Comparison and Insight into Long-Channel MOSFET Drain Current Models

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Abstract-- In this paper we provide an insight into the drain current model for long-channel MOSFET devices. A new method to perform the integral of the rigorous Pao-Sah dual integral current is derived. From it, we demonstrate the error of the traditional charge sheet models in predicting the drain current compared with Pao-Sah’s dual integral model, also provide the reason that Brews’ charge sheet model fails to pass the self consistency tests reported previously. Three charge-sheet approximation models are tested in order to find a simple yet accurate drain current model for surface potential-based compact models.

Keywords-- MOSFET models, charge-sheet approximation, drain current model

I. INTRODUCTION

At present there are varieties of non Vth based compact models for MOSFETs which can be mainly divided into two groups: (1), The surface potential-based models, like PSP, HISIM, PUNSIM; and (2), The charge-based models, like EKV, ACM, BSIM5. The surface potential-based models are mostly based on Brews’ charge sheet model [1]. It is said that the charge sheet model agrees well with Pao-Sah model [2]. But the model assumption that the current consists of drift and diffusion appears to be somewhat drastic while the exact mathematic link between Pao-Sah model and the charge sheet model is not evident so far. Recently it is proved that the predicted channel current by Brews’s model is always less than that by Pao-Sah model [3]. Also, C.Galup-Montoro et al. test the consistency of the MOSFET models in [4] and they draw the conclusion that Brews’ charge sheet model cannot pass the self consistency tests reported previously. Three charge-sheet approximation models are tested in order to find a simple yet accurate drain current model for surface potential-based compact models.

II. EQUIVALENT FORMULATION OF PAO-SAH DUAL INTEGRAL

In this paper we normalize various quantities, for example: $\phi = \frac{kT}{q}$ for voltage, $C_n\phi_i = \frac{C_nkT}{q}$ for charge, and $I = \frac{\mu W}{L} C_n\phi_i^2$ for current.

The combination of the 1-D Poisson’s equation along the normal direction to the channel and Gauss law yields to the surface potential equation:

\[
U_g - U_s = \gamma [U_s - 1 + \exp(U_s - V) - \exp(U_s - V)] \tag{1}
\]

where $U_g = U_s - U_p$, means the effective gate voltage, $\gamma = \sqrt{2q\epsilon_s N_s/C_n\phi_i}$ is the body factor after normalizing, and other symbols have their usual meanings. For simplicity we neglect some terms in the above expression and obtain the following general formula that is widely used in the compact model:

\[
(U_g - U_s) = \gamma [U_s - 1 + \exp(U_s - 2U_p - V)] \tag{2}
\]

Deriving the quasi Fermi-potential from (2) yields to:

\[
V = U_s - 2U_p - \ln\left(\frac{U_g - U_p}{\gamma} - (U_s - U_p)\right) \tag{3}
\]

Notice that it is common to express the depletion
charge density $Q_B$ as
\[ Q_B = \gamma \sqrt{U_s - 1} \]  
(4)

and the inversion charge density $Q_I$ is written as
\[ Q_I = Q_T - Q_B = (U_s - U_d) - \gamma \sqrt{U_s - 1} \]  
(5)
in a charge sheet based model. $Q_T$ represents the total space charge density.

Then one can rewrite (3):
\[ V = U_s - \mathcal{X}_j - \ln\left(\frac{Q}{\gamma}\right) = U_s - \mathcal{X}_j - \ln(Q + 2Q_s) + 2\gamma \]  
(6)

Differentiating (6) leads to the gradient of the quasi Fermi-potential that is useful in the drain current:
\[ \frac{dV}{dy} = \frac{dU_s}{dy} - \frac{1}{\gamma} \frac{dQ}{dy} - \frac{1}{\gamma} \frac{d(Q + 2Q_s)}{dy} \]  
(7)

The Pao-Sah’s current equation after normalizing is in the following form:
\[ \frac{I}{L} = \frac{Q}{\gamma} \frac{dV}{dy} \]  
(8)

Substitute (7) into (8) one can obtain the following expression easily:
\[ \frac{I}{L} = \frac{Q}{\gamma} \frac{dU_s}{dy} - \frac{1}{\gamma} \frac{dQ}{dy} - \frac{Q}{\gamma} \frac{d(Q + 2Q_s)}{dy} \]  
(9)

Performing the above integral along the channel from the source edge to the drain edge, we get:
\[ I = \left[ Q dU_s - \frac{Q}{2\gamma} (Q + 2Q_s) \right]_{U_s}^{U_d} \]  
(10)

(10) is derived directly from the surface potential equation (2) and Pao-Sah’s dual integral current equation (8), there is no doubt that the new model is self-consistent. Obviously Brews’s model omitted the last term of the above current equation (9). The approximation in Brews’s charge sheet model, neglecting a positive integral term, always causes the channel current being less than the exact value predicted by Pao-Sah’s model [3]. Also, that is why the compact model based on Brews’s one cannot pass the self-consistency test [4].

Further, when MOSFET works in the depletion mode or the weak inversion mode, the last term of (7) is relative small and the relationship between inversion charge and surface potential is about $\frac{d\ln Q_I}{dU_s} = 1$ for a constant quasi Fermi-potential along the channel. On the other hand, when strong inversion mode comes, the effect of the depletion charge is relative small and the relation $\frac{d\ln Q_I}{dU_s} = \frac{1}{2}$ holds.

It agrees with the benchmark test on charge-sheet models performed in [3].

If integration of (10) is finished completely one can find the result will be the same with the model proposed by VAN DE WIELE in 1979 [5]. But (10) is more clear and direct than VAN’s model.

Unfortunately, the complete integral seems too complex and is not suitable for compact model due to its high request in the computation time.

### III. MODEL TEST AND COMPARISON

In order to apply the new insight of Eq.(10) to drain current model without introducing new parameters, we need to simplify the above obtained current expression from MOSFET device physics. In this section we will try to test some approximations in solving the integration (10) based on numeric calculation. The Pao-Sah’s current model is used as the Golden Reference, calculated from Pierret’s method of a single integration [6]. Also we assume $\mu_s = 550cm^2/V\cdot S$, the constant electron mobility in the surface inversion channel.

#### A. The first approximation and result

The first one approximation to (10) can be obtained by approximating $Q_s$ to in the last integral, since it does not affect the general relationship of $Q_s$ vs $U_s$ in [3]. Upon this approximation we can perform the integral in (10) and obtain the following current expression:
\[ I = \left[ Q_d U_s - \frac{Q}{2\gamma} (Q + 2Q_s) \right]_{U_s}^{U_d} \]  
(11)

The f function is the following form:
\[ f(U_s) = \sum_{Q_s} dQ_s f(U_s) + \sum_{U_s} \frac{Q_s}{Q_d} dU_s = f(U_s) \]  
(11)

where $U_d$ and $U_s$ are surface potentials is the source and drain edges, evaluated from (2) for $V = V_{SB}$ and $V = V_{SD}$, respectively.

In Fig.1 we compare the result from (11) with Brews’ charge sheet model and Pao-Sah model.
B. The second approximation and result

A more accurate one is to approximate \( Q_I + 2Q_B \) to \( Q_I + Q_B \) in the last integral, then one can rewrite the above (11) in a new form:

\[
I = \int_{\theta_0}^{\theta_1} \frac{Q_I}{Q_B} \frac{dU_I}{dQ} + \frac{Q_I}{Q_B} \frac{dU_I}{dQ} = g(U_I) - g(U_O) \quad (12)
\]

The \( g \) function is:

\[
g(U) = \left( U + 2U_I - \frac{1}{2} U_I^2 \right)^2 + 2\gamma \left( U - U_I \right)^2 + 2\gamma \left( U + U_I \right)^2 - \frac{2U}{U - U_I + U_I}
\]

\[
\gamma = \frac{1}{U - U_I + U_I - \frac{1}{2}} \left[ \frac{U - U_I - \frac{1}{2}}{4} + \frac{U - U_I - \frac{1}{2}}{4} \right]
\]

Fig.1 The comparison between Pao-Sah’s model, Brews’s model and approximate one for (a) transfer characteristics (b) output characteristics.

It is shown that both Brews’s model and Eq. (11) agree with Pao-Sah’s model in depletion region and weak inversion region. But there are still some relative errors. We will explain it later. In strong inversion region, as can be seen in Fig.1 (b), the relative error brought by Brews model and our model is about +3% and -0.5%, for \( N_A=5\times10^{17} \) cm\(^{-3} \), about 4% and ± 0.15%, for \( N_A=1\times10^{16} \) cm\(^{-3} \), respectively. The relative error is defined as: \( \eta = \frac{I_{\text{Brews}} - I_{\text{approx}}}{I_{\text{Brews}}} \). A more light doping leads to a small body factor \( \gamma \), the last term of (10) become more important, thus larger errors of Brews’ charge sheet model are observed, identical with the result in [6].

C. The third approximation and result

Based on the recognition of the fact that the depletion charge’s effect in the last term of (9) is not significant in strong inversion, also the surface potential obtained rigorously from (2) and from the depletion approximation are almost the same in weak inversion [7], we approximate the depletion charge density \( Q_d = \gamma \sqrt{U_w} - 1 \) in the last term of (9) and obtain another simple arithmetic of (10):

\[
I = \int_{\theta_0}^{\theta_1} \frac{Q_I}{Q_B} \frac{dU_I}{dQ} + \frac{Q_I}{Q_B} \frac{dU_I}{dQ} = \frac{\sqrt{U_w - 1}}{U_w - U_I + U_I - 1} \quad (13)
\]

where \( h \) function is:

\[
h = \left( U + 2U_I - \frac{1}{2} U_I^2 \right)^2 + 2\gamma \left( U - U_I \right)^2 + 2\gamma \left( U + U_I \right)^2 - \frac{2U}{U - U_I + U_I - 1}
\]

and the depletion approximation surface potential:

\[
U_w' = U_w' + \frac{\gamma^2}{2} - \gamma \left( U - U_I + U_I - 1 \right)
\]
IV. CONCLUSION

In this paper we provide an insight into the drain current model for long channel MOSFET. The reason of Brews’ charge sheet model’s error in predicting the drain current compared with Pao-Sah model and its disability to pass the self-consistency test is explained from the proposed method. Then we test three approximation methods to simplify the complex integral of the drain current formula. The conclusion is that to preserve current model’s accuracy and self-consistency, we should take the complete form of the current formula (9). Some approximations must be needed to trade off between accuracy and complexity.

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