Electronic properties in two dimensional quantum rings consisting of one and two nanoelements

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ABSTRACT

The single-electron spectrum is studied in the two-dimensional quantum rings ($a << R$, a, R - inner and outer radii of the ring, respectively). It is shown that under certain circumstances, the system establishes quantum states that are close to the well known problem in quantum mechanics such as the fall of the particles on the center in the influence of a single central force, where, however, the presence of the inner boundary stabilizes the new ground state.

Keywords: sectorial wells, quantum ring, falling to centre, Kronig-Penney model. persistent current

1 INTRODUCTION

The quantum rings are investigated due to their properties, non-simply connected topology and the growing interest is supported essentially by the progress in the fabrication of the assembled double ring structures and devices that can be used with wide range of applications in nanoelectronics. It is known that the electronic spectrum of many meso- and nano-systems essentially depends on the details of the geometry. An interesting example of such systems is the one-dimensional ballistic quantum ring in the presence of transverse magnetic field [1]. The presence of Aharonov-Bohm flux leads to the destruction of the symmetry of the "left - right", which results in possible existence of persistent currents in this system [2]. The physical quantities characterizing the quantum ring depend periodically on the magnetic flux. It is important from the geometry point of view notice that the systems with double bound states can also exhibit an interesting physical behavior [3,4]. Here we consider the two-dimensional (2D) - quantum rings with radii $a << R$. In addition, the ring can contain either one or two quantum well sectors (see Fig. 1). In the case of a one well geometry the system consists of two adjacent truncated sectors and in the case of two wells – there are four sectors. In both cases, under certain conditions, this gives rise to states similar to the problem of the fall in the center of the particle [5], but the presence of the inner boundary trims endless oscillations of the wave function, causing a formation of a new ground state. We here study a one electron energy spectrum the dependence on the geometries of the rings (size, shape and etc.) and show that the spectrum of electron states in the 2D ring is qualitatively different from that of 1D - ring. The solution of this problem is also important to classical physics, since many forces such as gravity or a two-body problem with forces along the line connecting the two bodies can be reduced to a central-force problem.

2 QUANTUM RING STUCTURES

The two possible quantum ring structures are shown schematically in Fig. 1 where electrons are confined in all spatial dimensions. As a potential interaction of the electron with a quantum well we choose the following

$$V(\varphi)=\begin{cases}0, & 0\le\varphi\le\varphi_0\\ U_0, & \varphi_0\le\varphi\le 2\pi\end{cases},\qquad V(\varphi+2\pi)=V(\varphi) \tag{1}$$

For the construction of quantum states we impose the following boundary conditions:

$$\psi_I(0,r)=\psi_{II}(2\pi,r);\;\; \psi_I'(0,r)=\psi_{II}'(2\pi,r)\quad \forall r \tag{2a}$$

$$\psi_I(\varphi_0,r)=\psi_{II}(\varphi_0,r);\;\; \psi_I'(\varphi_0,r)=\psi_{II}'(\varphi_0,r)\quad \forall r, \tag{2b}$$

where the indices I and II numerates $(0,\varphi_0)$ and $(\varphi_0, 2\pi-\varphi_0)$ regions, respectively. For simplicity, we can ignore the possible difference of the effective masses in I and II regions. Moreover, we consider a rigid boundaries, i.e.,

$$\psi_I(\varphi,R)=\psi_{II}(\varphi,R)=\psi_I(\varphi,a)=\psi_{II}(\varphi,a)=0,\quad \forall\varphi. \tag{3}$$

Schrödinger equation of this problem is separable, but in general it is difficult to satisfy to the boundary conditions (2) and (3), so the adiabatic approximation is used to get solution. The wave function is searched in the form $\psi(r,\varphi)=\psi_1(r,\varphi)\psi_2(r)$, where $\psi_1(r,\varphi)$ satisfies the equation

$$-\frac{\hbar^2}{2mr^2}\frac{\partial^2\psi_1}{\partial\varphi^2}+V(\varphi)\psi_1=\varepsilon\psi_1 \qquad (4)$$

In quantum rings (Fig. 1), electrons are confined to move in one- or two-sector quantum wells separated by an angle φ_0 in space.

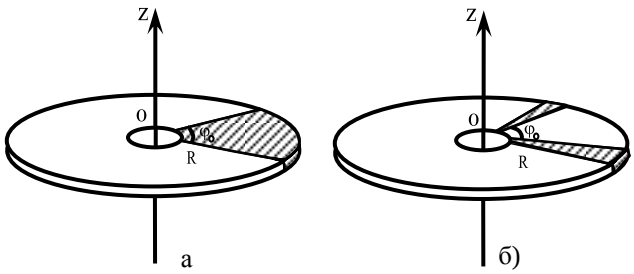

Figure 1: A schematic picture of the quantum ring with a) one and b) two sectored

After substitution into the Schrödinger equation and simple transformations, we obtain an equation for the wave function $\psi_2(r)$

$$-\frac{\hbar^2}{2m}\Delta_r\psi_2+\varepsilon(r)\psi_2=E\psi_2+\hat{C}\psi_2 \qquad (5)$$

where Δ_r - the radial part of the 2D, $\hat{C}$ - Laplacian operator of nonadiabaticity [6]. Further we can neglect term $\hat{C}\psi_2$, by considering the conditions under which it is possible. Thus, the system can be divided into a "fast" (azimuthal degree of freedom) and "slow" (radial) motions. Solving (4) with boundary conditions (2) the dispersion equation is reduced to

$$ch(\mu b)\cos(\lambda\varphi_0)+\frac{\mu^2-\lambda^2}{2\mu\lambda}sh(\mu b)\sin(\lambda\varphi_0)=1,$$

$$b=2\pi-\varphi_0 \qquad (6)$$

the solution of which coincides with the dispersion equation for the Kronig-Penney model with a single zero value of the Bloch wave number. Hence the behavior of azimuthal degree of freedom equivalent to the behavior of a particle in an infinite periodic potential, described as quadrupeds Bloch wave functions with the only difference: these states correspond to the center of the Brillouin zone. In (6) the following notations are introduced

$$\lambda=\frac{1}{\hbar}\sqrt{2m\varepsilon},\quad \mu=\frac{1}{\hbar}\sqrt{2m(U-\varepsilon)} \qquad (7)$$

Solution of (6) is chosen in the form $\varepsilon=U-\delta,\ \ \delta<<U$, i.e., looking for a weakly bound state. The result is

$$\varepsilon(r)=U-\frac{4\hbar^2}{m\varphi_0^2r^2}, \qquad (8)$$

which represents the effective potential field acting on the radial degree of freedom ("slow" subsystem). Then, solving the equation (5) with (8), for the internal (bound) states:

$$\psi_2(r)\sim\sin\left(\beta\ln\frac{r}{a}\right),$$

$$E=U-\frac{2\hbar^2}{ma^2}\exp\left[\frac{1}{\beta}\left(\frac{\pi}{2}+2\alpha-2\pi n\right)\right] \qquad (9)$$

$\alpha=\tilde{A}(1+i\beta),\ \beta=2^{\frac{3}{2}}/\varphi_0,\ \tilde{A}(z)$ - Gamma - Euler function - runs through the integer numbers.

For the above well (unbound) states we get

$$\psi_2(r)\sim sh\left(\beta\ln\frac{r}{a}\right),$$

$$E=U+\frac{\hbar^2}{2mR^2}\left(\xi+\beta\ln\frac{a}{2R}+\pi n\right)^2,$$

$$tg\xi=th\frac{\pi\beta}{2}. \qquad (10)$$

The expression $\frac{1}{R}<<k<<\frac{1}{a}$ for the wave functions in (9) and (10) is valid close to the inner edge of the ring, and, in addition, that in (8) and (8) give only thee part of the spectrum.

From comparison of (8) and (9) one can see that the quantum states inside and outside of the well are very different. Thus, the intra-well states oscillate and these oscillations are terminated at the inner boundary implies that there is a new ground state. The time dependent wave function does not oscillate out of well region, and the spectrum resembles the spectrum of the infinitely deep well with a shifted quantum number.

3 TWO SECTORIAL QUANTUM RINGS

Let us briefly consider a model with two-sector quantum wells separated by an angle φ_0. Because of the circular symmetry of the system we can write it in form

$$U(r,\varphi)=-\frac{UR}{r}\sum_{n=-\infty}^{+\infty}\left[\delta(\varphi-2\pi n)+\delta(\varphi-\varphi_0-2\pi n)\right], \qquad (11)$$

where U - the typical depth of the well, R - the outer radius of the quantum ring. The qualitative picture of the electron behavior in this system is the following: on a long distance from the inner edge of the electron is localized on

the azimuthal degree of freedom at the level ε_0 in a single δ potential well. By approaching the inner boundary the wells are getting closer and the tunnel level splitting ε_0 increase. The lower level continues to go down as one approaches to the inner boundary by reducing the tunneling length, i.e., $r\varphi_0$. Thus, this implies that the penetration of the electron into the inner boundary region is energetically favorable. On the other hand, the existence of the internal boundary suppresses the process of penetration, which results in a new ground state of the system.

As in the first problem here is used an adiabatic approximation to solve a problem. The states corresponding to the azimuthal degrees of freedom are constructed by taking into account the boundary conditions

$$\psi_1'(+0)-\psi_1'(-0)+\alpha\psi_1(0)=0,$$

$$\psi_1'(\varphi_0+0)-\psi_1'(\varphi_0-0)+\alpha\psi_1(\varphi_0)=0$$

$$\psi_1^{(I)}(0)=\psi_1^{(II)}(0);$$

$$\psi_1^{(I)}(\varphi_0)=\psi_1^{(II)}(\varphi_0),\quad \alpha=\frac{2mUR}{\hbar^2}. \qquad (12)$$

Then the effective potential of the field for the radial degree of freedom is

$$E_0(r)=\varepsilon_0-\frac{2\hbar^6}{m^3U^2\varphi_0^2r^4},$$

$$\varepsilon_0=\frac{2mU^2R^2}{\hbar^2}. \qquad (13)$$

The solution of the Schrödinger equation for the radial degree of freedom is given with the boundary condition, $\psi_2(a)=\psi_2(R)=0,$ leads to the following final energy spectrum of the system: there is one energy level that is below the localization energy ε_0

$$E=\varepsilon_0-\frac{\hbar^2}{8mR^2}\ln^2 ctg\xi,$$

$$\xi=\frac{\beta^{1/2}}{a}-\frac{\pi}{4},$$

$$\beta=\frac{4\hbar^4}{m^2U_0^2R^2\varphi_0^4}. \qquad (14)$$

The wave function corresponding to this energy level near the inner boundary has a form

$$\psi(r)\sim\sin\beta^{1/2}\left(\frac{1}{a}-\frac{1}{r}\right). \qquad (15)$$

$$E_n=\varepsilon_0+\frac{\hbar^2}{2mR^2}\left(\frac{\beta^{1/2}}{a}+\pi n\right)^2,\quad n=0,\pm1,... \qquad (16)$$

4 CONCLUSION

The above two problems can be combined as follows: the ground state of these two systems due to the two factors one is the competition of particle incident on the center, and strong repulsion on the inside edge of the ring. In both cases the condition for the applicability of the adiabatic approximation is $\varphi_0 << 2\pi$ and the shallower depth of the quantum well. In a problem with one well, this leads to the existence of a single level, which due to tunneling is splitted into two, one of which remains in the well. It is known that one attracting δ - well contains one level, then the adiabatic condition leads to a rather large tunnel splitting of this level, i.e., to the large value of the tunneling frequency - this is what makes the azimuthal degree of freedom "fast". As a result, the radial degree of freedom turns out to be under the influence of an attractive effective central potential, $U_{eff}\sim -r^{-\gamma}$ $(\gamma=2;4)$, which leads to the emergence of radial states close to falling to the center. In both calculated cases, the energy spectrum has the following features: the dispersion is not symmetric $\varepsilon(n)=\varepsilon(-n)$ (in the second problem it is not related to the ground state). This spectrum asymmetry may lead to the existence of persistent currents in the two-dimensional ring. In one-dimensional rings similar situation may occur only in the presence of the transverse magnetic field (flux) in the system. The physical properties of such quantum rings with central potential can be modeled and continuously tuned by changing the geometry (radiuses and angles), making them ideal candidates for applications in future electronic devices.

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