

# Transient Analysis on Nonlinear Model of Electrostatically Actuated Nanomechanical Switch

Il Kwang Kim, Dong Hee Han, and Soo Il Lee\*

University of Seoul, Jeonnong-dong, Dongdaemun-gu, Seoul, 130-743, Korea,  
leesoil@uos.ac.kr

## ABSTRACT

This research predicted the transient behaviors of a nano-switch. The nano-switch consists of a cantilevered carbon nanotube incorporating the electrostatic forces and intermolecular attractive and repulsive forces between the CNT and graphene ground plane. To investigate the transient pull-in/out behaviors with large deflection of the CNT and different excitation frequency, our model includes geometric and inertial nonlinearities of the CNT. To solve this problem, we used the Galerkin discretization method and the numerical time integration technique. As a result, we identified that the nonlinear effect of CNT affects the pull-in behaviors calculating near the pull-in voltages with linear and nonlinear model. The nonlinear model had the lower pull-in voltage than that of the linear model. But, there are no significant differences between the linear and nonlinear model on pull-out behaviors. The contact bounce prevented the perfect pull-in when the nano-switch was switched quickly.

**Keywords:** carbon nanotube, nano-switch, nonlinear modeling

## 1 INTRODUCTION

The carbon nanotube (CNT) based nano-devices take advantages of the excellent mechanical and electrical properties such as the low density, high aspect ratio and high conductivity. In nano-scale, the electrostatic forces [1] and intermolecular forces [2] play important roles in actuating nanoelectromechanical devices like a CNT based nano-switch. Dequesnes *et al.* [1, 3] showed the possibility for the nano-switch on the basis of experimental studies [4, 5]. They predicted the static pull-in voltage of nano-switch with linear [1], nonlinear beam theory and molecular dynamics [3]. Ke *et al.* suggested the On/Off nano-switch model calculating static pull-in/out voltages [6, 7] and considered the electrostatic charge distribution [8] and concentrations [9] effects on CNT. Raseckh and Khadem [10] studied the pull-in behaviors with the nonlinearities of nanocantilever beam according to the ratio of beam length and gap between the beam and electrode.

Those previous studies are focused on the static analysis in steady-state or only dynamic pull-in analysis. However, nano-switches which are operated by On/Off signal inputs have dynamic pull-in/out behaviors. The CNT of nano-

switch has different responses as applying DC voltage with linear and nonlinear beam theory. To design the stable On/Off switching in dynamic range is very important to predict the contact bounce [11] at pull-in/out with transient analysis. Our nano-switch model includes geometric and inertial nonlinearities of the CNT based on our previous study [12] to investigate transient pull-in/out behaviors with the large deflection of the CNT and excitation time period variations of pulse wave. In order to solve this problem, we used the Galerkin discretization method and the numerical time integration technique.

## 2 NANO-SWITCH MODELING

The schematic diagram of a pulse wave actuated nano-switch is shown in Fig. 1. The Fig. 1 shows the force equilibrium incorporating the elastic force of CNT and external forces such as electrostatic forces and intermolecular attractive and repulsive forces between the multi-walled CNT (MWCNT) and graphene ground plane. We assumed that the MWCNT is a non-hollow cylindrical beam and considered the intermolecular attractive (van der Waals) and repulsive forces based on Lennard-Jones potential model [2]. From the configuration of CNT switch, we derived the nonlinear equation of motion using Hamilton's principle. Then, we discretized the non-dimensionalized equation using one-mode approximation to simulate the nano-switch. The approximate deflection  $w(x, t)$  of the CNT is written as

$$w(x, t) = y_1(t)\phi_1(x) \quad (1)$$

where  $y_1(t)$  and  $\phi_1(x)$  are temporal and normalized spatial function of the corresponding the 1<sup>st</sup> mode of the cantilever beam, respectively. After using Eq. (1), we obtained the ordinary differential equation of the nano-switch in Eq. (2).

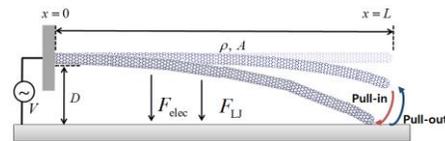


Figure 1: The electrostatically actuated nanomechanical switch.

$$\ddot{y}_1(t) + c\dot{y}_1(t) + \omega_1^2 y_1(t) + \eta_G \gamma_1 y_1(t)^3 + \eta_I \gamma_1 (y_1(t)^2 \ddot{y}_1(t) + y_1(t) \dot{y}_1(t)^2) \quad (2a)$$

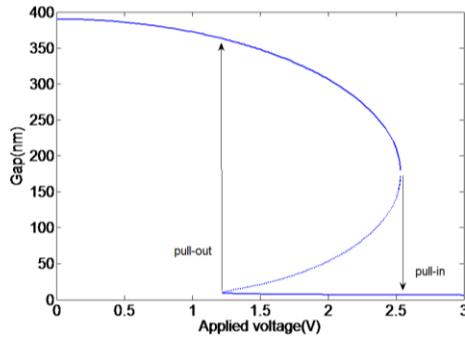
$$= F_{\text{elec}}(x, t) + F_{\text{LJ}}^{\text{attraction}}(x, t) + F_{\text{LJ}}^{\text{repulsion}}(x, t),$$

$$c = \frac{(\beta_1 L)^2}{Q}, \quad \omega_1 = (\beta_1 L)^2 = 3.516, \quad \gamma_1 = \left(\frac{D}{L}\right)^2, \quad (2b)$$

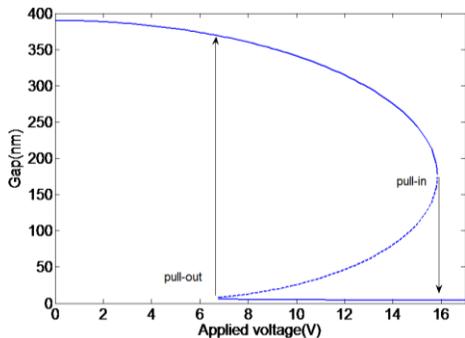
where  $\eta_G$  (40.441) and  $\eta_I$  (4.597) denote the coefficients of the geometric and inertial nonlinearity of the CNT, respectively.

### 3 SIMULATION RESULTS

To predict the behaviors of the nano-switch, we calculated the static pull-in/out voltages using the minimum potential energy profiles and obtained the dynamic pull-in voltages with linear and nonlinear beam theory by MATLAB. We set two cases in different ratio of the CNT length to initial gap to find the nonlinearity influenced on the behaviors (Case 1:  $L = 2500$  nm, Case 2:  $L = 1000$  nm with  $R = 5.45$  nm,  $D = 390$  nm, and  $Q = 150$ ). The static pull-in/out voltages of each case are shown as Fig. 2.



(a) Case 1:  $V_{\text{spi}} = 2.52$  V,  $V_{\text{spo}} = 1.22$  V



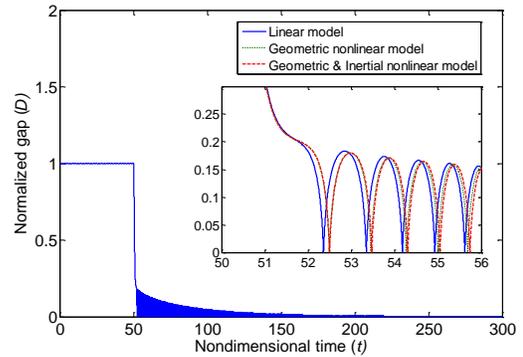
(b) Case 2:  $V_{\text{spi}} = 15.85$  V,  $V_{\text{spo}} = 6.75$  V

Figure 2: The static pull-in/out voltages of the Case 1 and Case 2.

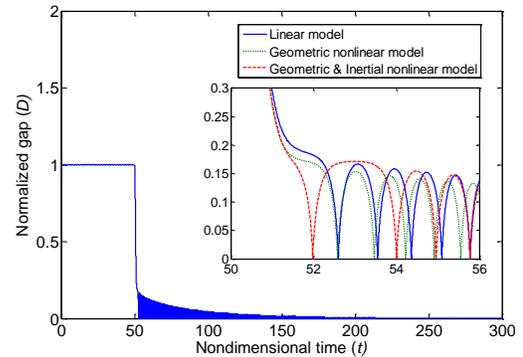
Case	Linear model	Geometric & Inertial nonlinear model
1	2.277	2.284
2	14.755	15.070

Table 1: Dynamic pull-in voltages (V).

Then, we identified that the nonlinear effect of CNT affects the pull-in behaviors calculating near the pull-in voltages with linear and nonlinear model of nano-switch as shown in Table 1 and Fig. 3. The nonlinear model had the lower pull-in voltages than those of the linear model. In case of Case 1, the dynamic pull-in voltage of our nonlinear model is similar to the previous theoretical studies [3, 6] and experimental study [5]. There are no significant differences between the linear and nonlinear model. But, in case of Case 2, the nonlinearities of CNT affect the dynamic pull-in voltage and behavior. Especially, the nonlinear model of Case 2 showed the retardation at pull-in with contact bounce. If the gap-length ratio has more than 0.3, the nonlinearity effect of CNT can no longer be ignored [10]. The gap-length ratios of Case 1 and Case 2 are 0.156 and 0.390, respectively. These results show that the nonlinear beam theory in nano-switch with the large deflection should be considered.

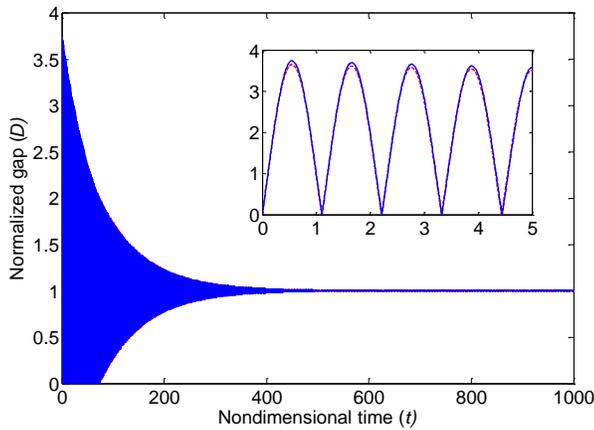


(a) Case 1

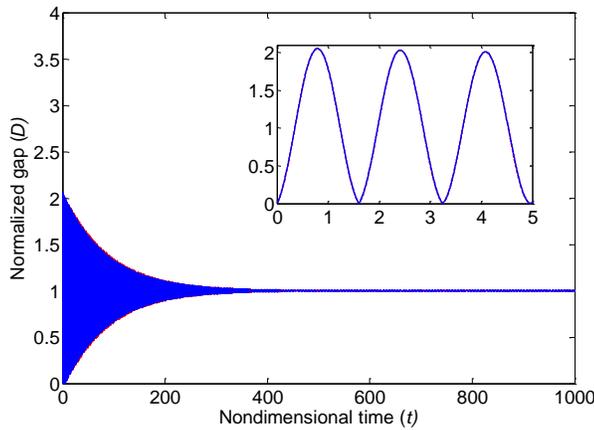


(b) Case 2

Figure 3: Pull-in behaviors at sudden input signal with dynamic pull-in voltages



(a) Case 1

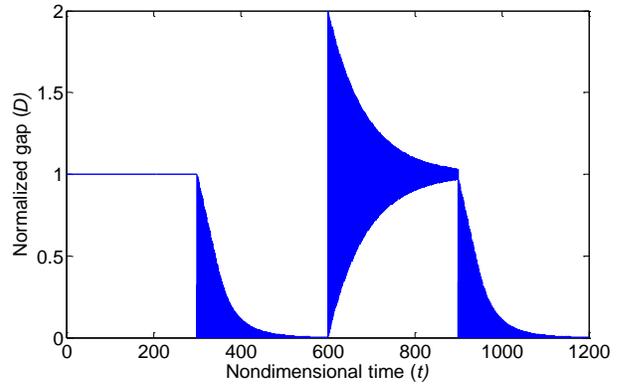


(b) Case 2

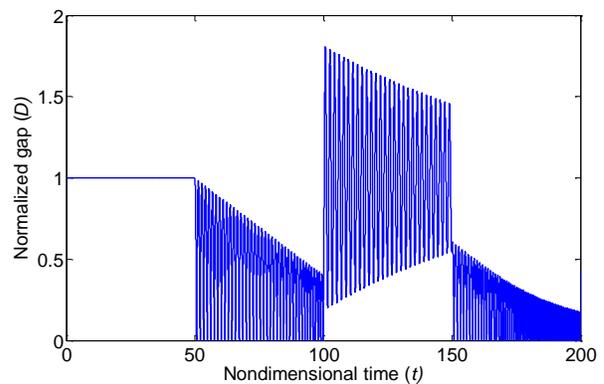
Figure 4: Pull-out behaviors at voltage 0 V after pull-in state. The blue solid and red dashed line denote the time history of linear and nonlinear model.

Figure 4 shows that there are no big differences between the linear and nonlinear model on pull-out behaviors at sudden Off signal after pull-in state. The Case 1 of nano-switch had the temporarily contact bounce with initial large amplitude which has over 3.5 times the gap even in pull-out state. This phenomenon shows that the nano-switch has an example of the unstable or unexpected response.

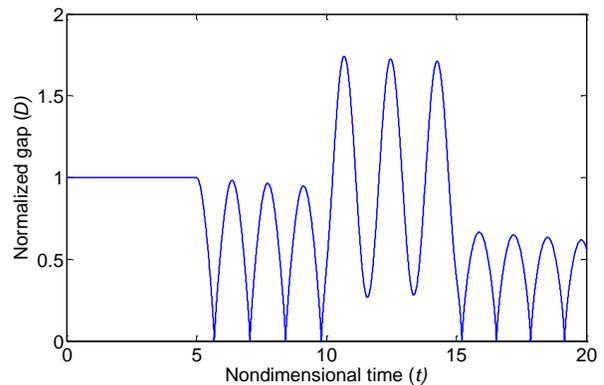
After pull-in and out analysis, we simulated the dynamic On/Off nano-switch actuated by pulse wave with 50% duty cycle. Figure 5 shows the On/Off behavior of nonlinear model of Case 1 with the different time periods of the input pulse wave (pull-in voltage = 3 V and pull-out voltage = 0 V). In the Fig. 5, the contact bounce prevented the perfect pull-in when the nano-switch was switched quickly. And, on the pull-out behaviors, the oscillating amplitude of CNT had twice as much as the gap between the CNT and the electrode.



(a) Nondimensional time period: 600



(b) Nondimensional time period: 100



(c) Nondimensional time period: 10

Figure 5: Transient behaviors of nonlinear model of Case 1 actuated by On/Off signal inputs with pull-in voltage 3 V, pull-out voltage 0 V and different periods.

## 4 CONCLUSIONS

We predicted the transient behaviors of a nano-switch with static and dynamic pull-in/out analysis. The nano-switch consists of a cantilevered MWCNT incorporating

the electrostatic forces and intermolecular attractive (van der Waals) and repulsive forces from Lennard-Jones potential model between the CNT and graphene ground plane. The CNT is bent differently in oscillating to the electrode as applying DC voltage with linear and nonlinear beam model. To design the stable On/Off switching in dynamic range is very important to predict the contact bounce at pull-in/out with transient analysis.

As a result, we obtained the static pull-in/out voltages using the minimum potential energy profiles. Then, we identified that the nonlinear effect of CNT affects the pull-in behaviors calculating near the dynamic pull-in voltages with linear and nonlinear model of nano-switch. The nonlinear model had the lower dynamic pull-in voltage than that of the linear model. But, there are no significant differences between the linear and nonlinear model on pull-out behaviors. The contact bounce prevented the perfect pull-in when the nano-switch was switched quickly as a dynamic On/Off switch. And, on the pull-out behaviors, the oscillating amplitude of CNT had twice as much as the gap between the CNT and the electrode. This shows that the packaging margin in nano-switch device would be needed more than twice of the gap. This study helps to understand the transient behaviors as well as to design the robust nano-switch.

## ACKNOWLEDGEMENTS

This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (MEST). (NRF-2012-0002982)

## REFERENCES

- [1] M. Dequesnes, S. V. Rotkin and N. R. Aluru, "Calculation of Pull-In Voltages for Carbon-Nanotube-Based Nanoelectromechanical Switches," *Nanotechnology*, 13, 120, 2002.
- [2] J. Israelachvili, "Intermolecular and Surface Forces," Academic Press, 155, 1992.
- [3] M. Dequesnes, Z. Tang and N. R. Aluru, "Static and Dynamic Analysis of Carbon Nanotube-Based Switches," *Journal of Engineering Materials and Technology*, 126, 230, 2004.
- [4] P. Kim and C. M. Lieber, "Nanotube Nanotweezers," *Science*, 286, 2148, 1999.
- [5] S. Akita, Y. Nakayama, S. Mizooka, Y. Takano, T. Okawa, Y. Miyatake, S. Yamanaka, M. Tsuji, and T. Nosaka, "Nanotweezers Consisting of Carbon Nanotubes Operating in an Atomic Force Microscope," *Applied Physics Letters*, 79, 1691, 2001.
- [6] C. H. Ke, N. Pugno, B. Peng and H. D. Espinosa, "Experiments and Modeling of Carbon Nanotube-Based NEMS Devices," *Journal of Mechanics and Physics of Solids*, 53, 1314, 2005.
- [7] C. H. Ke and H. D. Espinosa, "Feedback Controlled Nanocantilever Device," *Applied Physics Letters*, 85, 681, 2004.
- [8] C. H. Ke and H. D. Espinosa, "Numerical Analysis of Nanotube-Based NEMS Devices—Part I: Electrostatic Charge Distribution on Multiwalled Nanotubes," *ASME Journal of Applied Mechanics*, 72, 721, 2005.
- [9] C. H. Ke, H. D. Espinosa and N. Pugno, "Numerical Analysis of Nanotube-Based NEMS Devices—Part II: Role of Finite Kinematics, Stretching and Charge Concentrations," *ASME Journal of Applied Mechanics*, 72, 726, 2005.
- [10] M. Rasekh and S. E. Khadem, "Pull-in Analysis of an Electrostatically Actuated Nano-Cantilever Beam with Nonlinearity in Curvature and Inertia," *International Journal of Mechanical Sciences*, 53, 108, 2011.
- [11] B. McCarthy, G. G. Adams, N. E. McGruer and D. Potter, "A Dynamic Model, Including Contact Bounce of an Electrostatically Actuated Microswitch," *Journal of Microelectromechanical Systems*, 11, 276, 2002.
- [12] I. K. Kim and S. I. Lee, "Nonlinear Dynamics Response of Cantilevered Carbon Nanotube Resonator by Electrostatic Excitation," *Transactions of the Korean Society for Noise and Vibration Engineering*, 21, 81, 2011.