Integration of memristors with MEMS in different circuit configurations

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ABSTRACT

The combination of MEMS and memristor is investigated analytically. This work simulates and analyzes the charge transfer interaction of basic circuits combining a MEMS parallel plate capacitor with a bipolar memristor in series and parallel circuit configurations. In both circuit configurations it is possible to observe that the displacement of the MEMS can be a function of the memresistance at low frequencies expressing the MEMS top electrode position in form of resistance instead of capacitance; however at high frequencies a pinched hysteresis loop between the memresistance and the displacement of the MEMS is shown.

***Keywords** Terms*—memristor, micro-electrical-mechanical systems, MEMS parallel plate capacitor, resistive switching.

1. INTRODUCTION

Leon Chua in 1971 proposed for the first time the memristor as the fourth passive circuit element theoretically, this element associates the flux (φ) and the charge (q)(1). Recently researchers from Hewlett-Packard (HP) linked the resistive switching in metal/metal-oxide/metal (MOM) structures to memristor behavior proposed by Chua (2). Since then, MOM structures have received much attention due to their potential application for high density non-volatile memories(3). Furthermore many applications beside non-volatile memories have been proposed including; memristors in chaotic circuits(4), amoeba's learning(5), neural synaptic emulation(6), reprogrammable and reconfigurable circuits(7), and more recently the integration of memristors with micro-electrical-mechanical systems (MEMS)(8). Interestingly, charge transfer is an operating principle that is common to both memristors and certain kinds of MEMS devices such as parallel plate capacitors. More specifically, electronic charge transfer plays a central role in the capacitive operating principle of MEMS capacitors. Similarly, memristors use the migration of ions to change their resistance in response to charge flow. However, unlike MEMS capacitors, memristors also demonstrate non-volatile memory of the amount of charge transferred. This ability makes the integration of memristors with MEMS capacitors highly intriguing to create capacitors with local memory storage in what can be called mems-capacitors.

While the general concept of integrating memristors with MEMS was introduced in Ref(8), the actual details of the integration are lacking. This paper investigates the combination of a memristor with a MEMS parallel plate capacitor and begins by presenting models for a MEMS parallel plate capacitor and for a memristor. The analysis consists of both series and parallel circuit configurations in order to explore the MEMS mechanical movement/capacitance variance as well as the memristor behavior.

2. MEMRISTOR MODEL

There are basically two types of resistive switching: bipolar which requires positive and negative voltage to switch between high and low resistance stateand unipolar which only requires two different levels of positive voltage to change the device resistance(9). The bipolar resistive switching behavior obserbed in TiO_2 structures (10) was linked to the Leo Chua's memristor device (1) by Hewlett-Packard (HP). However since 1962 a large variety of MOM structures have shown either unipolar (11)or bipolar resistive switching (3).

The basic mathematical model for a voltage controlled memristor proposed by Chua is the following:

$$v = R(w,i)i \tag{1}$$

$$\frac{dw}{dt} = f(w,i) \tag{2}$$

Where *R* is the memresistance which is a function of the state variable *w* in the model presented by HP. *w* represents the doped portion of the total thickness according to the HP model and its derivative can be a function of current. According to the HP model, the voltage applied to the memristor will move *w*- doping and undoping the oxide film (TiO_2), depending on the voltage polarity. The following equations describes this phenomenon

$$v(t) = \left(R_{on}\frac{w(t)}{D} + R_{off}\left(1 - \frac{w(t)}{D}\right)\right)\mathrm{i(t)} \tag{3}$$

$$\frac{dw(t)}{dt} = \mu_V \frac{\mathrm{R_{on}}}{\mathrm{D}}\mathrm{i(t)} \tag{4}$$

Where D is the total oxide thickness, μ_v is the average ion mobility, R_{on} is the low resistance (LR), and R_{off} is the high resistance (HR). From these equations it is possible to observe that if $w = D$ the memristor is in low resistance state (LRS) and if $w = 0$ the memristor is in high resistance state (HRS). Figure 1 shows a graphic representation of these equations where V is a voltmeter and A is an ammeter.

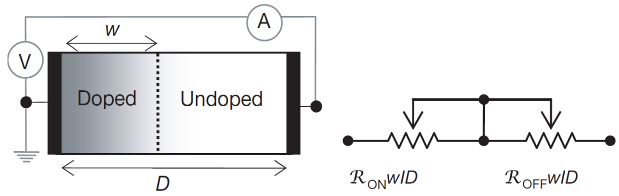

Figure 1. Graphic representation for the equations 3 and 4. Reprinted by permission from Macmillan Publishers Ltd: Nature Ref (10), copyright (2008)

3. PARALLEL PLATE MEMS

In this work the dynamic model for the MEMS parallel plate capacitor depicted in Figure 2 to make circuit configurations. Moreover, it is important to tune the MEMS behavior to the memristor characteristics so that the devices can achieve adequate coupling in their respective circuit configuration.

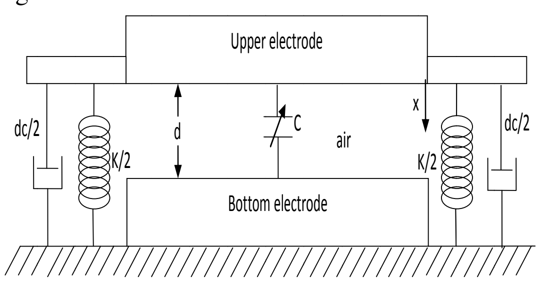

Figure 2. MEMS parallel plate capacitor structure

In this model, the bottom electrode is fixed while the top electrode is able to move in response the forces acting on it. Therefore all the forces acting in the top electrode are considered in the following equation which represents the top plate motion through time:

$$m\frac{d^2x_1}{dt^2}+d_c\frac{dx_1}{dt}+kx_1=\frac{\varepsilon_0 A V_{MEMS}^2}{2(d-x_1)^2} \tag{5}$$

Equation 5 represents the dynamics of the MEMS where: k is the spring constant, dc is the damping constant, d is the equilibrium distance between plates, x_1 is the displacement of the upper electrode, $\varepsilon_0$is the vacuum permittivity, A is the area of the electrodes (assuming both electrodes with the same area), m is the mass of the top electrode, and V_{MEMS} is the voltage applied to the electrodes.

4. MEMS AND MEMRISTOR IN A SERIES CIRCUIT

Figure 3 shows a circuit configuration of a MEMS connected in series with a memristor, where the MEMS is represented by a variable capacitor.

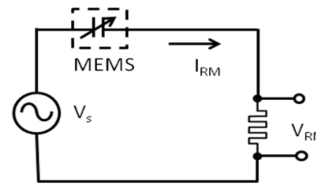

Figure 3. MEMS and Memristor series circuit

From the series circuit in Figure 3, the following equations are established using Kirchhoff's voltage and current laws:

$$Vs = V_{MEMS} + V_{RM} \tag{6}$$

$$I_{MEMS} = I_{RM} \tag{7}$$

The current that crosses through the MEMS parallel plate capacitor is given by:

$$I_{MEMS} = \frac{d(CV_{MEMS})}{dt} \tag{8}$$

However, the MEMS device has a capacitance which is able to change with time since it is a function of the displacement between the bottom and top electrodes. In turn, the top electrode changes over time because it is a function of the voltage applied to the electrodes, according to equation 5. The current through the MEMS capacitor, I_{MEMS}, is therefore given as follows:

$$I_{MEMS} = C\frac{dV_{MEMS}}{dt}+V_{MEMS}\frac{dC}{dt} \tag{9}$$

The capacitance and the derivative of the capacitance are given by:

$$C=\frac{\varepsilon_0 A}{d-x_1} \tag{10}$$

$$\frac{dC}{dt}=\left(\frac{\varepsilon_0 A}{(d-x_1)^2}\right)\frac{dx_1}{dt} \tag{11}$$

Combining equations 3 to 11 and making $x_2=\frac{dx_1}{dt}$ (x_2 is the velocity of the upper electrode) in order to lower the order of equation 5 from second order to first order, the following set of equations are obtained:

$$\frac{dx_1}{dt}=x_2 \tag{12}$$

$$\frac{dx_2}{dt}=\frac{\varepsilon_0 A V_{MEMS}^2}{2m(d-x_1)^2}-\frac{d_c}{m}x_2-\frac{k}{m}x_1 \tag{13}$$

$$\frac{dV_{MEMS}}{dt}=\frac{(Vs-V_{MEMS})(d-x_1)}{\varepsilon_0 A\left(R_{on}\frac{w(t)}{D}+R_{of}f\left(1-\frac{w(t)}{D}\right)\right)}-\frac{V_{MEMS}x_2}{(d-x_1)} \tag{14}$$

$$\frac{dw(t)}{dt}=Ron\mu_V\frac{(Vs-V_{MEMS})}{D\left(Ron\frac{w(t)}{D}+Roff\left(1-\frac{w(t)}{D}\right)\right)} \tag{15}$$

Since the circuit is in a series configuration, V_{RM} and V_{MEMS} are unknown; however equation 14 can be obtained by combining equations 3, 9, 10, and 11. Finally equation 15 can be obtained from equation 3, 4, and 6. In this way there are four equations and four unknowns.

These equations were solved simultaneously using the function ode23s from MATLAB. Where V_S is a sinusoidal input voltage given by $V_S = \sqrt{8kd^3/27A\varepsilon_0}\sin(\omega t)$. The maximum amplitude of the input voltage was set to the MEMS pull-in voltage in order to avoid the unstable situation where the top electrode moves continuously towards the bottom electrode until making contact with it.

In this case the parameters for the parallel plate capacitor were obtained from devices characterized at Universidad Autonoma de Ciudad Juarez: ε_0 = 8.854x10^{-12} F/m, d = 5x10^{-6} m, k = 0.3125 N/m, A = (300x10^{-6})^2 m^2, dc = 5x10^{-5}, ρ = 2329 kg/m^3, m = ρ *A *2x10^{-6} Kg, and the memristor parameters are R_{on} = 100 Ω, R_{off} = 10000 Ω, μ_v = 1.5x10^{-10} m^2/(Vs), and D = 0.02x10^{-4} m. The choice of parameters is important because they describe the behavior of the devices individually and how well they couple with each other in a circuit. Also the following initial conditions were used; $x_1(0) = 0$, $x_2(0) = 0$, which means that the upper MEMS electrode is initially stationary at its equilibrium position, the initial voltage on the memristor is zero, $V_{MEMS}(0) = 0$, and the boundary between the un-doped and doped regions in the memristor is initially at the center of the device, w(0) = D/2. Using Eqn (3), the memresistance is given by $R_{MR}(t) = R_{on}w(t)/D + R_{off}(1 - w(t)/D)$. Using the device parameters and initial conditions, the initial memresistance is $R_{MR}(0)$ = 5,050 Ohms.

Results are shown in Figure 4, where the memresistance R_{MR} and the displacement x_1 of the MEMS upper electrode are analyzed as a function of time and with respect to each other at two discrete frequencies; f = 4 Hz and f = 4,000 Hz. At f = 4 Hz, it is possible to observe in Figure 4(a) that the frequency of the displacement is twice the memresistance frequency and that they are in phase. The difference in the frequency is due to the V_{MEMS}^2 dependence of the displacement on the voltage which has a rectifying effect. In contrast, R_{MR} has a linear dependence on the voltage where a positive voltage will deplete the oxide in the metal-oxide and reduce R_{MR} while a negative voltage will oxidize the memristor and increase the resistance. This results in a quadratic form for the displacement-memresistance curve shown in Figure 4(c).

At f = 4,000 Hz the fluctuation of the upper electrode displacement is much smaller compared to the fluctuation at lower frequency (f = 4 Hz) as shown in Figure 4(b). This is due to the MEMS physical properties that affect the electro-mechanical reaction time of the upper electrode. Moreover, a phase shift is observed between the displacement and memresistance. The phase shift creates a pinched hysteresis loop in the displacement-memresistance curve (Figure 4(c)). In either case the change in the memresistance is small due to the circuit configuration and the memristor behavior.

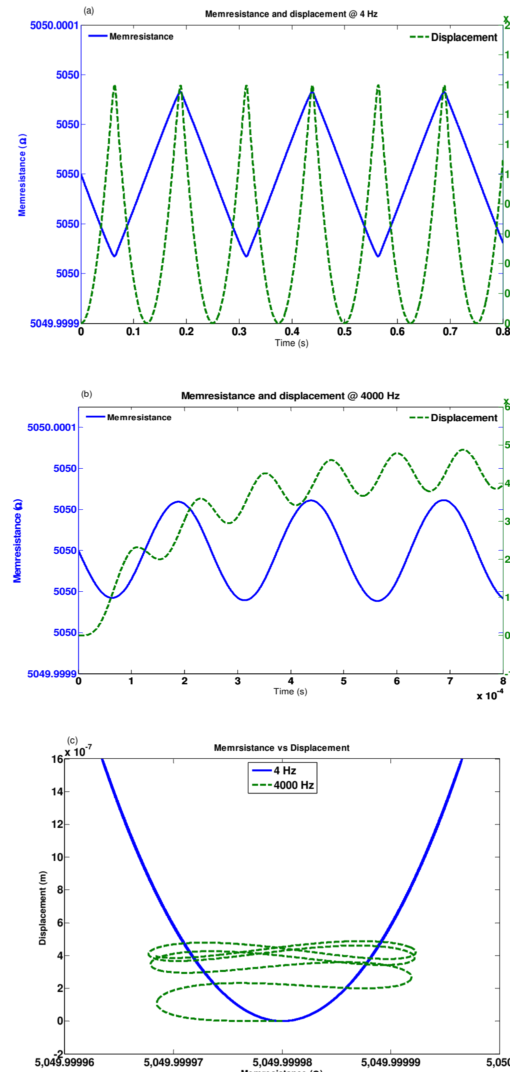

Figure 4. Simulation results for MEMS and Memristor series circuit. (a) Memresistance vs MEMS top contact displacement at 4 Hz, (b) Memresistance and MEMS displacement through time at 4 Hz, and (c) Memresistance versus MEMS displacement at 4 Hz and

At low frequencies the MEMS capacitor behaves as an open circuit and the voltage seen by the memristor is very small. On other hand at high frequencies the MEMS capacitor behaves more like a short circuit however due to the low mobility μ_v the memristor is not able to respond to high frequencies. In other to overcome the small resistance change, the amplification of either the memristor voltage or the circuit current is suggested.

5. MEMS AND BIPOLAR MEMRISTOR IN A PARALLEL CIRCUIT

Similar to the series circuit, now a parallel circuit of MEMS capacitor with a memristor will be analyzed as shown in Figure 5.

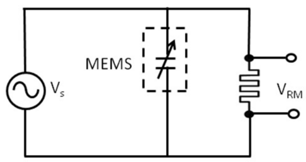

Figure 5. MEMS and Memristor parallel configuration circuit.

Performing a similar analysis as in the series circuit the following equations are derived:

$$\frac{dx_1}{dt} = x_2 \tag{16}$$

$$\frac{dx_2}{dt} = \frac{\varepsilon_0 A V_S^2}{2m(d - x_1)^2} - \frac{d_c x_2}{m} - \frac{k x_1}{m} \tag{17}$$

$$\frac{dw(t)}{dt} = \frac{R_{on}\mu_V}{D\left(R_{on}\frac{w(t)}{D} + R_{off}\left(1 - \frac{w(t)}{D}\right)\right)} Vs \tag{18}$$

Using the same values for all the parameters as in the serial circuit, these equations were solved with the ode23s from MATLAB. For this case the initial conditions were as follows: $x_1(0) = 0, x_2(0) = 0,$ and $R_{MR}(0) = R_{off} = 10000$ Ohms. Figure 6 shows the memresistance versus the MEMS displacement. At low frequencies the displacement is a function of the memresistance similar to the series circuit, however in this case the change in the memresistance is ~200 Ohms. In the other hand the inset in Figure 6 (which is a magnification of the dotted line) shows hysteresis loop at high frequencies with a memresistance fluctuation of ~2 Ohms. Thus the memresistance in this configuration shows a bigger change than the series configuration.

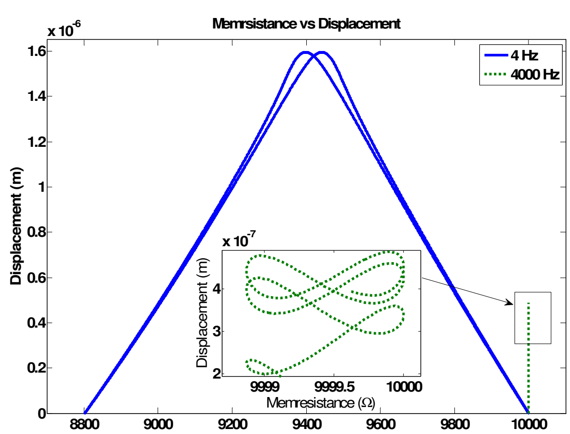

Figure 6. Memresistance vs MEMS top contact displacement at 4 Hz and 4000 Hz

6. CONCLUSIONS

The coupling of a memristor with a MEMS capacitor in series and parallel circuits was analyzed in this paper. At low frequencies the memresistance can be used to sense the position of the MEMS upper electrode for both circuit configurations. However the resistance change in the memristor is much larger in the parallel case than the series case, therefore the circuit current or the memristor voltage for the series circuit need to be amplified in order to obtain a bigger change in the memresistance. At high frequencies a hysteresis loop is observed between the memresistance and the MEMS displacement for both circuit configurations, however the change in the memresistance is small due to the small ion mobility in the memristor.

7. ACKNOWLEDGEMENTS

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