On the velocity of spontaneous capillary flows in confined and open microchannels

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ABSTRACT

Capillary microfluidics or capillarics is gaining importance in the biotechnological domain. It combines the advantages of capillary actuation which does not require pumps or syringes to move the fluids, with low-cost fabrication, user-friendliness, portability and telemedicine compatibility.

In this work, we present expressions of the spontaneous capillary flow velocity in different geometrical configurations. It is shown that relatively large velocities— at the scale of microsystems— can be reached by capillary microflows. Consequently transport distances can be important.

Keywords: spontaneous capillary flow, capillarics, capillary force, microgroove, suspended channels.

1 INTRODUCTION

In biotechnology and medicine, point of care (POC) and home care systems are progressively gaining momentum. Such systems must be easily portable, user friendly, robust and low-cost. These systems considerably improve the comfort of patients and the rapidity of the diagnostics. [1]

The conventional microfluidic systems use fluids moved by pumps, syringes or electric means, and are not the best solutions when portability and low-cost are an issue. Such systems are bulky and/or requires costly, external appliances.

On the other hand, capillary-based systems fulfill the requirements for POC and home-care [2,3]. In these systems, fluids are moved by capillary and surface tension forces. The self-motion of the fluid is called spontaneous capillary flow (SCF). In fact in such systems, the energy source for the fluid motion is the capillary force exerted by the triple line at the front end of the flow. This contrasts with pump or syringe-driven flows where the motor of the motion is a pump or a syringe placed at the back end of the flow.

Moreover, the energy source for capillary-based systems is "packed in" under the form of an adequate surface energy of the walls, while the energy source is external to the biochip when dealing with active systems using pumps, syringes or other electrical means.

Many different geometries of capillary channels exist (figure 1). Capillary systems can be confined, i.e. use closed microchannels [2,3], or they can have an open-surface and the microchannel has the shape of a groove etched in a solid substrate [4-6] or they can even be "suspended", i.e. the liquids flow between vertical walls without supporting walls at the bottom [7,8].

For the applications in medicine and biotechnology, it is of utmost importance to determine the sample liquid velocities in such capillary systems. Especially it is essential to know if sufficiently high velocities can be reached in these channels. Its is also of importance to know if a system can be totally filled by the sample liquid in an adequate time lapse.

In this work, we derive differential equations for the velocity in capillary channels that collapse in closed form expressions when inertia can be neglected—which is frequent at the microscale. Four geometries are investigated and compared: (1) rectangular, confined channels, (2) rectangular, open-surface U-grooves, (3) triangular, open surface V-grooves, (4) rectangular, suspended channels (figure 1).

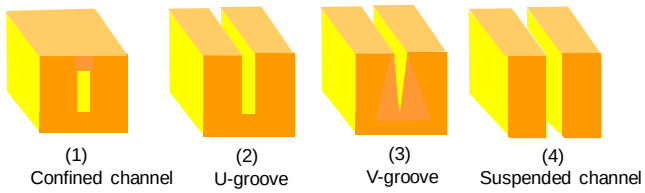

Figure 1. Different morphologies of capillary channels: (1) rectangular, confined; (2) U-groove; (3) V-groove; (4) rectangular suspended.

It is shown that the velocities are functions of the inverse of the square root of time, and that they are the product of the square root of a "physical" velocity and of a "geometrical" velocity. Depending on the morphology of

the channel, relatively large velocities can be obtained, at least in the first few millimeters of the channel.

2 THE DYNAMICS OF SCF

We place ourselves in the simplified case where the liquid boundaries are a solid wall with a Young contact angle θ, and—in the case of open channels—a free boundary with air. From a dynamic standpoint, the average velocity of the open microflow can be determined using a balance between capillary forces, surface tension forces and friction with walls [4-6,9-11].

The capillary force reads [10,11]

$$F_{cap} = p_W \gamma \cos\theta - p_F \gamma \ , \tag{1}$$

where γ is the surface tension between the liquid and air, p_W and p_F are respectively the wetted and free perimeters in a cross-section of the channel, as shown in figure 2.

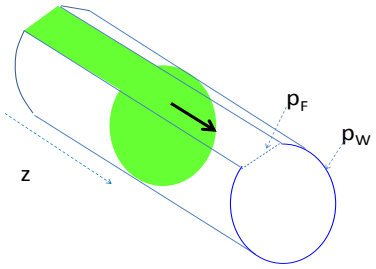

Figure 2. Sketch of a capillary flow, with the free and wetted perimeters.

On the opther hand, the friction force is

$$F_{drag} = \tau S = \tau\, p_W\, z(t), \tag{2}$$

where τ is the wall friction, S the wetted surface and z the distance of the interface from inlet which depends on the time t.

The force balance on the fluid flow is then

$$m\frac{dV}{dt} = F_{cap} - F_{drag} \ , \tag{3}$$

where m is the mass of the fluid in the channel and V the average velocity. The mass of fluid being proportional to the penetration distance, (3) can be written under the form

$$\rho\, z(t) S_c \frac{dV}{dt} = p_W \gamma \cos\theta - p_F \gamma - \tau\, p_W\, z(t), \tag{4}$$

where S_c is the cross-section area and ρ the volumic mass of the fluid. The Reynolds number of the fluid being small, the flow is highly laminar and the flow profile is "Poiseuille-like" in the channel. The friction τ then depends on the geometry of the channel.

2.1. Closed rectangular channels

In such channels, of width w and height h, the friction can approximately be expressed as

$$\begin{aligned} F_{drag} &= 6\mu\frac{V}{w} z(t)\,(2h) + 6\mu\frac{V}{h} z(t)\,(2w) \\ &= 12\,\mu V\, z(t)\left(\frac{w}{h}+\frac{h}{w}\right) \end{aligned} \tag{5}$$

where μ is the dynamic viscosty of the fluid. On the other hand, the capillary force is

$$F_{cap} = 2\gamma\cos\theta\,(w+h) \ . \tag{6}$$

The velocity is then the solution of the differential equation

$$\frac{1}{2}\rho\frac{dV^2}{dt} = 2\gamma\cos\theta\left(\frac{1}{w}+\frac{1}{h}\right)\frac{1}{t} - 12\,\mu V^2\left(\frac{1}{w^2}+\frac{1}{h^2}\right). \tag{7}$$

Note that, because the channel is closed, w and h are reversible in relation (7). Inertia can usually be neglected and we find a closed form solution for the velocity

$$V = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}\frac{q\cos\theta(1+q)}{6(1+q^2)}} = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}}\ \sqrt{f_{geom,1}}\ , \tag{8}$$

where q is the aspect ratio $q=w/h$, and $f_{geom,1}$ a non-dimensional function that depends on the geometry (and on the contact angle θ). Relation (8) shows that the velocity is the product of the square root of a "physical velocity" $\sqrt{\gamma/\mu}$ —which depends on the properties of the fluid— with the square root of a "geometrical velocity" $\sqrt{h/t}$ and with a characteristic non-dimensional function $\sqrt{f_{geom,1}}$.

The penetration distance is then given by considering that $V = dz/dt$. Integration leads to

$$z = 2\sqrt{\frac{\gamma}{\mu}}\ \sqrt{ht}\ \sqrt{f_{geom,1}}\ . \tag{9}$$

The penetration distance varies as the square root of time. Finally, eleminating the time from (8) and (9)

produces the relation between the velocity and the penetration distance

$$V = 2\frac{\gamma}{\mu}\frac{h}{z} f_{geom,1} \qquad (10)$$

2.2. Open rectangular U-grooves

The same approach applies for open, rectangular U-grooves [8,10,12]. The transposition of relation (8) in this case is

$$V = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}\frac{\left[(q+2)\cos\theta - q\right]}{6\left(\frac{2}{q}+q\right)}} = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}}\sqrt{f_{geom,2}}\ , \qquad (11)$$

relation which has the same form as (7) with a different geometrical function $f_{geom,2}$ .The penetration as a function of time is then

$$z = 2\sqrt{\frac{\gamma}{\mu}}\sqrt{ht}\sqrt{f_{geom,2}}\ , \qquad (12)$$

and

$$V = 2\frac{\gamma}{\mu}\frac{h}{z} f_{geom,2} \qquad (13)$$

We have checked the relations (11) and (12) using a long winding U-groove etched in a PMMA substrate, hydrophilically coated with a silica layer deposed by CVD (figure 3A). Figures 3B and C show that SCF can be obtained at very small scales using silicon substrates coated with an oxide layer.

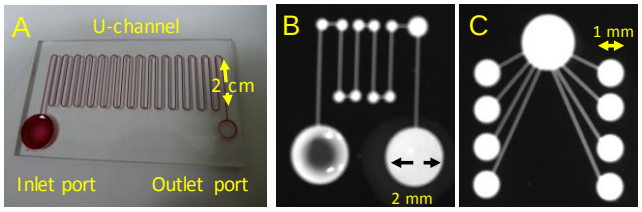

Figure 3. A: SCF in a winding U-groove channel etched in PMMA (w=300 µm, h=1 mm); B and C: SCF in a U-grooves etched in silicon (w=50 µm, h=200 µm).

2.3. OpenV-grooves

The case of V-grooves—with moderate values of the dihedral angle—is well documented in the literature [4-6]. In the case of a V-groove of semi-angle α, it has been shown that the square of the velocity of the flow is

$$V = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}}\sqrt{\frac{(\cos\theta - \cos\alpha)}{8\pi\sin\alpha}} = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}}\sqrt{f_{geom,3}}\ . \qquad (14)$$

Relation (13) also has the same structure as (8) and (11). Note that the angle α can be related to the aspect ratio by the relation $\alpha = \arctan(q/2)$. The penetration distance is given by

$$z = 2\sqrt{\frac{\gamma}{\mu}}\sqrt{ht}\sqrt{f_{geom,3}}\ , \qquad (15)$$

And the velocity as a function of the penetration distance is

$$V = 2\frac{\gamma}{\mu}\frac{h}{z} f_{geom,3} \qquad (16)$$

2.4. Suspended rectangular channels

A suspended microflow is a flow that uses capillary forces and surface tension to fill and maintain a fluid in microscale structures devoid of a ceiling and floor (figure 4A). These flows have the advantage to be accessible on both sides, from above and from below.

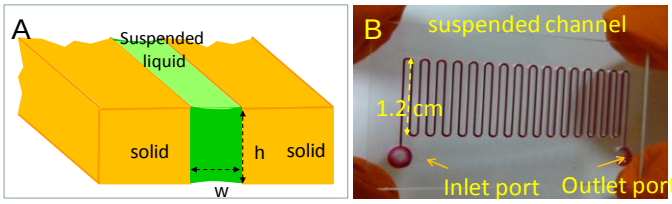

Figure 4. A: sketch of a suspended flow; B: SCF in a suspended channel (w=300 µm, h= 1 mm).

The dynamics of this kind of microflow is derived in the same manner as for U and V-grooves, and we find

$$V = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}}\sqrt{\frac{\left[q\left(\cos\theta - q\right)\right]}{6}} = \sqrt{\frac{\gamma}{\mu}\frac{h}{t}}\sqrt{f_{geom,4}}\ . \qquad (17)$$

Similarly,

$$z = 2\sqrt{\frac{\gamma}{\mu}}\sqrt{ht}\sqrt{f_{geom,4}}\ , \qquad (18)$$

and

$$V = 2\frac{\gamma}{\mu}\frac{h}{z} f_{geom,4}\ . \qquad (19)$$

We have checked the relations (17) and (18) using a long winding suspended channel etched in a PMMA substrate, hydrophilically coated with a silica layer deposed by CVD (figure 4B).

3 COMPARISON

Comparing the different geometrical configurations, it is the geometrical function which makes the velocity difference (figure 5). Figure 6 shows the penetration distance in the channel as a function of the elapsed time for two different aspect ratios q=0.19 (α= 5°) and q=0.60 (α= 17°) and a contact angle of 35°. The liquid is water, with a surface tension γ= 72 mN/m, and a dynamic viscosity μ=0.001 Pa.s. The closed, open-U and suspended channels stay always in the same order for the velocity magnitude. On the other hand, the velocity in the V-groove varies considerably.

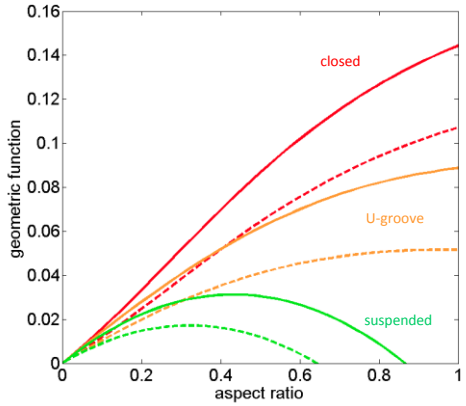

Figure 5. Comparison of the geometrical functions $f_{geom,1}$, $f_{geom,2}$ and $f_{geom,4}$. Function $f_{geom,3}$ is not plotted since it is not applicable when $q \rightarrow 0$ (or $\alpha \rightarrow 0$) because a thin filament forms in the groove corner. The continuous and dotted lines correspond respectively to a contact angle of 30° and 50°.

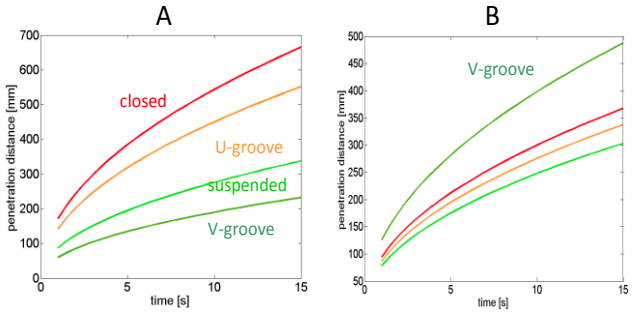

Figure 6. Comparison of the penetration distance between the different geometries. A: aspect ratio q=0.19. B: aspect ratio q=0.60. In both cases the contact angle is 35°.

Note that reltively high velocities are obtained for water by capillary means (V ~ 1 to 6 cm/s in a channel of dimensions w~200 µm and h~1 mm).

4 CONCLUSION AND PERSPECTIVES

In this work, the capillary velocities in closed channels, U and V-grooves and suspended channels have been investigated. Closed form expressions for the velocities with a dependency as $\sqrt{1/t}$ have been found, in agreement with the Wasburn-Lucas approach. These expressions can easily be extrapolated to composite walls of different materials [12]—which is often the case with closed channels where the cover can be made of a different material.

It also seems that there is a need to investigate further the velocity in very narrow V-grooves since the usual expression of the literature diverges when the groove angle decreases towards zero. Moreover, the non-Newtonian aspect is still to be investigated since biological fluids have often a shear-thinning behavior, and their viscosity increases with the decrease of the velocity in the capillary channel.

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