A NOVEL APPROACH FOR MODELING MECHANICAL BEHAVIOR OF POROUS MEDIA

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ABSTRACT

A multi-scale novel homogenization technique is introduced to model mechanical behavior of open-cell porous media. The proposed method consists of primarily four components. The first component is based on two assumptions. First, a random porous structure can be approximated by superimposing regular grids that are interacting with each other at "junction" points. The second component consists of replacing each grid by an equivalent continuum. The forces at the junction points are also replaced by interacting body forces. The third component is to represent the equivalent media by single medium by expressing the "average stresses" in the elastic mixture in terms of the "average displacement" It is discussed how to extract the information about the geometrical and mechanical properties of the grids by comparing the analytical and experimental data for the dispersion of waves propagating in porous medium.

Keywords: porous media, equivalent continuum, gradient elasticity, wave propagation, dispersion

1 INTRODUCTION

Open cell porous media can be found in the nature as light-weight structural elements. Bones in all mammals are typical example for such materials. Open cell porous media are also used as man-made materials for light-weight structural elements, high specific bending stiffness and strengths and as thermal insulators. It is an ongoing active research field to develop methods to predict the mechanical behavior of open cell porous media.

Open cell porous media can be envisioned as a collection of randomly interconnected struts. This collection is often considered that the struts form open cells. The properties, especially the geometrical (cross-section and length) properties of struts may exhibit significant variations from cell to cell,. even within a cell. Mechanical (Young's modulus and Poisson ratio) can also vary within certain limits. The complexity of the structure of the open cell porous media poses difficulty for modeling their mechanical behavior. Finite Element Analysis (FEA) has been proven to be useful [1, 2] to model the mechanical behavior of open cell porous media. Li et al. [1] studied the mechanical response of an open cell porous medium with various cell size and strut cross sectional area. Roberts and Garboczi [2] modeled the mechanical response of open cell porous media created using techniques developed in structural engineering. Although FEM has demonstrated to be efficient in modeling the mechanical behavior of open cell porous media, it can be time-consuming and expensive.

Studies on wave propagation phenomena in porous media have been an attractive research field after WW II [3-5] and it is still an active field. The efforts on the mechanical behavior of open cell porous media resulted in successful analytical approaches [5a,b]. Among others mixture theory has become an useful analytical tool. Truesdell and Toupin [6] developed an axiomatic mixture theory for interacting elastic continua such that each point of space is simultaneously occupied by all constituents of the mixture. It has been used with a remarkable success to model the mechanical behavior of composite materials [7]. In this approaches, it is also assumed that the interactions between the constituents of a mixture are accounted for as interaction forces in the appropriate field equations.

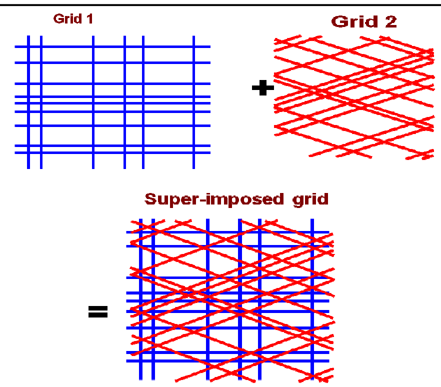
The success of the mixture theory is the main motive of this study which is a need to establish an efficient and accurate modeling approach to predict the mechanical response of open-cell porous materials as a function of the material microstructure. In this study a homogenization process to model the mechanical behavior of the open cell porous materials is introduced. The method is introduced in the next section. The wave propagation in a porous medium is discussed which is followed by a conclusion.

2 AN HOMOGENIZATION METHOD

The method proposed here is an improved version of the homogenization technique called "rule of mixture" [8]. Our method differs from the classical "rule of mixtures" on the following points. The major difference of our multi-scale homogenization method is that it is based on an assumption that a open cell porous medium which is random in geometrical and mechanical properties can be represented by superimposed regular grids. Also, interaction between the constituents of the mixture is taken into account. Second, a multi-scale homogenization process is applied where each constituent is replaced by an equivalent continua and the mixture is represented by a single media employing appropriate "average displacement field" and "average stress field". The final outcome of this method is a constitutive equation of "nonlocal" or "gradient dependent" where the constitutive constants are explicitly expressed in terms of the microstructural parameters of the open cell porous media. Our early works in this field are reported somewhere else [9-12].

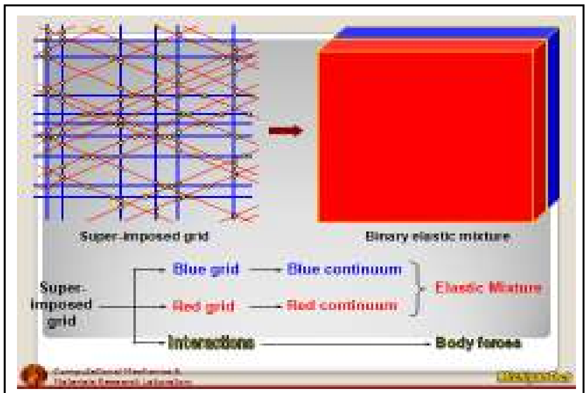
Our method consists of primarily four steps.

Step 1 is an assumption that an open cell porous media can be approximated as geometrically and mechanically close as desired by superimposing regular grids. This assumption is actually a mathematical conjecture which needs a rigorous proof. A proof of this conjecture formulating the problem as a multivariable Fourier series is under development. In this step, the interaction forces between the regular grids are also taken into account.



grids

Step 2 is to replace each regular grid by an equivalent media. For the fundamentals of such a process one can consult Gibson and Ashby [5] if the bending and torsion response of the grids are ignored. Recently, an homogenization process taking into account the bending and torsion strength using micropolar elasticity [13] has also been suggested. In this step, the interaction forces between the grids are homogenized as body forces proportional with the difference of the displacements of the constituents.



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There are several methods developed to determine the properties of the equivalent media. For example Zhang [14] reported the following results for a rectilinear grid.

Young's modulus

$$\frac{1}{E} = \frac{1}{E_s}\frac{(R-1)(9R^3 - 119R^2 + 45R - 15)}{80R^3}$$

and the Poisson ratio

$$\nu = \frac{3}{2}\frac{13R^3 - 13R^2 - 15R + 5}{9R^3 - 119R^2 + 45R - 15}$$

Here,

$$R = \frac{a}{l}$$

where a, l is the side length of the square cross-section and the length of the struts, respectively. E_s is the Young's modulus of the strut's material, and $\nu_s = 0.3$ is the Poisson ratio.

Equivalent media constitute na interacting binary elastic mixture and its elastic behavior can be represented with the following equations:

$$\sigma^1_{ij,j} = 2\alpha(1-\nu)(u^1_i - u^2_i) \quad (1)$$

$$\sigma^2_{ij,j} = -2\alpha\nu(u^1_i - u^2_i) \quad (2)$$

Step 3 consists of defining appropriate average and difference displacements and average stresses for the interacting elastic mixture.

Average Displacement:

$$u_i = \nu u^1_i + (1-\nu)u^2_i \quad (3)$$

Difference Displacement

$$v_i = (u^1_i - u^2_i)/2 \quad (4)$$

Average Stress

$$\sigma_{ij} = \nu\sigma^1_{ij} + (1-\nu)\sigma^2_{ij} \quad (5)$$

The outcome of this step, is relationship for the average stress in terms of average and difference displacements and a relationship between the average displacement and the difference displacements. The rest of this process consists of eliminating the difference displacement between two field equations of each medium and expressing the average stress in terms of the average displacement.

Step 4 is about eliminating the difference displacement between the difference displacement field and the average displacement field. The resulting equation is a nonlocal relationship between the average stress field and the average displacement field. The difference displacement satisfies the following partial differential equation.

$$\alpha\frac{2}{\mu'}\left\{2\frac{\lambda''\mu' - \lambda'\mu''}{3\lambda' + 2\mu'}v_{k,k}\delta_{ij} + \mu''(v_{i,j} + v_{j,i})\right\} - (1-2\nu)(v_{i,j} + v_{j,i})_{,ll} = (u_{i,j} + u_{j,i})_{,ll} \quad (6)$$

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The average stress can be expressed as follows.

$$\sigma_{ij} = [\nu\lambda_1 + (1-\nu)\lambda_2]u_{k,k}\delta_{ij} + [\nu\mu_1 + (1-\nu)\mu_2](u_{i,j} + u_{j,i}) + 2\nu(1-\nu)\{(\lambda_1 - \lambda_2)v_{k,k}\delta_{ij} + (\mu_1 - \mu_2)(v_{i,j} + v_{j,i})\} \tag{7}$$

If the difference displacement field is solved from Eq. (6) using Green formalism and is substituted above, the resulting constitutive equation is obtained. Please note that this constitutive relation is of nonlocal form. Although, if it is assumed that

$$\nabla^2(v_{i,j} + v_{j,i}) \ll (v_{i,j} + v_{j,i}) \tag{8}$$

The resulting constitutive equation becomes

C

$$\sigma_{ij} = [\nu\lambda_1 + (1-\nu)\lambda_2]u_{k,k}\delta_{ij} + [\nu\mu_1 + (1-\nu)\mu_2](u_{i,j} + u_{j,i}) + \nu(1-\nu) \times \left\{\frac{1}{\alpha}\frac{\mu''(\lambda_1-\lambda_2)(3\lambda'+2\mu') + 2(\mu_1-\mu_2)(\lambda'\mu''-\lambda''\mu')}{\mu''(3\lambda''+2\mu'')}u_{k,kll}\delta_{ij} + \frac{(\mu_1-\mu_2)}{\alpha}\frac{\mu'}{\mu''}(u_{i,j}+u_{j,i})_{,ll}\right\} \tag{9}$$

In these expressions, the following terms are used.

$$\lambda' = \lambda_1\lambda_2 + 2\lambda_1\mu_2 + 2\lambda_2\mu_1 + 2\mu_1\mu_2$$

$$\lambda'' = -\nu\lambda_1 + (1-\nu)\lambda_2 \quad , \quad \mu' = \mu_1\mu_2$$

$$\mu'' = -\nu\mu_1 + (1-\nu)\mu_2$$

Please note the final result is a constitutive equation of gradient depedent type.

3 DISPERSION RELATION

The mechanical and geometrical properties of the grids used in modelling an open-cell porous media can be determined by comparing the theoretical and experimental dispersion relation. This idea is demonstrated below in a simple manner.

Let usconsider the following displacement field.

$$u(x,y,z,t) = Aexp[i(kx - \omega t)] \tag{10}$$

$$v(x,y,z,t) = 0 \ , \ w(x,y,z,t) = 0$$

Here, k, ω are the wave number and the frequency, respectively. The corresponding strain and stress are

$$\varepsilon(x,t) = Aexp[i(kx - \omega t)] \tag{11}$$

$$\sigma(x,t) = (\lambda + 2\mu)\left[\varepsilon(x,t) - C\frac{\partial^2\varepsilon(x,t)}{\partial x^2}\right] \tag{12}$$

Here, the material constant C is defined in accordance with the constitutive equation given by Eq. (9). Substituting this stress field into the equation of motion, that is

$$\frac{\partial\sigma(x,t)}{\partial x} = \rho\frac{\partial^2 u(x,t)}{\partial t^2} \tag{13}$$

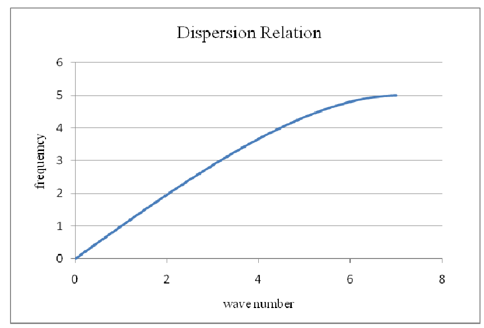
yields

$$k^2[1 + C^2k^2] = C_L^2\omega^2 \tag{14}$$

Here, ρ is the mass density, and

$$C_L = \sqrt{\frac{\rho}{\lambda+2\mu}}$$

is the phase velocity of the longitudinal waves. This relationship is called the *dispersion relation* for the propagation of a longitudinal harmonic wave in an elastic media where the constitutive relation is defined by Eq. (9). This dispersion relation is depicted in the figure below.



ion for a porous medium

Let us assume a set of experimental data of wave number-frequency is given.

$$\{k_i, \omega_i\}_{i=1}^n \tag{15}$$

The material constants (λ, μ, ρ, C) can be determined by using the following scheme. Let us consider the following sum.

$$S = \sum_{i=1}^n \left\{\omega - \frac{1}{C_L}k_i\sqrt{1 + C^2k_i^2}\right\}^2 \tag{16}$$

The material constants (λ, μ, ρ, C) can be determined by making this sum minimum, which can be attained by imposing the follwing conditions.

$$\frac{\partial S}{\partial\lambda} = 0$$

$$\frac{\partial S}{\partial\mu} = 0$$

$$\frac{\partial S}{\partial\rho} = 0$$

$$\frac{\partial S}{\partial C} = 0 \tag{17}$$

4 CONCLUSION

An An homogenization process for open cell porous media is proposed. The resulted constitutive equation is of nonlocal (or gradient dependent, it if is approximated). character. It has been also shown that wave propagation in nonlocal (an in gradient dependent) elasticity is dispersive. This property can be used to determine the properties of bones which will be main goal of the forthcoming paper.

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