Boundary Condition Independence of Cauer RC Ladder Compact Thermal Models

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ABSTRACT

This paper presents an experimental study aimed at the investigation of the influence of nonlinearities in the heat equation on the element values of compact thermal models generated in the form Cauer RC ladders. Particular attention is paid to the analysis of the impact of variable boundary conditions. The considerations are illustrated based on the experimental results obtained for discrete silicon carbide power devices.

Keywords: compact RC ladder thermal models, structure functions, time constant spectra

1 INTRODUCTION

Compact thermal modeling of electronic systems has become one of the most important fields of research over the last two decades. Compact Thermal Models (CTMs) are approximate representations which preserve only basic characteristic features of a detailed distributed model. The precise definition of CTMs does not exist, but one can say that CTMs contain a limited number of components which is considerably lower than in the original distributed ones. CTMs of electronic systems can be derived adopting the structural or the behavioral approach. The first one consists in the classic reduction of the system of equations resulting from the detailed distributed model whereas in the latter approach the model is fitted by some optimisation routine to the observed behavior of an entire system [1]-[3].

The structural approach was adopted in the DELPHI and PROFIT research projects which were aimed at the development of standard methodologies to generate static and dynamic CTMs. Both these projects contributed to the elaboration of new guidelines for the generation of CTMs which were included in the JEDEC standards [4]-[6].

The DELPHI style CTMs are derived from a detailed model created in a numerical simulation environment. First, the node locations for a CTM are chosen. Then, given a set of teaching boundary conditions expressed in terms of the heat transfer coefficient, the detailed model is repeatedly solved to generate the heat flux and temperature data for all node locations. Next, the detailed model is reduced using a nonlinear optimization routine. Finally, the reduced model is validated performing simulations for another larger test set of boundary conditions. Such a procedure assures that generated CTMs produce fairly accurate simulation results in all realistic cooling conditions thus they can be regarded as Boundary Condition Independent (BCI) ones.

Unfortunately, such thermal models cannot be derived when a validated detailed model is not available. Then, the behavioral approach to the generation of CTMs remains the only choice. These CTMs are usually generated in the form of ladder RC circuits based on the analysis of the measured transient temperature responses of a system and the values of model elements are found by simple fitting of multiple exponential curves to the thermal responses [7]-[8].

However, a more adequate technique to generate CTMs is probably the Network Identification by Deconvolution (NID) method, presented briefly in the following section, which was developed by Szekely [9] based on the earlier theoretical works published in [10]. This method, used also in this paper, offers a possible advantage over the DELPHI methodology because, as it was demonstrated in [11], the CTM elements can be assigned certain physical meaning. Nevertheless, to our knowledge no one discussed so far the issue of boundary condition dependence of such RC ladder models.

2 CAUER RC LADDER MODELS

The NID method is based on the concept known from the signal theory that transient responses of a linear system to an arbitrary excitation can be found as the convolution of this excitation and a characteristic function $w(t)$ or $a(t)$ which are system time responses to the Dirac-delta function $\delta(t)$ and the unit-step function $h(t)$ respectively. Conversely, if the temperature response for a given excitation is known, it is possible to determine the corresponding characteristic functions performing a deconvolution. This basic property of linear systems is used for the identification of thermal models from system transient thermal responses in the NID method.

The transient thermal response of a system to the unit step excitation can be expressed by the following formula:

$$a(t) = \int_0^\infty R_{th}(\tau)\left[1 - \exp\left(-t/\tau\right)\right]d\tau \qquad (1)$$

The quantity R_{th}, expressed in (K/J), appearing in the integral can be regarded as the spectral density of thermal resistance and commonly it is known as the time constant spectrum of an electronic system. When the time variables t and τ are replaced by their logarithmic counterparts z and ξ and $w_z(z)$ denotes $exp(z\text{-}exp(z))$, the following convolution formula can be obtained from equation (1) to compute the time derivative of the response $a(t)$:

$$\frac{d}{dz}a(z) = \int_{0}^{\infty} R_{th}(\xi) \cdot w_z(z-\xi)d\xi \tag{2}$$

Hence, according to the NID method thermal systems can be identified by determining their thermal resistance spectral densities. This is achieved by the recording of their transient temperature responses which after the substitution of the logarithmic time variable have to be differentiated. Finally, the spectral density can be found performing the numerical deconvolution.

The key issue in the thermal identification of electronic systems is the time resolution of the recorded data since they usually consist of many components having different geometry and thermal properties. Consequently, transient responses are superposition of numerous exponential curves corresponding to particular system components. Therefore, in order to characterize properly such systems their thermal responses have to be recorded using equidistant sampling on the logarithmic time scale, preferably starting with time intervals of individual microseconds and covering a few decades in time. Only then the entire thermal resistance spectrum of the response can be properly identified.

Then, the time constant spectra of thermal responses can divided into individual RC segments corresponding to the stages in the heat flow path such as the semiconductor chip, package or heat sink. Alternatively, they can be processed further in order to obtain new representations of a system. Firstly, with finite accuracy the time constant spectra can be discretized to obtain the Foster RC ladder representation, which cannot be physically correct since the capacitances form a direct path between a heat source and the ambient implying the infinite speed of heat diffusion. Thus, the ladder should be converted to the Cauer form, in which all thermal capacitances are connected to the ambient, which is already physically correct.

The Cauer networks can be represented by the so-called cumulative structure functions $C_\Sigma(R_\Sigma)$, shown in the next section, which constitute a kind of a thermal resistance and capacitance map for the entire heat-flow path. The origins of these curves correspond to the location where power is dissipated and singularities at the end can be associated to the ambient. Structure functions are constructed simply by adding up the individual thermal resistor and capacitor values along Cauer RC ladders. Theoretically, the plateaus in these curves can be related to a certain mass of material from where thermal capacitance values can be estimated.

The main problem during the creation of a CTM from these curves is how to divide them into individual segments so that the particular model components could have some physical interpretation. Certainly, the generation of CTMs by the simple division of time constant spectra with a fixed number of points per a decade definitely does not preserve any physical meaning of model element values. Also the approach based on the analysis of deflection points in the thermal structure functions presented in [12] might produce erroneous results.

On the contrary, in [13] the authors demonstrated that the analysis of the time constant spectra is more suitable for this purpose since it is less sensitive to errors and produces more stable results. The major advantage of this approach consists in the fact that the time constant spectra are divided into individual regions before the conversion of the ladder from the Foster to the Cauer form, which is a numerically unstable operation, and therefore it will be used in the next section of the paper for the generation of CTMs.

3 EXPERIMENTAL RESULTS

The experiments were conducted for a commercial dual SiC power diode CSD20030 in the TO-247 package. The heating curves of this device were recorded with the T3Ster equipment. All the time constant spectra and the structure functions presented in this paper were computed with the software provided by the thermal tester manufacturer.

During the measurements four different configurations were considered. First the device was cooled by the free convection in air without a heat sink. Then, the package was submerged in a recipient with water. Next a large heat sink was loosely attached to the heat slug and the assembly was cooled in air. Finally, the heat sink was firmly attached with the application of thermal grease. This experimental procedure allowed the recording of data in various cooling conditions, hence allowing the investigation of boundary condition independence of Cauer RC ladder CTMs.

The recorded heating curves are presented in Figure 1. As can be seen all the curves are almost identical during the first second of heating. Then, they diverge significantly and the differences in the steady state temperature rise values exceed even 40 K. Therefore, it should be expected that the RC stages in the Cauer ladder corresponding to the higher frequencies are identical and that the only differences could be observed in the stages reflecting the heat exchange with surrounding ambient. Only then the generated CTMs might be considered boundary condition independent. Another important observation is that the curves diverge also for low heating times what might be caused by measurement noise.

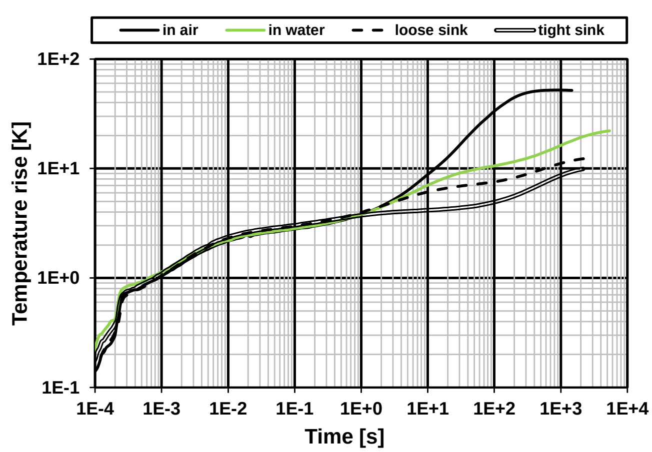

Figure 1: Recorded heating curves.

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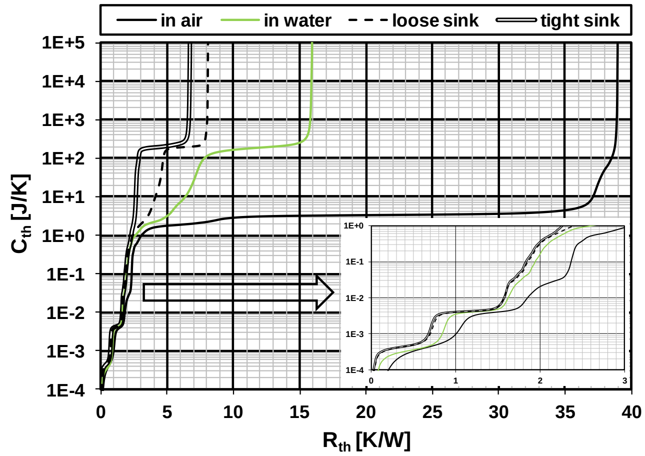

Figure 2: Cumulative structure functions.

The cumulative structure functions calculated for the recorded heating curves, presented in Figure 2, reveal that the cooling conditions indeed in all the considered cases were considerably different and the junction-to-ambient thermal resistance varied from 7 K/W to 39 K/W. This resistance was particularly reduced by the addition of the heat sink, even not perfectly attached.

The long plateau in the curve computed for the diode cooled in air without any heat sink corresponds the most probably to the thermal capacitance of the package whereas the shorter ones in the other curves at almost 200 J/K, reflect the capacitances of the heat sink or water.

Unfortunately, in the region close to the diode junction the structure functions are not identical, see the small inset at the right bottom corner of the figure, and even for the thermal resistance of 2 K/W, for which the curves were supposed to be identical, thermal capacitance values differ by more than an order of magnitude what might be the evidence that the generated Cauer RC ladder CTM cannot be boundary condition independent.

The time constant spectra computed for the recorded curves are presented in Figure 3. Analyzing this figure one can see that in all cases there exists a large maximum round 250 μs which could be attributed to the semiconductor structure. This is followed by a more blurred region which ends at some 1-2 s. Then, more distinct peaks appear in the spectra. According to the earlier analysis of time responses these peaks reflect the heat transfer processes from the package to the ambient. As can be seen the thermal steady state with the heat sink and in water is achieved only after over half an hour due to large thermal capacitance of water and the heat sink. On the other hand the device alone will reach its steady state only after a couple of minutes.

The generation of the Cauer RC ladder CTMs began with the division of the time constant spectra into four ranges at the locations of the distinct local minima visible in the spectra. Then, the thermal resistance values in the particular ranges were added up and assigned to the time constant for which the maximal resistance value in a given range occurred.

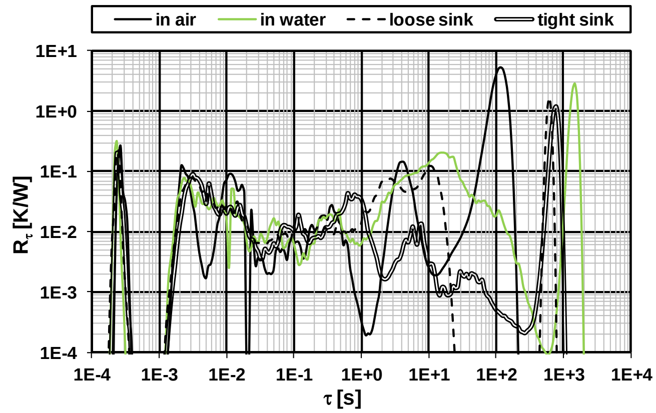

Figure 3: Time constant spectra.

Next, the resulting four-stage Foster RC ladder CTMs were converted into the respective Cauer counterparts using the recursive algorithm suggested in [14]. The final element values of the Cauer RC ladder CTMs obtained for all the considered cases are presented in Table 1. Analyzing the values provided in the table, it should be observed that the element values for the first two stages are similar but not identical and that apparently they depend on the cooling conditions which should not occur in the case of true BCI CTMs. Nevertheless, the thermal capacitance values always correspond to the plateaus visible in the inset of Figure 2.

in air		
τ (s)	R (K/W)	C (J/K)
2.87E-04	6.19E-01	4.64E-04
9.81E-03	1.73E+00	5.67E-03
8.04E+00	4.72E+00	1.71E+00
5.62E+01	3.14E+01	1.79E+00
in water		
τ (s)	R (K/W)	C (J/K)
2.79E-04	6.95E-01	4.01E-04
6.19E-03	1.50E+00	4.13E-03
1.61E+01	5.05E+00	3.19E+00
1.42E+03	8.53E+00	1.67E+02
loose sink		
τ (s)	R (K/W)	C (J/K)
2.81E-04	6.36E-01	4.42E-04
5.07E-03	1.47E+00	3.46E-03
1.11E+01	2.39E+00	4.62E+00
6.54E+02	3.45E+00	1.89E+02
tight sink		
τ (s)	R (K/W)	C (J/K)
2.86E-04	6.87E-01	4.17E-04
2.69E-03	1.05E+00	2.57E-03
8.06E-01	7.92E-01	1.02E+00
7.76E+02	4.09E+00	1.90E+02

Table 1: Cauer RC ladder CTM element values.

Moreover, it can be observed that in the second stage both the resistance and the capacitance values decrease with improved cooling. This suggests that generated heat is not only more efficiently removed from the semiconductor but also that the heat transfer rate is much higher.

Looking at the last two stages of the generated models, one can notice that loosening of the screw fixing the heat sink significantly increases the resistance of the third stage by some 1.6 K/W what agrees well with the total measured increase of the junction-to-ambient resistance between the last two cases. The capacitance of this stage is also greatly increased due to the thin air gap introduced in the main heat flow path.

The resistance of the last ladder stage reflects the heat exchange with the ambient and its capacitance supposedly corresponds to the package heat sink or water. Indeed, the estimated thermal capacitance values are close to the real ones since the measured volume of water in the recipient was 40 cm^3 and the volume of the aluminum heat sink was around 80 cm^3. Knowing the surface of the water recipient and the surface of the heat sink it would possible to find the values of heat exchange coefficients.

The last two values of the thermal capacitance for air cooled device without the heat sink represent the package. The first value of 1.71 J/K corresponds the most probably to the heat slug which has a direct thermal path to the die and the value of 1.79 J/K reflects the remaining parts of the package. Altogether it gives the total estimated capacitance of the package equal to 3.5 J/K what is a reasonable value and is close to the value of 3.2 J/K estimated for the device cooled in water.

4 CONCLUSIONS

This paper discussed the problem of boundary condition independence of Cauer RC ladder CTMs. Unfortunately, the general conclusion is that such models, when derived with the NID method, do depend on the cooling conditions mainly because the theory is based on the assumption of the linearity of an analyzed system, which is not true mainly due to the temperature dependence of the heat exchange coefficient.

Nevertheless, the methodology for the creation of Cauer RC ladder CTMs proposed by the authors has several advantages. Compared to the original NID method, the discretisation of the time constant spectra is performed before the actual transformation to the Cauer RC ladder representation. Consequently, the conversion is carried out for a small number of RC stages and the resulting values of CTM elements are more stable.

Moreover, as demonstrated on the practical example, the proposed method for the division of the time constant spectra at the location of their local minima allows the assignment of physical meaning to all the model elements, what is extremely important because it renders possible to perform parametric analyses a systems.

The CTMs with physically meaningful element values are beneficial both for manufacturers of electronic system components and their customers. Providing such CTMs the manufacturers do not have to reveal any information on the technology and their customers are also satisfied because such simple thermal models predict junction temperature with an acceptable accuracy in negligible simulation time.

Furthermore, these models can be easily implemented in any standard circuit simulators, such as SPICE or Saber, for joint multi-domain electronic system analysis allowing the reliable prediction of dynamic thermal behavior of any system.

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