Numerical modeling of squeeze-film damping in micro-mirrors including rarefaction effects

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ABSTRACT

In many MEMS applications, the fluid causing squeeze-film damping is rarefied. Thus, the Navier-Stokes equation is no longer effective for correct modeling of the fluid behavior. However, a common way to take into account fluid rarefaction is the introduction of a slight modification in its classical formulation. In particular, the fluid viscosity is substituted with an effective viscosity. The present paper deals with some squeeze-film damping problems, involving torsion micromirrors, operating at varying pressures, for which experimental data are already reported in the literature. In particular, such cases are studied by numerical solving a full 3D Navier-Stokes equation, where the effective viscosity is introduced and computed according to different expressions, included in the literature. Furthermore, the damping coefficient of the same micromirrors is computed by the use of an analytical formula, derived from the Navier-Stokes equation. Also in this case, an effective viscosity is considered instead of the traditional fluid viscosity, which is computed according to the same models considered in the numerical analysis. Then, the numerical results are compared with the analytical and the experimental data. From such comparison, the numerical values show better agreement with experiments than the analytical ones.

Keywords: squeeze-film damping, rarefaction, micromirror, FEM

1 INTRODUCTION

Squeeze-film air damping is a typical dissipating phenomenon, which affects dynamic performance of many MEMS-based devices, such as microaccelerometers, microgyroscopes, and micromirrors. It arises when a thin layer of fluid, like air, turns to be confined between two solid walls, which are in reciprocal movement. The nature of such reciprocal movement can be either parallel, like in microaccelerometers, or rotational, like in torsion micromirrors. However, in either case, when the two walls move apart from each other or come close to each other, the fluid is sucked into/pulled out of its tight channel and this causes an even significant pressure field to be present. This pressure field is then responsible of a resistive force, which prevents the walls' movement.

If the fluid can be considered as continuum, its flow can be described through the classical Navier-Stokes equation. However, such model fails when the fluid results to be rarefied. In these circumstances, other approaches than the Navier-Stokes equation should be followed.

The parameter conventionally used to characterize fluid rarefaction is the Knudsen number, K_n. It is defined as the ratio of the mean free path of the fluid molecules (λ) to the characteristic length of the fluid channel. In particular, the mean free path is related to the ambient pressure as [1]:

$$\lambda = \frac{R \cdot T}{\pi\sqrt{2}d^2 \cdot N_A \cdot p} \tag{1}$$

In the above R is the gas constant, T is the temperature, d is the molecule diameter, N_A the Avogadro number, and p the ambient pressure.

When K_n<0.1 (high pressure regime), the fluid flow can be modeled through the classical Navier-Stokes equation, which sometimes can be simplified in the form of the Reynolds equation (e.g., out-of-plane flow, inertial effects, and thermal gradients are negligible) [2].

When K_n>0.1 (transition and free molecular regime), the main cause of dissipation is no longer related to the interaction among fluid particles, but to the interaction of the fluid molecules with the confining walls. Thus, the classical Navier-Stokes equation is no longer valid [2].

During the last years a variety of analytical models have been proposed to model the squeeze-film air damping phenomenon [2]. However, these are effective in a strict number of cases, when the problem geometry is regular (e.g., rectangular shape, regular pattern of perforations). A numerical approach could be more useful or even necessary, when geometries become complex, as it could be in real applications [3].

In this work, a commercial finite element software, Comsol Multiphsics, is adopted to solve some squeeze-film damping problems, which are reported in the literature [4]. They include two torsion micromirrors, moving at both high and low pressure regimes.

2 ANALYTICAL MODELING OF SQUEEZE-FILM DAMPING IN RAREFIED REGIME

A common practice to model fluid rarefaction suggests the introduction of a slight modification inside the usual formulation of the Navier-Stokes equation. In particular, the classical fluid viscosity (μ) is substituted with an effective viscosity term (μ_{eff}), for which the literature provides a variety of expressions. Here, we consider those which were proven to work better [5-6]. In particular, these are three formulations, proposed by Veijola et al. [7], Li et al. [8], and Pandey and Pratap [5], respectively.

Veijola et al. computed the effective viscosity as [7]:

$$\mu_{eV} = \frac{\mu}{1 + 9.638 \cdot K_n^{1.159}} \tag{2}$$

Li proposed instead the following expression [8]:

$$\mu_{eL} = \frac{\mu}{1 + 3a\sqrt{\pi}/D + 6bD^c} \tag{3}$$

where a=0.01807, b=1.35355 and c=-1.17468, and D is:

$$D = \sqrt{\pi}/(2K_n)$$

Pandey et al. improved Li's model to achieve better agreement with experimental data and proposed [5]:

$$\mu_{eP} = \frac{\mu}{1 + 3a\sqrt{\pi}/(1.4D) + 6b(1.4D)^c} \tag{4}$$

where a, b, c, and D are defined as before.

For computation of the damping coefficient affecting a considered geometry, one of the above reported expressions for the effective viscosity can then be implemented inside the Navier-Stokes equation, or, if available, in closed formulas, derived from this one [2]. For example, for a plate rotating with respect to the substrate, the analytical formula for computing the damping coefficient is [9]:

$$c_a = \frac{192\mu L w^5}{\pi^6 h^3} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} \frac{1}{m^2 n^2 [m^2\eta^2 + n^2]} \tag{5}$$

In the above, h is the air gap, μ is the fluid viscosity, $\eta=w/L$, being L and w the plate length and width, respectively.

Equation (5) was derived for torsion plates working at high pressure. Nevertheless, this latter can be adapted for low pressure regimes, if the fluid viscosity is substituted with the effective viscosity, computed according to one of the previously mentioned expressions.

3 NUMERICAL MODELING OF SQUEEZE-FILM DAMPING IN RAREFIED REGIME

A commercial finite element software (Comsol Multiphysics) was adopted to solve some squeeze-film damping problems, already reported in the literature [4]. In particular, such software allows for performance of analysis contemporarily defined in different physical domains. Thus, it is very suitable for the problems herein considered, which involve both structural mechanics and fluid dynamics.

By the means of Comsol, a full 3D incompressible Navier-Stokes equation was solved on a workstation with the following technical features: RAM 16 GB, Intel(R) CORE(TM) i7 CPU 860 @ 2.80 GHz.

The mesh is automatically generated by the software, and consists of tetrahedral elements. Figure 1 and 2 shows the typical pressure fields over a square torsion mirror with 500 µm side, thickness of 30 µm, air gap of 13 µm, rotating around the x-axis. Here, blue indicates the region where the fluid is sucked into its tight gap (low pressure area), whereas red indicates the region where the fluid is pulled out (high pressure area).

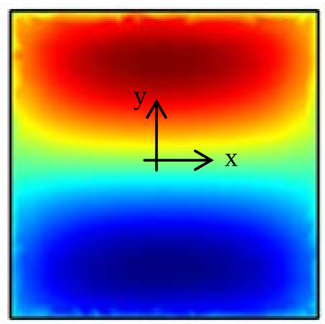


Figure 1: Typical pressure distribution over a torsion micromirror.

The damping coefficient determined through numerical analysis (c_n) was computed as [4]:

$$c_n = \frac{\int p(x,y) \cdot y \, dxdy}{\omega} \tag{6}$$

Where $p(x,y)$ is the pressure field inside the fluid film, and $\omega=2\pi f$, being f the frequency of the torsion movement.

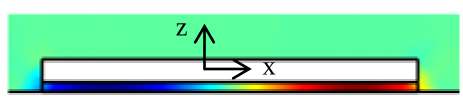


Figure 2: Typical pressure field inside the fluid confined between a torsion micromirror and the substrate.

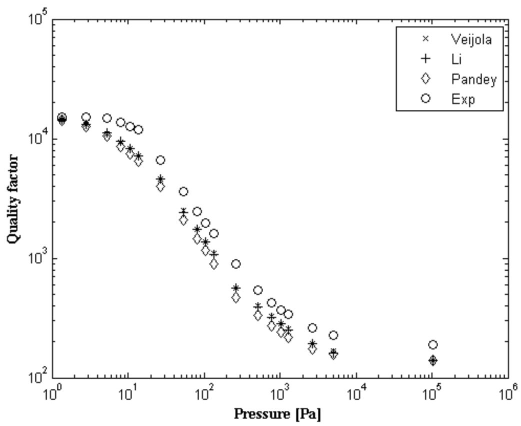
From the damping coefficient is then possible to compute the corresponding quality factor as:

$$Q_n = \frac{c_n}{2I_x\omega}$$

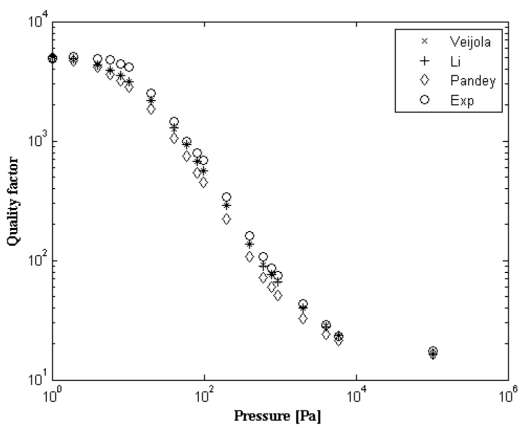
being I_x the moment of inertia of the square plate around the x-axis.

4 COMPARISON OF ANALYTICAL AND NUMERICAL MODELING WITH EXPERIMENTAL DATA

The squeeze-film damping problems herein considered are those experimentally investigated in [4]. They refer to a couple of silicon square torsion micromirrors, with side length of 500 μm, thickness of 30 μm, initial air gap of 28 and 13 μm, respectively, and natural frequency of 13092.56 Hz and 12824.87 Hz, respectively. In [4], the damping coefficient characterizing both plates is experimentally determined at varying pressures, ranging from the atmospheric value to few Pa.



(a)

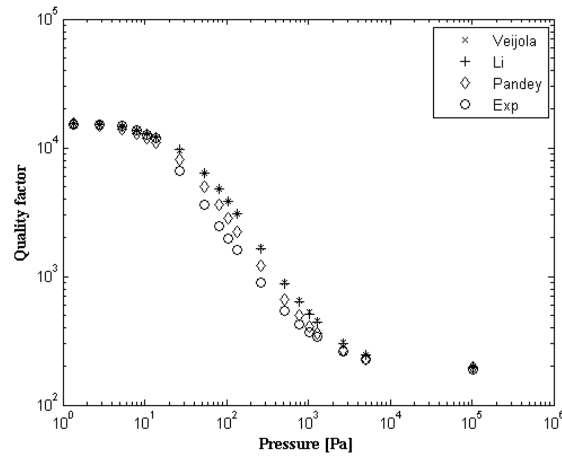


(b)

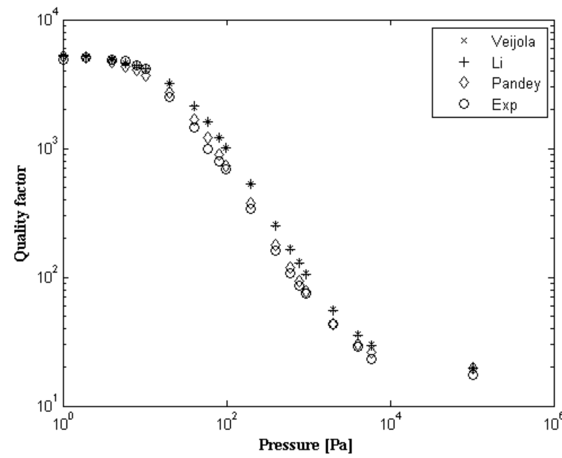
Figure 3: Comparison between the experimental damping coefficient and the corresponding numerical results for the first (a) and the second (b) mirror reported in [4] at varying pressure regimes.

The same cases reported in [4] were studied by the means of Comsol Multiphysics. In particular, three sets of numerical data were obtained, each corresponding to the implementation of one of expressions (2), (3), and (4) for the effective viscosity. Figure 3 reports a comparison between the numerical and the experimental data for the first (figure 3a) and the second (figure 3b) micromirror at varying pressure regimes.

Furthermore, equation (5) was considered for computation of the damping coefficient for each plate at different pressures. Also in this case, three sets of analytical results were obtained, each corresponding to expression (2), (3), and (4) for the effective viscosity. Figure 4 reports a comparison between these analytical results and the experimental data, at varying pressure regimes.



(a)



(b)

Figure 4: Comparison between the experimental damping coefficient and the corresponding analytical results for the first (a) and second (b) typology of plate reported in [4] at varying pressure regimes.

In particular, if either Li's or Veijola's models are considered, numerical simulations underestimate the experimental quality factor by 28% and 11% (average values) for the first and the second micromirror, respectively, while with equation (5) there is an overestimation of 37% and 30% (average differences). Thus, the numerical approach provides better agreement with the experiments than the analytical approach. Such better agreement is related to the border effects which are considered within the numerical analysis, but which are neglected in the analytical approach.

A separate discussion deserves Pandey and Pratap's model. Their expression for the effective viscosity was derived from Li's model with the introduction of a scaling factor equal to 1.4 (equation 4). Such factor was identified by minimizing the difference between the experimental values, related to both the torsion mirrors herein considered, and the corresponding analytical results obtained from equation (5), while implementing Li's model for the effective viscosity.

Thus, the difference between the experimental data and the analytical results according to equation (5) with Pandey and Pratap'e model for the effective viscosity is as small as 14% and 5% (average value) for the first and the second mirror, respectively. However, such difference increases when the same model is implemented within numerical simulations. In fact, in these cases, the average difference is 23% for the first mirror and 27% for the second mirror. The increase in the average difference can be explained, since Pandey and Pratap's model for the effective viscosity already takes into account the border effects. Thus, if this model is then implemented within the numerical analysis, the border effects are considered twice. Furthermore, such border effects already included in Pandey and Pratap's model are specific for the geometries herein considered. Thus, if their model is adopted for other geometries in combination with equation (5), the corresponding results could not be as effective as in the present case.

Finally, both the results, which have been found in the present work and those presented in [10], involving plates moving parallel to the substrate, suggest that the expression of the effective viscosity can be further improved to achieve better agreement with experiments.

5 CONCLUSION

In this work, a numerical approach was tested for determining the quality factor of two torsion micromirrors at varying pressures, ranging from atmospheric pressure to few Pa. In order to correctly model the fluid behavior, the fluid viscosity in the Navier-Stokes equation was substituted with an effective viscosity, computed according to three different expressions reported in the literature. The numerical results were then compared with both the results obtained from analytical solution of the Navier-Stokes equation and the experimental data already available in the literature. From such comparison, it was observed that the numerical analysis provided results showing better agreement with experiments than the results obtained by analytical modeling, which can however be considered only in a restricted number of cases. Furthermore, the results showed that it is possible to further improve the expressions for the effective viscosity included in the literature, in order to achieve even better agreement with experiments. Thus, in future developments, the numerical approach herein considered will be used by the authors to achieve this aim.

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