an Efficient Iterative Grid Selection Strategy for Time-Mapped Harmonic Balance Method

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ABSTRACT

In this paper, a new iterative grid selection strategy for the Time-Mapped Harmonic Balance (TMHB) method is presented. This strategy is referred to the adaptive grid methods, which tracks the rapid transitions and constructs a smooth time-map function. The theoretical analysis and simulation results of this grid selection strategy are presented. The results demonstrate that this new strategy improves the numerical stability of the TMHB method and achieves accurate results based on the improved TMHB method.

Keyword: Time-Mapped Harmonic Balance, grid selection strategy , rapid transition

1 INTRODUCTION

Harmonic Balance (HB) methods are the frequency-domain simulation algorithms for circuit simulation. For circuits where the nonlinearities are moderate, HB methods are the preferred algorithms over the time-domain methods such as shooting methods, because they can compute the periodic steady-state with spectral accuracy [1-3]. The preconditioned matrix-implicit Krylov-subspace algorithms have made these methods more efficient for the large circuits [4]. However, for the strongly nonlinear circuits with rapid transitions, the HB methods need a large number of harmonics to present the accurate circuit solution. The Time-Mapped Harmonic Balance method (TMHB) is able to solve this kind of problem, which uses non-uniform grids to resolve the rapid transitions [5-6]. One significant weakness of the method is the grid selection strategies which may not well distribute the grid points and make the algorithm unstable [7].

In this paper, a new iterative grid selection based on adaptive grid methods is developed. It produces a very stable and simple grid generation which yields a better result in the TMHB algorithm. The next section overviews the TMHB algorithm. In Section 3 the theoretical analysis of iterative grid selection strategy is demonstrated and a new iterative grid selection strategy is proposed. In Section 4 the results of the new iterative grid selection strategy are presented. Finally, a conclusion is given in Section 5.

2 TIME-MAPPED HARMONIC BALANCE

For time domain the circuit equations are given by

$$\frac{dq(v(t))}{dt} + f(v(t)) + u(t) = 0 \tag{1}$$

where $u(t)$ is the vector of input source, $v(t)$ is the vector of node voltage, and $f(v(t))$ is the vector of node currents and $q(v(t))$ is the node charge (or fluxes). All these vectors are of size *N*. The periodic steady-state solution of (1) satisfies $v(t+T) = v(t)$.

The TMHB method utilizes a non-uniform grid of M time points to describe the solution waveforms which have rapid transitions by mapping and solving the circuit problem in a new pseudo-time domain. This method uses the time-map function λ to convert a uniform grid in pseudo-time $\hat{t}$ to non-uniform grid in real time t. The time-map function concentrates time points in the sharp regions of the solution waveforms, making the solution waveform $v(t)$ much smoother when viewed in pseudo-time. Then HB methods are used in pseudo-time where the reduction in the truncation error of the pseudo Fourier series approximation is acquired. The time-map function is constructed as:

$$t = \lambda(\hat{t}) = \hat{t} + \sum_{k=-J}^{J} \phi_k e^{j2\pi k f \hat{t}} \tag{2}$$

where *2J+1=S*, S is the sampled non-uniform time points used to construct the time-map function. This construction guarantees its derivative with spectral accuracy in the TMHB method.

In order to describe (1) in pseudo-time $\hat{t}$, using

$$dt = \lambda^{'}(\hat{t})d\hat{t} \tag{3}$$

Substituting (2), (3) into (1):

$$\frac{1}{\lambda^{'}(\hat{t})}\frac{d}{d\hat{t}}q(v(\lambda(\hat{t}))) + f(v(\lambda(\hat{t}))) + u(\lambda(\hat{t})) = 0 \tag{4}$$

The solution waveform $v(t)$ is represented as

$$v(t) = v(\lambda(\hat{t})) = \sum_{k=-K}^{K} \hat{V}_k e^{j2\pi kf\hat{t}} \tag{5}$$

where K is the number of harmonics in the truncation.

Substituting the Fourier representations of $v(t)$ and $u(t)$ in (4), which is expanded in frequency-domain yielding NM nonlinear algebraic equations as:

$$\hat{F}(\hat{V}) = \Gamma\Lambda\Gamma^{-1}\Omega q(\Gamma^{-1}\hat{V}) + \Gamma f(\Gamma^{-1}\hat{V}) + \Gamma u = 0 \tag{6}$$

where Ω is a diagonal matrix expressing time derivation in frequency domain, Γ represents time to frequency translation matrix which in practice is implemented using numerically efficient FFT, Λ is the diagonal matrix

$$\Lambda = \begin{bmatrix} \frac{1}{\lambda^{'}(\hat{t}_1)}I_N & & & & \\ & \frac{1}{\lambda^{'}(\hat{t}_2)}I_N & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots & \\ & & & & & \frac{1}{\lambda^{'}(\hat{t}_M)}I_N \end{bmatrix} \tag{7}$$

The NM nonlinear algebraic equations (6) are solved by applying Newton's method. The iteration equation is

$$J(\hat{V}^{(l)})(\hat{V}^{(l+1)} - \hat{V}^{(l)}) = -\hat{F}(\hat{V}^{(l)}) \tag{8}$$

where l is the Newton iteration index. The *l-th* Newton Jacobian is

$$J(\hat{V}^{(l)}) = \Gamma\Lambda\Gamma^{-1}\Omega\Gamma C\Gamma^{-1} + \Gamma G\Gamma^{-1} \tag{9}$$

In the above equation C is a block diagonal matrix with blocks $C_i = {dq(v(\lambda(\hat{t}_i)))}/{dv}$ and G is a block diagonal matrix with blocks $G_i = {df(v(\lambda(\hat{t}_i)))}/{dv}$. The iteration equation is solved by Krylov-Subspace methods [8].

3 ITERATIVE GRID SELECTION STRATEGY

The time-map function λ, which maps the uniform grid in pseudo-time to the non-uniform grid in the real time, critically affects the efficiency of the TMHB method. In order to preserve the spectral accuracy of the TMHB method, the time-map function must be smooth enough, at least as smooth as the functions describing the device models. Nastov develops an iterative grid selection strategy to determine S non-uniform real time points which are used to construct the time-map function [6].

The initial solution is obtained by solving (1) with a loose convergence tolerance. The time step is set as $h_m = t_{m+1} - t_m$ and h_m needs to satisfy

$$\sum_{m=1}^{S} h_m = T \tag{10}$$

The monitor function *R* which describes the changes of the initial guess solution is

$$R_m = \frac{T}{h_m}\max_j\left(\frac{|v_j(t_{m+1}) - v_j(t_m)|}{F_j}\right) \tag{11}$$

where v_j is the initial guess solution of the j-th circuit equation, and F_j are the solution weights.

The time step h_m adapts in the following way:

$$R_m h_m = \text{constant} \tag{12}$$

for m =1, 2, …, (S-1).

Solving the nonlinear system of (10) and (12) by Newton iteration, we get the real distribution of the grid and construct the time-map function.

This grid selection strategy presents some limits: ① the time step changes fast during the regions with rapid transitions which causes numerical instability. ② The monitor function features the regions which do not have real rapid transitions and makes h_m change irregularly. This brings error during the transform between time domain and frequency domain.

A new iterative grid selection is developed by constructing a new monitor function which is referred to the adaptive grid methods based on [9-10]. Determining the sharp features (rapid transitions) is carried out as follows: the value of R_m in each grid from (11) is compared with the values in each of the adjacent grids. If the value of R_m is the maximum of the three, and bigger than the cut-off value, then the feature is accepted. We let

$$S_j = R_m^{\ j} \tag{13}$$

where j is the index of R_m which satisfies the requirements above.

The smooth monitor function is defined：

$$S_m = 1 + \sum_{j=1}^{n_f} \frac{S_j}{\sqrt{\mu^2 (t_m - x_j)^2 + 1}} \tag{14}$$

where n_f is the number of sharp features, x_j is the time point corresponding to the real sharp feature of S_j, μ is the parameter determining the smoothness of the grid.

Replacing R_m with S_m in (12):

$$S_m h_m = \text{constant} \tag{15}$$

for m =1, 2, …, (S-1).

Then the appropriate grid is got by solving the nonlinear system of (10) and (15) using the Newton iteration. Combining this non-uniform grid with its corresponding uniform grid, a smooth time-map function λ is obtained by (2).

Some remarks can be made. Firstly, the predefined cut-off value is used to determine the number of the real sharp features. For circuits where the nonlinearities are moderate, the value of R_m will be all smaller than the cut-off value. There will be no sharp features and obtain a set of uniform grids. Secondly, the parameter μ is used to determine the smoothness of the grid, which is as increased as the nonlinearities of the circuits. Thirdly, the strategy proposed can get a smooth monitor function, which only features the rapid transitions and do not contain the normal changes in the solution waveform.

4 NUMERICAL EXPERIMENT

The new iterative grid selection strategy introduced in the previous section is implemented in our MATLAB-based circuit simulator. Then the result is showed on the strongly nonlinear power-supply circuit (Fig.1) which is common used in circuit simulation [11-13]. The Nastov's iterative grid selection strategy (TMHB-1) and the iterative grid selection strategy we proposed (TMHB-2) are compared in Fig.2. The number of grid points for the iterative strategy is S = 50, and the exact solution is computed using standard HB method with K = 500. The plots demonstrate that the new iterative grid selection strategy (TMHB-2) is more successful than the Nastov's iterative grid selection strategy (TMHB-1).

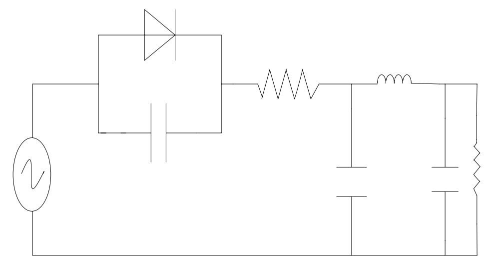

Fig.1 Power-supply circuit Parameters:C1=1 μ F, C2=C3=1mF,L1=0.1H,R1=5 Ω ,R2=1k Ω .

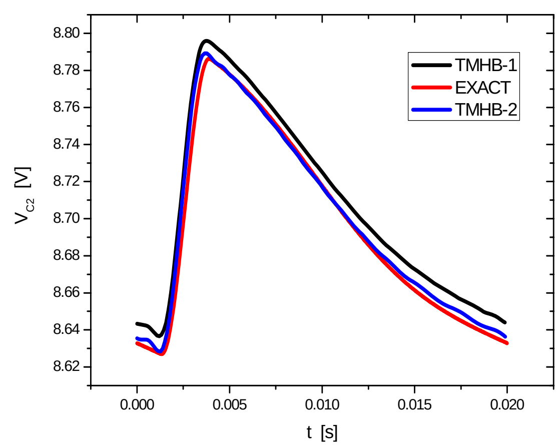

Fig.2 Steady-state response of VC2 in power supply computed with K=15.

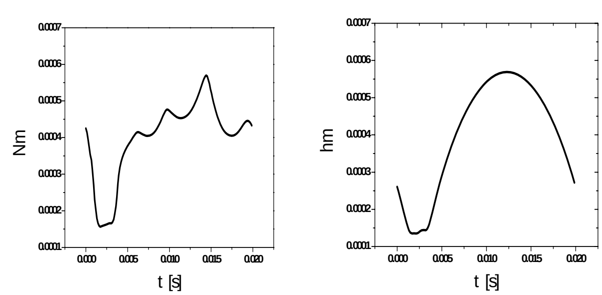

Fig.3The time-steps of iterative grid selection strategies: (A)TMHB-1; (B) TMHB-2.

Fig.3 and 4 show the detail of time-steps, it is illustrated that the Nastov's iterative grid selection strategy excessively increases grids in the regions which do not have sharp features and the ratios of neighboring time-steps are bigger which can cause numerical instability. The strategy we proposed solved this problem: the set of grids that our strategy generates not only catches the sharp features, but also changes much smoother and more stable with smaller ratios of neighboring time-steps.

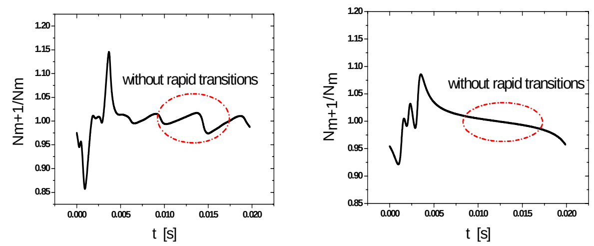

Fig.4 The ratios of neighboring time-steps of iterative grid selection strategies: (A) TMHB-1; (B) TMHB-2.

5 CONCLUSION

In this paper an efficient iterative grid selection strategy is proposed for the Time-Mapped Harmonic Balance method. This strategy is compared with the Nastov's iterative grid selection strategy and the results show the algorithm is more stable and accurate. This new iterative grid selection strategy makes the Time-Mapped Harmonic Balance method more practical.

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