Electro-Thermo-Mechanical Beam Model

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ABSTRACT

In this paper we present our compact electro-thermo-mechanical beam model. Compact models are important for efficiently modeling parametric performances of a large number of multicomponent systems. There are some finite element analysis tools like COMSOL, Intellisense and lumped models in Sugar, Coventoreware that can model and simulate thermodynamic phenomena's. But what is new about our compact model is that along with the features offered by the above models, ours can support energy exchange by radiation or between itself and neighboring components. Our compact model considers mechanical beams subject to various energy transfer mechanisms, properties, and boundary conditions. Energy transfer mechanisms can include: conduction, convection to ambient, radiation due to the radiation source, and interaction with the substrate. Thermal expansion is determined by computing the average temperature of the beam and by applying equivalent end node forces to elongate the beam. As a test case we model the deflection of a U-shaped thermal actuator subject to all the energies listed above. The results compare well with experiment.

1. INTRODUCTION

An Electro-Thermo-Mechanical beam model is important for efficiently computing the deflection of an assemblage of mechanical beams that are subject to one or more applied voltages, currents, thermal radiation sources, or energy exchanges between the beams and neighboring components, underlying substrate, or the ambient. Thermodynamic phenomena is commonly modeled and simulated using finite element analysis (FEA) tools such as COMSOL, Coventorware [1-2] and Intellisense [3]. Although FEA is highly informative, it is computationally expensive, and can be prohibitive for a large system of components or interacting devices. Although lumped elements are not as informative as FEA, their effective performance often compares well to FEA. Existing compact models for electro thermal actuators (ETA) have been limited in utility. E.g., Sugar's [4] previous ETA model required the user to know the average temperature, and Coventorware's lumped ETA does not support energy exchange by radiation or between itself and neighboring components.

Beyond Joule heating, we appear to be the first to report on a compact hybrid MEMS model that also accommodates energy transfer by radiation, convection, and other forms of conduction. We found no such compact model in the MEMS literature. Our model can be used for both sensing and actuation. When used as a sensor, applied heating from a variety of sources generates a thermal expansion that changes the lumped beam's geometry and resistance, which can be sensed by a change in current, voltage, or capacitance. When used as an actuator, an applied current or voltage generates a small thermal expansion, which can be magnified to achieve a useful displacement.

The rest of the paper is organized as follows. In Section 2 we describe our model by providing the assumptions, governing equations, boundary conditions, and the options available for the user. We provide our results in Section 3. We explore some applications in Section 4. We validate our model in Section 5. Finally we conclude our discussion in Section 6.

2. MODEL DESCRIPTION

Our modeling assumptions, equations, boundary conditions, and parameters are as follows.

2.1 Assumptions

The following assumptions were made for developing our model.

i) The axial dimension, length *L*, is much larger than with *w* and thickness *h*. This allows us to approximate the conduction as being one-dimensional along the beam's length [5-6].

ii) Steady-state condition, such that energy storage rate is zero.

2.2 Governing equations

To obtain the governing equations of our compact model, energy balance relationships are applied to its control volume. See Figure 1. This gives $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$ where $\dot{E}_{in}$ denotes the rate of energy entering the system and it includes $-k\, dT/dx$ from conductive contacts, $\varepsilon\sigma T_{source}^4$ from radiation source, $h_{conv} T_\infty$ due to convection, $\dot{E}_{out}$ is the rate of energy leaving the system which includes $-k\, dT/dx + \nabla\left(-k\, dT/dx\right)dx$ to conductive contact, $S\left(T - T_{substrate}\right)/hR_T$ to the substrate, $\varepsilon\sigma T^4$ to the ambient, and $h_{conv} T$ which is convection to ambient, $\dot{E}_{gen}$ is the rate of energy generated within the system due to joule heating and it's given by $\dot{E}_{gen} = V^2/\rho L^2$, $\dot{E}_{st}$ is the rate of energy stored in the system where *st* stands for 'stored energy', which is zero from the second assumption. Hence the equation reduces to

$$k\frac{d^2T}{dx^2} + \frac{V^2}{\rho L^2} = \frac{S\left(T - T_s\right)}{hR_T} + \varepsilon\sigma\left(T^4 - T_{source}^4\right) + h_{conv}\left(T - T_\infty\right)$$

The shape factor *S* accounts for heat transfer between nearby components [5-7], *h* is the gap between structures, $R_T(h)$ is the thermal resistance [8] from a nearby components, ε is emissivity, σ is Stefan Boltzmann constant, T_{source} is the temperature of a radiation source, *k* is thermal conductivity, *V* the applied voltage, $\rho(T)$ is the resistivity, *L* is beam length, $T_{substrate}$ is the average substrate temperature, and $T(x)$ is the element temperature for $0 \le x \le L$. The average of $T(x)$ is used for thermal expansion, which is produced by equivalent node forces.

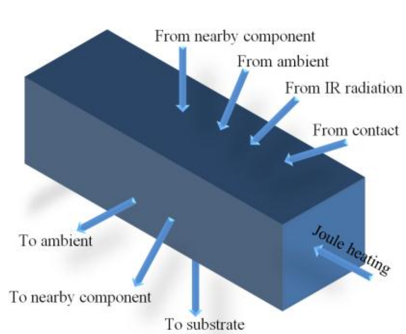

Figure 1: A Lumped control volume showing the various sources and sinks.

2.3 Boundary Conditions

There are three types of boundary conditions, Dirichlet, Neumann, and continuity.

2.3.1 Dirichlet boundary condition

The Dirichlet boundary condition prescribes a state upon a surface boundary. The state of temperature upon this surface boundary is $T\left(x_b\right) = T_b$, where x_b is the location of the boundary surface.

2.3.2 Neumann boundary condition

The Neumann boundary condition prescribes a flux at a boundary surface. We model a heat flux as $q_b^{"} = -k\, dT/dx\big|_{x=x_b}$ where the known heat flux which is rate heat transfer per unit area, at the boundary is $q_b^{"}$.

2.3.3 Continuity condition

The continuity condition prescribes the equality of conditions on the immediate sides of the boundary. The temperature and flux equate at the boundary. $T\left(x\right)\big|_{x=x_{bL}} = T\left(x\right)\big|_{x=x_{bR}}, -k\, dT/dx\big|_{x=x_{bL}} = -k\, dT/dx\big|_{x=x_{bR}}$ where the subscripts *bL* and *bR* stand for the immediate left and right of the boundary.

2.4 Options Available for the User

The user can choose

i) Variable resistivity ρ.
ii) Variable thermal conductivity *k*.
iii) Any combination of available boundary conditions.
iv) With or without thermal radiation.
v) With or without convection.

3. RESULTS

We explore our model by simulating its temperature profile along its axial length. Figure 2 shows the temperature distribution for various lengths of a beam 2.5 µm wide, 2 µm thick, 1 µm above the substrate, with an applied voltage of 0.3V and a dirichlet boundary condition. Figure 3 shows the average temperature for this beam.

NSTI-Nanotech 2012, www.nsti.org, ISBN 978-1-4665-6275-2 Vol. 2, 2012

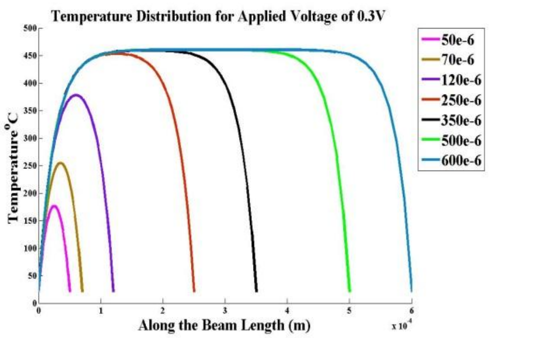

Figure 2: Temperature distribution for beams of various lengths. Here the boundary is considered to be at substrate temperature.

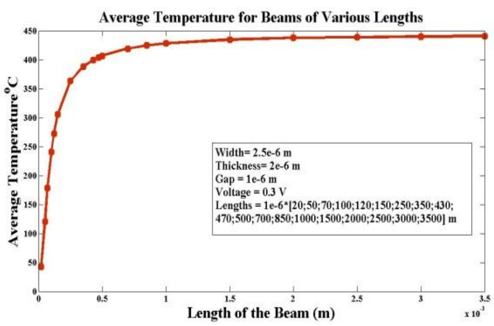

Figure 3: Average temperature for beams of various lengths. Here the boundary is considered to be at substrate temperature.

Figure 4 shows the average temperature of a beam 20 µm long, 2.5 µm wide, 2 µm thick and 1 µm above the substrate. From the plot we can see that as the voltage increases the average temperature increases.

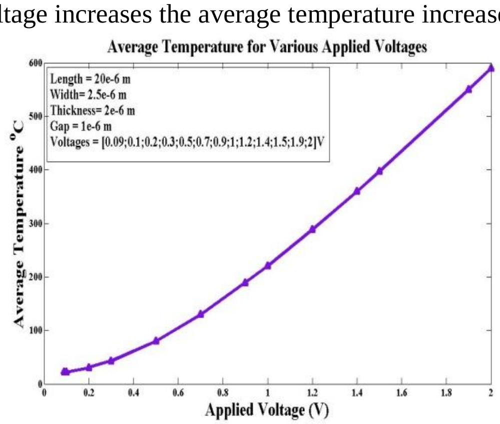

Figure 4: Average temperature of a beam for various applied voltages. Here the boundary is considered to be at substrate temperature.

For the above cases the resistivity is assumed to vary with temperature as $\rho(T) = 1e-5\left(1+1.23e-3\left(T - T_{substrate}\right)\right)$ [6], [8]. Figure 5 shows the average temperature of beam for various lengths with Neumann condition at the boundary, varying resistivity and conductivity [6].

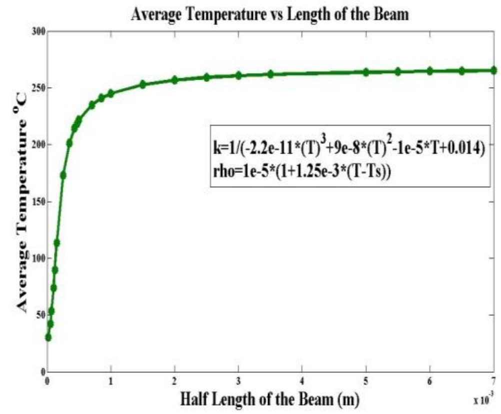

Figure 5: Average temperature for beams of various lengths. Here a Neumann condition with varying resistivity and thermal conductivity is applied.

4. APPLICATIONS

In this section we apply our model to get the lateral deflection of a U-shaped actuator. Figure 6 shows the lateral deflection of a U-shaped actuator due to Joule heating form an applied voltage. There is energy exchange between the structure and an underlying substrate at 20^0C. The anchored ends are assumed to be at the substrate temperature. Figure 7 shows the deflection with inclusion of convection and radiation in addition to the above conditions.

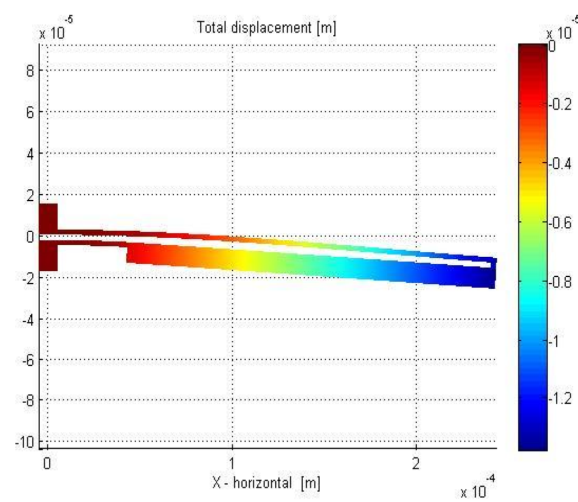

Figure 6: Deflection of a U-shaped actuator due to Joule heating form an applied voltage of 2.4V. There is energy exchange between the structure and an

underlying substrate at 20^{0}C, anchors at 20^{0}C, The simulation was performed by using our online MEMS simulator called Sugar.

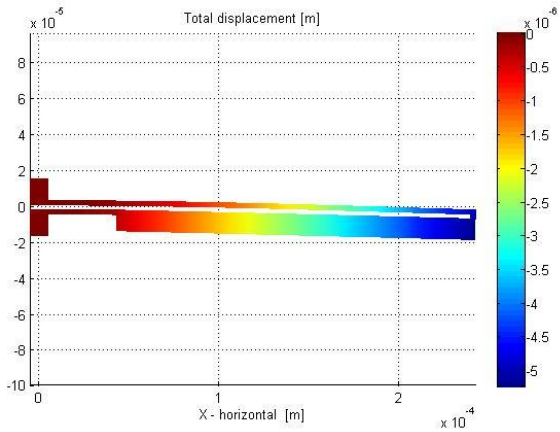

Figure 7: Deflection of a U-shaped actuator due to Joule heating form an applied voltage of 2.4V. There is energy exchange between the structure and an underlying substrate at 20^{0}C, anchors at 20^{0}C, energy exchange between the hot and cold arms of the structure, and surrounding ambient at 20^{0}C. The simulation was performed by using our online MEMS simulator called Sugar.

5. VALIDATION

The results were compared with the experimental results reported by Huang in [8].From the Figures 8 and 9 we can see that the results match really well.

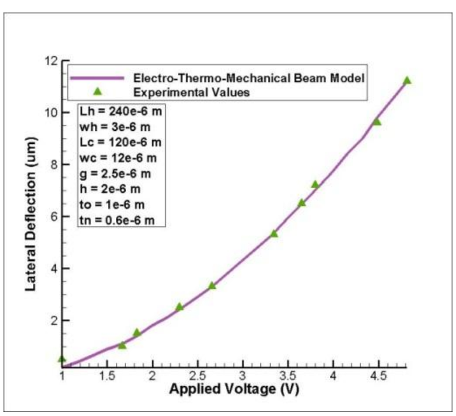

Figure 8. Lateral deflection of U-Shaped Actuator vs Applied Voltage. This plot shows that our model predicts results which are very close to the experimental values.

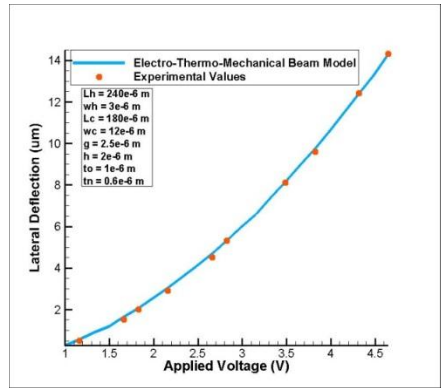

Figure 9. Lateral deflection of U-Shaped Actuator vs Applied Voltage. In this case the cold arm is 60 µm longer than the previous case. This plot shows that our model predicts results which are very close to the experimental values.

6. CONCLUSIONS

In this paper we presented our compact hybrid electro-thermo-mechanical beam model which is needed for efficiently computing the deflection of an assemblage of mechanical beams that are subject to one or more applied voltages, currents, thermal radiation sources, or energy exchanges between the beams and neighboring components, underlying substrate, or the ambient. We presented our model and validated it. The user may select various energy transfer mechanisms, properties and boundary conditions.

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