Parametric Study on the Effect of Radiation Heat Loss and Non-Constant Heating in the Electrothermal Technique for Thermal Property Measurement of Micro/Nanoscale Fibers

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ABSTRACT

This paper presents a parametric study of the effects of radiation heat loss and non-constant heating power on the measurement accuracy of thermal properties of micro/nanoscale fibers using the Transient Electrothermal (TET) technique. Measurement deviation was found to have linear relationships to both the radiation loss and the non-constant heat generation. For practical use, an experimental designer/user can estimate the error associated with these sources from the resulting diagrams. The findings of this study are beneficial for the accurate measurement of thermal properties of micro/nanoscale filament-like samples, including electrical conductors and nonconductors. These results are especially important in the current development of synthetic spider silk for quantifying and improving its thermal properties.

Keywords: electrothermal technique, fiber measurement, thermal conductivity, thermal diffusivity

1 INTRODUCTION

The general thermal property measurement can be conducted either in a steady state or transient manner depending on the sample geometry and constitution. For the thin disk shape sample, laser flash [1] based method can be used to extract the diffusivity. For the wire shape material with a diameter in the order of micron to nanometer scale, the conventional techniques are difficult to employ.

The Transient Electrothermal method was developed by a group of researchers [2-3] to measure thermal conductivity and diffusivity of carbon nanotubes and natural spider silks. In this technique, a fiber-like sample is suspended between two heat sinks. A transient temperature profile is induced in the sample through constant, DC electrical heating of the sample (or its coating). The transient and the final steady-state average temperatures of the sample is recorded, which allows extraction of thermal diffusivity and thermal conductivity, respectively. In the original derivation of the TET theory, assumptions were made to neglect radiation heat loss and non-constant heating power caused by electrical resistivity change, to simplify the solution, resulting in a reduced model. However, these two effects were found to become significant as the sample diameter is reduced.

The objective of this research is to establish criteria for the significance of radiation heat loss and non-constant heating effects for applicability of the reduced model. Experimentally, the range of variability of the parameters associated with these effects is limited, making it difficult to render clear trends or criteria. In the study, numerical "experiments" using the finite element software, COMSOL, were used to yield analysis data while parametrically varying material properties, geometries, and working conditions. The reduced and our full models were then used to extract thermal conductivity and diffusivity from the numerical data.

2 THEORETICAL MODEL FOR THE TET

The full governing equation for the 1D (large aspect ratio, L/D where L is length [m] and D is diameter [m]) heat transfer problem in the TET method is written as

$$\frac{1}{\alpha}\frac{\partial \Delta T}{\partial t} = \frac{\partial^2 \Delta T}{\partial x^2} - H_e{}^2 \Delta T + \frac{q_0'''}{k} \qquad (1)$$

where $H_e{}^2 = \left[(h_r + h_c)A_l - I^2 R'\right] / V_s k$; t is time [s]; α is thermal diffusivity of the sample [m^2/s], $\alpha = k/(\rho c_p)$ (ρ is the density [kg/m^3] and c_p is the heat capacity [J/kg/K]); ΔT is the temperature rise above the environment T_0 [K]; V_s is the volume equal to $\pi D^2 L/4$ for a cylinder sample; k is the thermal conductivity [W/m/K]; h_r is the linearized radiation heat transfer coefficient equal to $4T_0^3 \varepsilon\sigma$ with ε representing emissivity and σ as the Stefan-Boltzmann constant ($\sigma = 5.67\text{x}10^{-8}$ W/m^2/K^4); h_c is the convective heat transfer coefficient for imperfect vacuum considerations [W/m^2/K]; A_l is the lateral surface area [m^2] equal to πDL; I is the current passed through the sample [A] and R' is the slope of the temperature/resistance relation [Ω/K], $R' = \alpha_T R_0$ (α_T is the temperature coefficient of resistivity [1/K] and R_0 [Ω]is the resistance measured at a current I_0 [A] small enough to avoid Joule heating.); q_0''' is the volumetric heat generation where $q_0''' = I^2 R_0 / V_s$ (R_0 can also be calculated by $R_0 = 4\rho_e L / \pi D^2$ where ρ_e is the electrical resistivity [Ωm]).

By solving Eq. (1), the thermal conductivity is implicitly expressed as

$$1 - 2\frac{\cosh(LH_e) - 1}{LH_e \sinh(LH_e)} = \frac{\pi D^2 L k H_e{}^2 (R_1 - R_0)}{4 I^2 R_0 R'} \qquad (2)$$

where I [A] and R_1[Ω] are the working current to induce Joule heating and steady state resistance, respectively.

If $LH_e \approx 0$, Eq. (2) becomes the reduced form obtained by previous researches.

$$k = \frac{I^2 R_0 R' L}{3\pi D^2 (R_1 - R_0)} \tag{3}$$

Thermal diffusivity is obtained by fitting the normalized temperature rise of Eq. (1).

$$\frac{\overline{\Delta T}}{\overline{\Delta T_\infty}} = \frac{\Delta V}{\Delta V_\infty} = 1 - \frac{8(LH_e)^2}{1 - 2\dfrac{\cosh(LH_e) - 1}{LH_e \sinh(LH_e)}} \sum_{n=1}^{\infty} \frac{e^{-\left[(2m-1)^2\pi^2 + (LH_e)^2\right]\alpha t / L^2}}{\left[(2m-1)\pi\right]^4 + \left[LH_e(2m-1)\pi\right]^2} \tag{4}$$

Similarly, if $LH_e \approx 0$, Eq. (4) turns to be the reduced model.

$$\frac{\Delta V}{\Delta V_\infty} = 1 - \frac{96}{\pi^4} \sum_{n=1}^{\infty} \frac{e^{-(2m-1)^2\pi^2\alpha t / L^2}}{(2m-1)^4} \tag{5}$$

3 NUMERICAL EXPERIMENT

Three materials were selected for the radiation heat loss and non-constant heating evaluations: carbon fiber (α=3.38x10^{-6} m^2/s, k=6.83 Wm^{-1}K^{-1}, ε=0.98), platinum (α=2.51x10^{-5} m^2/s, k=71.6 Wm^{-1}K^{-1}, ε=0.05) and spider silk (α=1.23x10^{-4} m^2/s, k=415.9 Wm^{-1}K^{-1}, ε=0.5). The length and diameter were varied for parametric studies.

The finite element software, COMSOL was employed to generate data. To separate the two effects, when radiation effect was evaluated, the current passing through the sample was adjusted so that the average temperautre rise is around 0.1K, thus the variable heat generation is minimized; when non-constant heating was investigated, the radiation term was switched off in the software. Data generated by COMSOL were saved for the full and reduced model evaluation.

4 RESULTS

Figure 1 presents the percentage error of measured thermal conductivity with respect to a dimensionless variable ($\lambda = 4\varepsilon\sigma T_0^3 L^2 D^{-1} k^{-1}$), which represents the ratio of radiation heat loss to conduction heat transfer. The percentage error of diffusivity has the same trend as that of conductivity. By changing geometry and materials, all of the parameters in λ were considered. If the reduced model was used, the percentage error increased linearly along the same line with the increase of λ, regardless of what material or geometry was used. With regards to sample preparation, the radiation heat loss can be reduced by reducing the length-to-diameter aspect ratio, but care must be exercised so that the 1D assumption is not violated. This also indicates that the radiation effect is more sensitive to the length than the diameter. The results shown in Figures 1 also demonstrate that if a sample has large emissivity and low thermal conductivity, the radiation heat loss is significant.

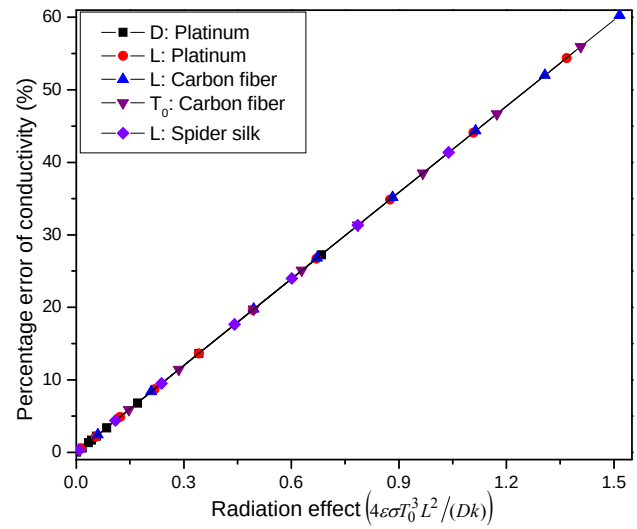

Figure 1 Percentage error with respect to radiation influence

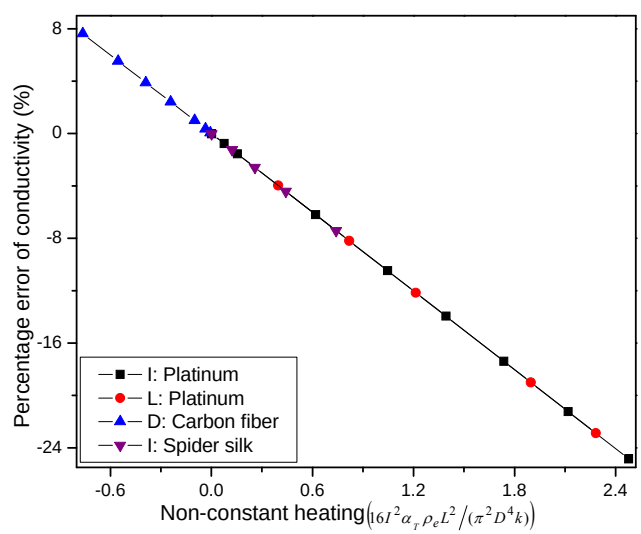

Figure 2 Percentage error with respect to non-constant heating influence

Figure 2 presents the percentage error of thermal conductivity with respect to a dimensionless variable γ ($\gamma = 16I^2\alpha_T\rho_e L^2\pi^{-2}D^{-4}k^{-1}$), which represents the ratio of

incremental Joule heating to conduction heat transfer. If the sample has a positive α_T, the resistance increases with temperature causing an increase of heat generation. As a result, the steady state temperature rises and the required time to steady state is larger (Eqs. 2 and 4). Reduced model under-predicts the heat input then the measured thermal conductivity and diffusivity values, causing negative errors. If the sample has a negative α_T (such as carbon fiber), positive error is generated.

If the reduced model is used, the magnitude of the percentage errors for both conductivity and diffusivity increase linearly with an increase of γ (conductivity was used in the plot), meaning that the value is increasingly under-predicted. Similar to the radiation effect, a larger thermal conductivity material causes lower error while keeping other parameters the same; alternatively, the percent error is more sensitive to diameter D than length L with respect to the non-constant heating effect, which is the opposite of the behavior of radiation heat loss considerations.

5 CONCLUSION

Parametric studies were performed on the radiation and non-constant heating condition. A linear percentage deviation was found with respect to dimensionless radiation ($\lambda=4\varepsilon\sigma T_o^3L^2D^{-2}k^{-1}$) and non-constant heating ($\gamma=16I^2\alpha_T\rho_eL^2\pi^{-2}D^{-4}k^{-1}$) effects if the reduced model is adopted. These relationships show that the reduced model can be applied with little error at sufficiently low λ and γ values. If λ and γ are large, full model should be employed to minimize the measurement bias error.

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