, 8]. As $\mathcal{X}$ and $\mathcal{Y}$ can be measured, one yields $\mathcal{P}(\mathcal{X})$, $\mathcal{H}(\mathcal{X})$, $\mathcal{H}(\mathcal{X};\mathcal{Y})$, $\mathcal{I}(\mathcal{X};\mathcal{Y})$ and other probabilistic measures and information estimates. This concept is substantiated by *natural* systems, as well as by quantum-effect optoelectronic and photonic devices which are widely deployed and commercialized [1]. A quantum-effect processing primitive $\mathcal{P}_j$ exhibits transductions $\mathcal{S}_j(\boldsymbol{\nu})$ on *detectable*, *measurable* and *processable* variables $\boldsymbol{\nu}_j$ yielding *distinguishable* and *computable* transforms $\mathcal{T}_j(\mathcal{S},\boldsymbol{\nu})$. These quantum transductions $\mathcal{S}_j(\boldsymbol{\nu})$ result in processing tasks.

Communication and Channel Capacity – For conventional and quantum-deterministic communication, the channel capacity of a stationary memoryless channel with finite input and output alphabets is

$$C=\max_{p_X(\cdot)} I(X;Y) \text{ and } \mathcal{C}=\max_{p_X(\cdot)}\mathcal{I}(\mathcal{X};\mathcal{Y}) \text{ [bits/symbol]}. \quad (5)$$

If the transition probabilities vary, for nonstationary memoryless channels

$$C=\max_{p_{X_1}(\cdot),\ldots,p_{X_n}(\cdot)} \tfrac{1}{n}\sum_{i=1}^{n} I(X_i;Y_i)=\tfrac{1}{n}\sum_{i=1}^{n} C_i, \text{ [bits/symbol]}. \quad (6)$$

Communication and Transmission Rate – Significant errors and distortions are observed in low-power nanoscaled microelectronic devices, nanoICs, nanowaveguides and ~100 Gbit/sec CMOS nanophotonics [1-4]. High-performance computing, communication and processing platforms must be designed despite errors, distortions, non-uniformity, inconsistencies, uncertainties, etc. Using the probability of a bit error p_b, the maximum transmission rate is

$$r_{\max}(p_b)=\frac{C}{1-H_2(p_b)},\ H_2(p_b)=-[p_b\log_2 p_b+(1-p_b)\log_2(1-p_b)], \quad (7)$$

where H_2 is the binary entropy function.

Distortion Measure – Consider a sequence $X_1,\ldots, X_n$ with $p(x)$ and a finite alphabet $\mathcal{A}$, $x\in\mathcal{A}$. Using the reproduced alphabet $\mathcal{A}_r$ with symbols $x_r\in\mathcal{A}_r$, the distortion measure $d{:}\,\mathcal{A}\times\mathcal{A}_r\rightarrow\mathbb{R}$ is finite, and

$$d_{\max}=\max_{x\in A,\,x_r\in A_r} d(x,x_r)<\infty. \quad (8)$$

The distortion depends on the sequences, encoding and decoding functions, etc. The rate distortion function $(r,\mathcal{D})$ for a source X with $d(x,x_r)$ can be defined as

$$r_I(\mathcal{D})=\min I(X;X_r), \quad (9)$$

$$I(X;X_r)=H(X)-H(X|X_r)\geq H(p)-H(\mathcal{D}),\ r_I(\mathcal{D})\geq H(p)-H(\mathcal{D}).$$

The minimum $\min I(X;X_r)$ must be found with respect to all condition distributions $p(x_r|x)$ for which the joint distribution $p(x,x_r)=p(x)p(x_r|x)$ satisfies the imposed distortion constraints.

4. Information-Theoretic Measures

The system-level communication and processing capabilities must be quantitatively evaluated for the microelectronics, CMOS-integrated nanophotonics, optoelectronics, as well as quantum communication and processing. We introduce consistent measures pertained to classical, quantum↔classical and quantum data processing.

Data Processing Capability – Using $I(X;Y)$ and $\boldsymbol{I}(\mathcal{X};\mathcal{Y})$, given by (2), we define the data processing capability measures as

$$D=BI(X;Y)r^{-1}(p_b)r_I^{-1}(\mathcal{D}),\ \boldsymbol{D}=\mathcal{R}\,\boldsymbol{I}(\mathcal{X};\mathcal{Y})\mathcal{r}^{-1}(p_b)\mathcal{r}_I^{-1}(\mathcal{D}),\qquad(10)$$

where B is the bandwidth; $\mathcal{R}$ is the quantum transduction rate in the *microscopic* processing system.

These quantitative D and $\boldsymbol{D}$ allow one to evaluate the processing performance and processing platforms capacity.

Data Processing Complexity – The entropies H and $\boldsymbol{H}$ define the data set complexity. A finite length of the string $x\in\{0, 1\}$ is denoted as $l(x)$. The Kolmogorov descriptive complexity $K_U(x)$ provides the minimal description length l of a string x with respect to a universal processor U within a processing realization p. That is,

$$K_U(x)=\min_{p:U(p)=x} l(p),\qquad(11)$$

where U is the computable function of two arguments x and p.

Binary strings are the words in the alphabet $\mathcal{A}=\{0, 1\}$. For any computable function, we have $U:\mathcal{A}\to\mathcal{A}$. The complexity of $x\in\mathcal{A}$ can be defined with respect to U. For any processor $\mathcal{P}$, one has

$$K_U(x)\geq cK_{\mathcal{P}}(x),\ \forall x,\ c>0,\qquad(12)$$

where c is the constant which depends on U and $\mathcal{P}$.

Recalling that $I(X;Y)=H(Y)-H(Y|X)$, we define the mutual complexity as

$$I_K(X;Y) = K_U(Y) - H(Y|X, K_U(X)).\qquad(13)$$

The data processing complexity measures are given as

$$L = H(X)K_U(x)I_K(X;Y),\ \boldsymbol{L} = \boldsymbol{H}\mathcal{K}_{\mathcal{U}}(x)\boldsymbol{I}_K(\mathcal{X};\mathcal{Y}).\qquad(14)$$

Data Quality – A Markov information source is a pair (M,f) of stationary Markov chain M and function f of reachable states s_k, and, $f(s_k):S\to A$. The transductions $\mathcal{S}_j(\boldsymbol{\nu})$ are on *detectable*, *measurable* and *processable* variables $\boldsymbol{\nu}_j$. The mapping $f(s_k)$ maps states S into the Markov chain to entities in the alphabet $\mathcal{A}$. To estimate the data quality of information sources Q and $\boldsymbol{Q}$, we use the positively-definite mutual information. The sequence of length n has a complexity $\sim O(n)$. The probability of an input p is $\sim 2^{-l(p)}$. The universal probability of a binary string x is

$$P_U(x)=\sum_{p:U(p)=x}2^{-l(p)}=\Pr\big(U(p)=x\big),\ P_U(x)\approx 2^{-K(x)}.\qquad(15)$$

We define the data quality measures as

$$Q = I(X;Y)P_U(x),\ \boldsymbol{Q}=\boldsymbol{I}(\mathcal{X};\mathcal{Y})\mathcal{P}_{\mathcal{U}}(x).\qquad(16)$$

Here, for two discrete random variables X and Y

$$I(X;Y)=\sum_{y\in Y}\sum_{x\in X}p(x,y)\ln\frac{p(x,y)}{p(x)p(y)},\qquad(17)$$

where $p(x,y)$ is the joint probability density function of X and Y; $p(x)$ and $p(y)$ are the marginal probability density functions of X and Y.

The proposed quantitative measures yield consistent estimates and metrics to evaluate robustness, effectiveness and capabilities of processing on extra-large-scale data.

Example 4. 1.

If X and Y are independent, $p(x,y)=p(x)p(y)$.

Hence $\ln\dfrac{p(x,y)}{p(x)p(y)}=\ln 1=0$, and $I(X;Y)=0$. ■

Example 4. 2.

Consider a *microscopic* system with two distinct unperturbed states $\mathcal{S}_{a,b}(\boldsymbol{\nu})$. Mathematically, this system is characterized by the wave functions ψ_a and ψ_b. Let these mathematical operators satisfy $\langle\psi_a|\psi_b\rangle=\delta_{ab}$ [7, 8]. Considering physical *microscopic* systems, the studied $\mathcal{S}_{a,b}(\boldsymbol{\nu})$ are exhibited by a great number of two-state quantum devices. '

The time-independent Schrödinger equations are

$\hat{H}_a\psi_a=E_a\psi_a$ and $\hat{H}_b\psi_b=E_b\psi_b$

with the Hamiltonian operators denoted as $\hat{H}_a$ and $\hat{H}_b$.

The wave function is $\Psi(t)=c_a\psi_a e^{-i\frac{E_a}{\hbar}t}+c_b\psi_b e^{-i\frac{E_b}{\hbar}t}$ with the probability amplitudes (c_a,c_b) which satisfy $|c_a|^2+|c_b|^2=1$. Using the perturbation theory [6-8], the dynamic evolutions of $c_a(t)$ and $c_b(t)$ are expressed as

$$\frac{dc_a}{dt}=-i\frac{1}{\hbar}H_{Eab}e^{-i\frac{E_b-E_a}{\hbar}t}c_b(t),\ \frac{dc_b}{dt}=-i\frac{1}{\hbar}H_{Eba}e^{i\frac{E_b-E_a}{\hbar}t}c_a(t).$$

The quantum transition probability $p_{a\to b}(t)$ defines the probability that the *microscopic* system, which started in $\mathcal{S}_a(\boldsymbol{\nu})$ will be found in $\mathcal{S}_b(\boldsymbol{\nu})$ at time t. These $\mathcal{S}_a(\boldsymbol{\nu})$ and $\mathcal{S}_b(\boldsymbol{\nu})$, mathematically are modeled using the corresponding wave functions ψ_a and ψ_b.

The probability is $p_{a\to b}(t)=|c_b(t)|^2$.

Then, $p_{a\to b}(t,\omega)=|\boldsymbol{\mathcal{E}}_{ab}|^2\left|\dfrac{1}{2\hbar}\left[\dfrac{e^{j(\omega_0+\omega)t}-1}{\omega_0+\omega}+\dfrac{e^{j(\omega_0-\omega)t}-1}{\omega_0-\omega}\right]\right|^2.$

Consider a sinusoidal excitation which leads to

$\hat{H}_E(t,\mathbf{r})=\boldsymbol{\mathcal{E}}(\mathbf{r})\cos\omega t$, or, $\hat{H}_{Eab}=\boldsymbol{\mathcal{E}}_{ab}\cos\omega t$, $\Pi_{ab}=\langle\psi_a|\boldsymbol{\mathcal{E}}|\psi_b\rangle$.

The theoretical quantum transition rate is $r=(dp/dt)$.

This r is relevant to the device quantum transition rate $\mathcal{R}$, which can be viewed as an analog to the bandwidth B. One recalls that the channel capacity is $C_{\max}=B\log_2[1+S/N]$, where S/N is the average signal-to-noise power ratio. The *natural* transition frequency is $\omega_0=(E_b-E_a)/\hbar$, $\omega_0\geq 0$.

Considering quantum communication and processing by a quantum device on optoelectronic quantum transductions $\mathcal{S}=[\mathcal{S}_a,\mathcal{S}_b]^T$, we study two-state quantum-mechanical switching $p_{a\to b}(t,\omega)$ at $f=1\times 10^{15}$ Hz [7, 8]. The three-dimensional plot for $p_{a\to b}(t,\omega)$ is shown in Figure 1.

For a serial optical photon communication, the *gross bit rate* r depends on the transmission time T_t, $r=1/T_t$.

For the parallel communication with N parallel chanels, the *gross bit rate* is $r=\sum_{i=1}^{N}T_i^{-1}\log_2 M_i$. Here, M_i is the number of symbols or modulation levels in the ith channel; T_i is the symbol duration time for the ith channel.

The quantum transductions $\mathcal{S}=[\mathcal{S}_a,\mathcal{S}_b]^T$ occur within $\sim 1\times10^{-15}$ sec [7, 8]. Let the maximum rate $r_{\max}$ varies from 1×10^{14} to 1×10^{15} pulse/sec, while the average rate r_0 changes from 1×10^{13} to 1×10^{14} pulse/sec. Assume that $r_{\min}$ is 1×10^{13} pulse/sec. The channel capacity $\mathcal{C}(r_0,r_{\max})$ is [7, 8]

$$\mathcal{C}=\frac{1}{\ln 2}\left((r_0-r_{\min})\ln\left(\frac{r_{\max}}{r_{\min}}\right)^{\frac{r_{\max}}{r_{\max}-r_{\min}}}-r_0\ln\left(\frac{r_0}{r_{\min}}\right)\right) \text{ if } r_{\min}\le r\le r_{\max}.$$

Figure 2 reports a three-dimensional plot for $\mathcal{C}(r_0,r_{\max})$. A very high channel capacity is achieved with $\mathcal{C}_{\max}=2.751\times10^{14}$ bits. The probability of a bit error p_b, depends on the derived quantum transition probability $p_{a\to b}(t)$, The reported analysis also lead to the maximum transmission rate $r_{\max}(p_b)$.

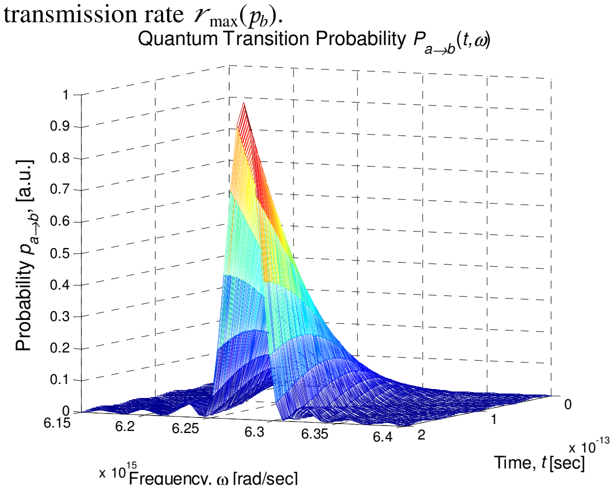

Figure 1. Quantum transition probability $p_{a\to b}(t,\omega)$

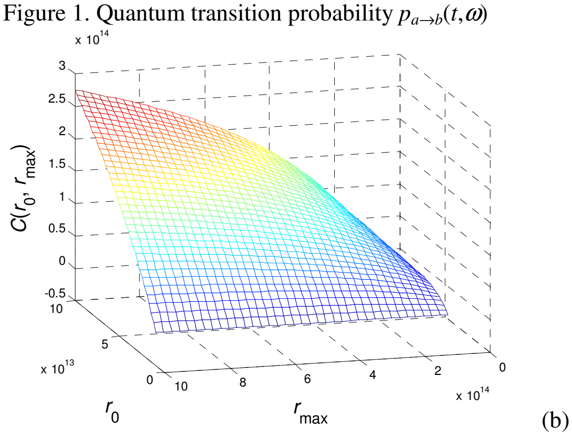

Figure 2. Three-dimensional plot for channel capacity $\mathcal{C}(r_0,r_{\max})$. ■

5. Conclusions

This paper further enabled quantum information theory by examining emerging paradigms of information fusion, communication, interfacing and processing. We proposed estimates and measures to assess classical, quantum and *hybrid* communication and processing. The information technology fundamentals and knowledge base foundations of engineering science were enabled in analysis and design of *microscopic–macroscopic* photonic and optoelectronic systems. A quantum↔classical consistency was achieved. High-fidelity modeling, heterogeneous simulations and data-intensive analysis were performed for nanoscale lasers, waveguides, detectors and processing hardware. Our results lead to enabling processing capabilities beyond classical solutions. The reported findings facilitate the developments in areas of a critical importance and significance, such as, quantum processing by *natural* and *engineered* systems, biotechnology, neurophotonics, etc.

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