Numerical Study on a Passive Micromixer with Sinusoidal Channel Walls

Arshad Afzal* and Kwang-Yong Kim**

*,** Department of Mechanical Engineering, Inha University, Incheon, 402-751, Rep. of Korea, kykim@inha.ac.kr

ABSTRACT

A computational study is carried out for a passive micromixer with wavy channel walls of sinusoidal variation for Reynolds numbers in the range $0.1 \leq Re \leq 30$. A mixing index is used to evaluate the performance of the micromixer. The mixing performance showed a strong dependence on the wavelength of the sinusoidal walls over the entire Re range considered. A shift in the center of rotation of Dean vortices toward inner sinusoidal wall for lower wavelength mixers was found to be an important factor significantly affecting the chaotic mechanism at higher Re. For a fixed mixing length, the sinusoidal design showed a remarkably improved mixing performance compared to a square-wave channel. This micromixer can easily be realized and integrated with microfluidic systems due to its simple in-plane structure.

Keywords: microfluidics, micromixer, Dean vortices, chaotic mixing.

1 INTRODUCTION

Mixing of liquid samples in microfluidic systems has attracted many researchers mainly due to small characteristic dimensions of the channels and absence of turbulence. Micromixers can be divided into two broad categories: active and passive. The active types require complex fabrication procedures and are difficult to incorporate with micro systems. In contrast, passive micromixer uses the geometry of the device to produce complex flow field for effective mixing. Depending on the mode of operation, different realizations of passive micromixers have been developed. A typical passive method to enhance the mixing process was proposed by Stroock et al. [1] using bas-relief structures on the floor of a channel. The patterned topography can be used to generate transverse flow that increases the interfacial area between the fluids to be mixed.

For flows through curved and bended channels, secondary flows evolve as a result of inertial and centrifugal effects. Liu et al. [2] studied mixing in a 3-D serpentine microchannel for a wide Reynolds number range of $6 \leq Re \leq 70$. Mixing in the serpentine channel was found to depend on the secondary flows originating at the bends. Howell et al.[ 3] showed the formation of Dean vortices by directing flow through a curved channel of square cross-section. Jiang et al. [4] used such secondary flows to induce chaotic mixing for flows through meandering channels. Vanka et al. [5] carried out a computational study to determine mixing rates in a curved square duct at low Reynolds numbers. Thus, there has been significant research on the generic problem of mixing via curved conduits.

Numerical simulation using computational fluid dynamics (CFD) have been carried out for a passive micromixer with sinusoidal walls proposed by Afzal and Kim [6]. With wavelength as the geometrical parameter, the micromixer is investigated for a wide Reynolds number range $0.1 \leq Re \leq 30$. Mixing index is employed to evaluate the performance of the micromixer.

2 PROBLEM DEFINITION AND QUANTIFICATION OF MIXING

A diagrammatic layout of the problem is given in Fig. 1. The profile of the main channel was generated using a sine function of the form $y = A \sin \left(\frac{2\pi x}{\lambda}\right)$ where $A$ is the amplitude, $\lambda$ is the pitch or wavelength, and $x$ and $y$ are the spatial coordinates. The value of $A$ was taken to be 0.2 mm.
The two sine curves have offset-\( y = 0.1 \) mm; i.e., the distance between any two opposite points on the respective curves is 0.1 mm. The value of \( H = 0.5 = 2a + 0.1 \) mm. The depth of the channel is 0.1 mm. The two inlets, Inlet 1 and Inlet 2, having a square cross-section dimension of 0.1 mm × 0.1 mm, are merged to the main channel with a T-joint as shown in Fig. 1. The axial lengths of the connecting channel \( L_o \) and exit channel \( L_e \) are fixed at 0.1 and 1.8 mm, respectively. The length of main channel \( L_c \) is a function of wavelength for a fixed mixing length. Simulations were carried out using ANSYS CFX-11 [7], a commercial computational fluid dynamics (CFD) package based on the finite volume method. The numerical analysis was carried out by solving the steady 3-D Navier-Stokes equations and advection-diffusion-type equation for the concentration field with appropriate boundary conditions.

\[
\vec{V} \cdot \vec{V} = 0
\]  

(1)

\[
(\vec{V} \cdot \vec{V}) \vec{V} = -\frac{1}{\rho} \nabla p + \nabla \vec{V}^2 \vec{V}
\]  

(2)

\[
(\vec{V} \cdot \vec{V})C_i = \alpha \nabla^2 C_i
\]  

(3)

where \( \vec{V} \) represents the fluid velocity, \( \rho \) and \( \nu \) are fluid density and kinematic viscosity, \( \alpha \) is the diffusivity coefficient and \( C_i \) is the concentration respectively. The properties of the working fluids, Ethanol and water, were measured at 20 ºC [8].

Hexahedral cells were used to discretize the computational domain. Using grid-independence test, a grid size of \( 3 \times 3 \times 3 \) μm was found to be accurate for spatial resolution. The SIMPLEC algorithm was used for pressure-velocity coupling. Multigrid techniques were used to accelerate the convergence behavior. The prediction accuracy of the numerical methods used in our study has been proven in previous works [9] for mixing in various micromixers. A variance-based method was employed to evaluate the mixing performance of the micromixer. The variance of the mass fraction of the mixture on a cross-sectional plane normal to the flow direction can be expressed mathematically as

\[
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (c_i - c_m)^2}
\]  

(4)

where \( N \) is the number of sampling points on the plane, \( c_i \) is the mass fraction at sampling points \( i \), and \( c_m \) is the optimal mixing mass fraction. Finally, the mixing index at any cross-sectional plane perpendicular to the axial direction is defined as

\[
M = 1 - \frac{\sigma^2}{\sigma_{max}^2}
\]  

(5)

Where \( \sigma_{max} \) is the maximum variance over the range of data. A higher value of \( M \) (close to 1) indicates a better mixing quality.

3 RESULTS AND DISCUSSION

For flows through curved conduits with circular or rectangular cross-sections, centrifugal force acts on fluid particles as a result of fluid inertia and curvature. However, fluid particles moving along the centerline experiences a higher centrifugal force (maximum axial velocity).
compared to the particles near the wall. On account of differential centrifugal forces, secondary flows are set up in the cross-sectional plane of the channel characterized by the presence of two-counter rotating vortices (known as Dean’s vortices) [10]. The Dean number \( \kappa \), is defined as \( \text{Re}(d/R)^{1/2} \), where \( d \) is the characteristic cross-sectional dimension of the channel and \( R \) is the radius of curvature. Dean number \( (\kappa) \) gives an idea of the intensity of the secondary motion. Two major aspects of the sinusoidal design of the micromixer are: Firstly, the turns at the highest (peak) and lowest (trough) points induce curvature to the path of the flow. Consequently, Dean vortices appear at the cross-sectional plane located at peaks and troughs for the entire range of Reynolds number. The direction of rotation of the vortices is reversed at the troughs due to change in the sign of curvature. The vortex pair existed even when the Reynolds number was very small. However, at very low flow rates \( (\text{Re} \leq 1) \), secondary flows are rather weak to perturb the interface and mixing is mainly by diffusion between adjacent concentration layers. As the strength of secondary flow increases with increasing Reynolds number \( (\text{Re} > 10) \), interface modulation commonly known as Dean’s vortices) [10]. The Dean number \( \kappa \), is defined as \( \text{Re}(d/R)^{1/2} \), where \( d \) is the characteristic cross-sectional dimension of the channel and \( R \) is the radius of curvature. Dean number \( (\kappa) \) gives an idea of the intensity of the secondary motion. Two major aspects of the sinusoidal design of the micromixer are: Firstly, the turns at the highest (peak) and lowest (trough) points induce curvature to the path of the flow. Consequently, Dean vortices appear at the cross-sectional plane located at peaks and troughs for the entire range of Reynolds number. The direction of rotation of the vortices is reversed at the troughs due to change in the sign of curvature. The vortex pair existed even when the Reynolds number was very small. However, at very low flow rates \( (\text{Re} \leq 1) \), secondary flows are rather weak to perturb the interface and mixing is mainly by diffusion between adjacent concentration layers. As the strength of secondary flow increases with increasing Reynolds number \( (\text{Re} > 10) \), interface modulation commonly known as Dean’s vortices) [10]. The Dean number \( \kappa \), is defined as \( \text{Re}(d/R)^{1/2} \), where \( d \) is the characteristic cross-sectional dimension of the channel and \( R \) is the radius of curvature. Dean number \( (\kappa) \) gives an idea of the intensity of the secondary motion. Two major aspects of the sinusoidal design of the micromixer are: Firstly, the turns at the highest (peak) and lowest (trough) points induce curvature to the path of the flow. Consequently, Dean vortices appear at the cross-sectional plane located at peaks and troughs for the entire range of Reynolds number. The direction of rotation of the vortices is reversed at the troughs due to change in the sign of curvature. The vortex pair existed even when the Reynolds number was very small. However, at very low flow rates \( (\text{Re} \leq 1) \), secondary flows are rather weak to perturb the interface and mixing is mainly by diffusion between adjacent concentration layers. As the strength of secondary flow increases with increasing Reynolds number \( (\text{Re} > 10) \), interface modulation commonly known as Dean’s vortices) [10]. The Dean number \( \kappa \), is defined as \( \text{Re}(d/R)^{1/2} \), where \( d \) is the characteristic cross-sectional dimension of the channel and \( R \) is the radius of curvature. Dean number \( (\kappa) \) gives an idea of the intensity of the secondary motion. Two major aspects of the sinusoidal design of the micromixer are: Firstly, the turns at the highest (peak) and lowest (trough) points induce curvature to the path of the flow. Consequently, Dean vortices appear at the cross-sectional plane located at peaks and troughs for the entire range of Reynolds number. The direction of rotation of the vortices is reversed at the troughs due to change in the sign of curvature. The vortex pair existed even when the Reynolds number was very small. However, at very low flow rates \( (\text{Re} \leq 1) \), secondary flows are rather weak to perturb the interface and mixing is mainly by diffusion between adjacent concentration layers. As the strength of secondary flow increases with increasing Reynolds number \( (\text{Re} > 10) \), interface modulation commonly known as
slow. However, lower wavelength channels ($\lambda = 0.56$ and $0.28$ mm) present advantage of the stronger and asymmetric Dean vortices resulting in enhanced mixing.

Fig. 5 shows mixing performance as a function of mixing length for three different wavelengths at $Re = 0.5$. For $Re \leq 1$, the secondary flow was weak and mixing was primarily by diffusion. Thus, for a similar path length traversed by the co-flowing fluids, a lower wavelength micromixer demonstrated a significantly improved performance due to reduction in diffusion distance in the channel cross-section. The mixing performance was evaluated in comparison to a square-wave channel of comparable dimensions ($\lambda = 0.28$ mm) for $Re = 0.5$ and $30$. The proposed sinusoidal mixer demonstrates significantly improve performance for both low and high Reynolds numbers (Fig. 4 and 5).

Fig. 6 shows the results obtained by the previous work [6] for the mixing performance of the micromixer for a wide Reynolds number range, $0.1 \leq Re \leq 30$. The fluid in the micromixer travel curved paths, therefore, a fixed mixing length was used for comparison at different wavelengths. Mixing was primarily by diffusion for $Re \leq 1$. Since, diffusive mixing is affected by the residence time of the fluid in the channel; the mixing quality deteriorates with increased Reynolds number at a fixed design wavelength. At higher Reynolds numbers ($Re > 10$), interface modulation known as stretching and folding occurs on account of secondary flow leading to increased area for diffusion and the mixing performance was significantly improved. However, due to weak secondary flow at $Re 10$ for $\lambda = 1.12$ mm, the mixing was primarily diffusion dominated. Although the mixing index is same for $\lambda = 0.56$ and $0.28$ mm ($Re = 30$) for a fixed mixing length, lower wavelength generates stronger secondary flow and mixing progresses relatively fast utilizing fewer number of turns (Fig. 5).

**4 CONCLUSIONS**

Numerical simulation based on Navier-stokes equations are carried out for a passive micromixer with sinusoidal walls for Reynolds number in the range $0.1 \leq Re \leq 30$. The mixing performance is found to sensitively increase as the wavelength of the sinusoidal channel walls decreases over the entire $Re$ range considered. A different chaotic mixing mechanism with a shift in the centre of rotation of Dean vortices towards inner sinusoidal wall was observed for lower wavelength microchannels. The sinusoidal design showed superior mixing performance compared to a square-wave channel for both diffusive and chaotic regimes.

**REFERENCES**