A comparative study of nanofluids exchanging heat with a thermal energy reservoir

J.A.R Parise*

*Pontifícia Universidade Católica do Rio de Janeiro, Department of Mechanical Engineering, Rua Marquês de São Vicente, 225, Gávea, 22453-900, Rio de Janeiro, RJ, Brazil, parise@puc-rio.br

ABSTRACT

For their enhanced thermal conductivity and heat transfer characteristics, considerable attention has been recently given to nanofluids, although a literature survey would show that much more attention has been given to the first parameter. However, one of the main applications of nanofluids as heat transfer fluids would be in heat exchangers, where the other thermal resistances, as well as the increased viscosity, would tend to reduce the nanofluid-induced heat transfer enhancement. A more realistic approach is here attempted, where a general model for the process of heat exchange between a nanofluid and a thermal energy reservoir is presented. A comparative analysis of the heat transfer rate and pumping power is carried out between a nanofluid and its pure base fluid. To this end, non-dimensional gain factors were defined, thus providing a more realistic and general view of the potential of nanofluids when applied in heat exchangers.

Keywords: nanofluids, heat transfer, pressure drop, heat exchanger

1 INTRODUCTION

Nanofluids are engineered colloids made of a base fluid and dispersed nanoparticles [1]. Yu et al [2], for example, list results from several authors showing the unequivocal thermal conductivity enhancement from nanofluids. For the heat transfer coefficient, however, enhancement ratios below unity were not uncommon. Other analyses of the heat transfer performance of nanofluids, as compared to their base fluid, so far have been carried out on the basis of the thermal conductivity and the heat transfer coefficient enhancements. However, the nanofluid-side heat transfer coefficient, or the Nusselt number, is a relevant yet incomplete indicator to describe the effect of a nanofluid in the thermal performance of a heat exchanger. As a first step to describe the effect of nanofluids is performing as heat transfer fluids, a comparative analysis of the heat transfer rate and pumping power is carried out for a Al₂O₃-water nanofluid exchanging heat with a constant temperature thermal energy reservoir, with one-pass fully developed laminar flow. For the purpose of the comparison, non-dimensional gain factors, for pressure drop and heat exchange rate, related to pumping power and thermal capacity, respectively, were defined.

2 MATHEMATICAL MODEL

For the comparative analysis of nanofluids for heat exchange applications, a known approach is employed, in which the fluid heat transfer and pressure drop equations are written in terms of two groups: one for operational and geometrical characteristics and the other for thermophysical properties. The first group is independent of the fluid used, and the latter is then used for the fluid comparison. This approach is here applied for Al₂O₃-water nanofluid.

2.1 Pumping Power

A pumping power gain factor, \( \xi_p \), is defined as:

\[
\xi_p = \left( \frac{\dot{W}_{p,nf}}{\dot{W}_{p,w}} \right)
\]

where the water and nanofluid pumping powers, \( \dot{W}_{p,w} \) and \( \dot{W}_{p,nf} \), are related to the fluid pressure drop, \( \Delta P_f \), by:

\[
\dot{W}_p = \left( \Delta P_f w A_c \right) / \eta_p
\]

where \( w \) is the fluid average velocity, \( A_c \), the conduit cross sectional area and \( \eta_p \), the pump overall efficiency. Assuming that these three variables remain unchanged between nanofluid and base fluid, equation (2) becomes:

\[
\xi_p = \left( \frac{\Delta P_{f,nf}}{\Delta P_{f,w}} \right)
\]

The frictional pressure drop is evaluated by:

\[
\Delta P_f = f \left( \frac{w^2}{2} \right) \rho \left( L_p / D \right)
\]

where \( L_p \) is the conduit total equivalent length, \( D \) is the conduit diameter (if not circular, the hydraulic diameter concept could be applied) and \( \rho \), the fluid density. It is assumed that pressure drop correlations for single-phase incompressible flow also apply to nanofluids [3]. The friction factor, \( f \), for laminar flow is:

\[
f = \left( 64 / Re \right) ; \quad Re = < 2.3 \times 10^1
\]
where \( Re \) is the Reynolds number and \( \mu \), the dynamic viscosity of the fluid.

\[
Re = \frac{(\rho \ w \ D)}{\mu} \tag{6}
\]

Substituting (5) and (6) in (4):

\[
\Delta P_f = 32 \ \mu \ w (L_f/D^2) \tag{7}
\]

and, therefore:

\[
\xi_f = \left( \frac{\mu_f}{\mu_w} \right) \tag{8}
\]

Equation (8) shows that, for laminar flow, the fluid viscosity is the key thermophysical property in the increment of pumping power due to nanofluid use.

### 2.2 Heat Exchange

Two gain factors, \( \xi_h \) and \( \xi_q \), defined as the ratios between the heat transfer coefficients, \( h_{nf} \) and \( h_w \), and heat exchange rates, \( \dot{Q}_{nf} \) and \( \dot{Q}_w \), respectively, are introduced:

\[
\xi_h = \left( \frac{h_{nf}}{h_w} \right) \tag{9}
\]

\[
\xi_q = \left( \frac{\dot{Q}_{nf}}{\dot{Q}_w} \right) \tag{10}
\]

For the purpose of comparison between heat transfer coefficients, the Sieder-Tate correlation [4], relating the Nusselt number to the Reynolds and Prandtl numbers, is used:

\[
Nu_{nf} = 1.98 \left[ \left( \frac{Re_{nf} Pr_{nf}}{L_q/D} \right) \right]^{0.14} \tag{11}
\]

where

\[
Nu = h D/k \tag{12}
\]

\[
Pr = \frac{\mu c_p}{k} \tag{13}
\]

The heat transfer coefficient is \( h \) and \( \mu \) is the fluid dynamic viscosity at the channel wall temperature. In the present work, for simplicity, and given the relatively low viscosities of the fluids involved, it is assumed that \( \left( \frac{\mu}{\mu_w} \right) = 1 \). By separating equation (11) into two groups of variables, one (composed by fluid properties) with those that vary with the change from pure water to nanofluid and the other with those that do not (heat exchanger geometry and operating conditions), the water-side film coefficient can be evaluated by:

\[
h_w = 1.86 \left( \frac{\rho_w k_w^2 c_{p,w}}{\mu_w} \right)^{1/3} \left[ \frac{w(D L_q)}{D} \right]^{1/3} \tag{14}
\]

where \( L_q \) is an equivalent heat transfer length, \( w \), the fluid velocity and \( \rho \), fluid density.

Velagapudi et al. [5] analyzed the internal flow of nanofluids through straight tubes of circular section. For laminar flow they proposed a modified version of the Sieder-Tate correlation. Constant 1.98 was obtained by regression analysis of experimental data from [6] and [7].

\[
Nu_{nf} = 1.98 \left[ \left( \frac{Re_{nf} Pr_{nf}}{L_q/D} \right) \right]^{0.14} \tag{15}
\]

and, analogously to equation (14),

\[
h_{nf} = 1.98 \left( \frac{\rho_{nf} k_{nf}^2 c_{p,nf}}{\mu_{nf}} \right)^{1/3} \left[ \frac{w(D L_q)}{D} \right]^{1/3} \tag{16}
\]

Therefore, taking (16) into (9), one has:

\[
\xi_q = 1.065 \left[ \left( \frac{\rho_{nf} k_{nf}^2 c_{p,nf}}{\rho_w k_w^2 c_{p,w}} \right) \right]^{1/3} \tag{17}
\]

Applying the \( \varepsilon-\)NTU method [4], the rate of heat exchange is:

\[
\dot{Q} = \dot{m} c_p \varepsilon \Delta T_{\text{max}} \tag{18}
\]

where \( \dot{m} = \rho w A \) is the fluid mass flow rate, \( c_p \), the fluid specific heat at constant pressure and \( \varepsilon \), the heat exchange effectiveness. The maximum temperature difference, \( \Delta T_{\text{max}} \), is the temperature difference between the inlet fluid (water or nanofluid) and the thermal energy reservoir. For heat exchange between a fluid and a constant temperature medium (which could be a thermal energy reservoir or a boiling or condensing pure fluid), the effectiveness is:

\[
\varepsilon = 1 - \exp \left\{- \left[ \frac{(h_A)}{(\dot{m} c_p)} \right] \right\} \tag{19}
\]

where \( A \) is the overall heat transfer area. Substituting equations (18) and (19) into (10):

\[
\xi_q = \frac{\rho_{nf} c_{p,nf} \rho_w}{c_{p,w}} \frac{1 - \exp \left\{- \left[ \frac{(h_A)}{(\dot{m} c_p)} \right] \right\}}{1 - \exp \left\{- \left[ \frac{(h_A)}{(\dot{m} c_p)} \right] \right\}} \tag{20}
\]

For single-pass flow, the heat transfer area to fluid total mass flow rate ratio, \( (A/\dot{m}) \), reduces to:

\[
(A/\dot{m}) = \frac{4 L_q}{(\rho D w)} \tag{21}
\]
Taking (21) into (20):

\[
\frac{\xi_{nf}}{\xi_{w}} = \frac{\rho_{nf} c_{p,nf}}{\rho_{w} c_{p,w}} \left[ 1 - \exp \left\{ - \left[ 4 \frac{L_{f}}{(Dw)} \right] \frac{h_{w}}{\left( \rho_{nf} c_{p,nf} \right)} \right\} \right]
\]

(22)

3 NANOFLUID CHARACTERIZATION

Assuming spherical and well dispersed particles, the thermal conductivity from the following equation [8,9]:

\[
k_{nf} = \frac{k_{p} + 2k_{w} + 2\phi_{p} \left( k_{p} - k_{w} \right)}{k_{w} + 2k_{w} - \phi_{p} \left( k_{p} - k_{w} \right)}
\]

(23)

where \( \phi_{p} \) and \( k_{p} \) are the particle volumetric fraction and thermal conductivity, respectively.

The correlation for the dynamic viscosity of \( \text{Al}_{2}\text{O}_{3}\)-water nanofluid is provided by Pak and Cho [6].

\[
\left( \frac{\mu_{nf}}{\mu_{w}} \right) = 1 + 39.11\phi_{p} + 533.9\phi_{p}^{2}
\]

(24)

Density and specific heat of the nanofluid were calculated from mass and energy balances, respectively.

\[
\rho_{nf} = \rho_{p}\phi_{p} + \rho_{w} \left( 1 - \phi_{p} \right)
\]

(25)

\[
c_{p,nf} = \left[ \frac{\rho_{p} c_{p,w} \left( 1 - \phi_{p} \right) + \rho_{p} c_{p,nf} \phi_{p}}{\rho_{nf}} \right]
\]

(26)

Thermophysical properties of \( \text{Al}_{2}\text{O}_{3}\) are compiled in [5].

4 RESULTS

The model was applied to a comparative analysis of \( \text{Al}_{2}\text{O}_{3}\)-water nanofluid in one-pass single-phase laminar flow exchanging heat with a thermal energy reservoir. Typical values were assumed for the process, namely: \( w = 0.2 \text{ m.s}^{-1}, L_{f} = 5 \text{ m and} D = 10 \text{ mm.} \) Thermophysical properties of both water and nanofluid were taken at the fluid inlet temperature, \( T = 20^o\text{C}. \) Plots were prepared for different values of particle volumetric fraction, \( (0 < \phi_{p} < 0.1). \)

Figure 1 shows the significant increase in pumping power, just above 10 times, for this particular application, when the volumetric fraction approaches 10%. Such gain in power consumption is due, as clearly seen in equations (8) and (24), to the increase of the fluid viscosity of a nanofluid. In certain applications, the relatively higher pumping power may become a relevant issue.

Figure 2 displays, in one single plot, the calculated gain, or enhancement, factors for the thermal conductivity, heat transfer coefficient and heat transfer rate. The thermal conductivity gain factor tends to 1 as the particle volumetric fraction approaches zero. This does not happen with the heat transfer coefficient enhancement curve. Such behaviors can both be explained, of course, by the nature of equations (17) and (23). In this respect, the heat transfer rate enhancement follows the heat transfer coefficient curve. Note that, for moderate volumetric fractions, the gain factor for the heat transfer coefficient surpasses the thermal conductivity, probably with the help of the also enhanced density of the nanofluid, in spite of a reduction in the specific heat. It can be observed that, in Figure 2, for this particular application, the heat transfer rate between fluid and thermal energy reservoir benefits from the thermal conductivity enhancement with the presence of nanoparticles, yet at a lesser rate.

Figure 1: Pumping power gain factor with single-phase laminar flow of \( \text{Al}_{2}\text{O}_{3}\)-water nanofluid as a function of particle volumetric fraction.

Figure 2: Gain factors for thermal conductivity, heat transfer coefficient and heat transfer rate with single-phase laminar flow of \( \text{Al}_{2}\text{O}_{3}\)-water nanofluid as a function of particle volumetric fraction.
5 CONCLUSION

Results presented to date in the literature have made clearer the trends and magnitudes of nanofluid heat transfer enhancement [2]. The thermal conductivity enhancement, for instance, in certain conditions and for certain materials, is considerable. However, in practical applications, it may be somehow counterbalanced by how other properties of the fluid that are equally affected by the increase of nanoparticle concentration, and by the heat transfer mechanism in itself. The present study took these effects into account, bringing, for a simple case, the comparison between nanofluid and its base fluid to a more realistic level. It is hoped that the present analysis can help contribute to a better understanding on of the potential of nanofluid applications in heat exchangers.

6 ACKNOWLEDGEMENTS

Thanks are due to CAPES, FAPERJ and CNPq, Brazilian research funding agencies, for financial support.

REFERENCES