Improved Modeling of the Comb Drive Levitation Effect

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ABSTRACT

Lumped comb drive models found in the literature ignore the electrostatic levitation effect, which decreases their accuracy. The levitation effect occurs when the substrate is within close proximity to the comb drive. Previously, modeling the levitation effect on a large number of comb drive fingers was limited to finite element analysis (FEA) and boundary element analysis (BEA). In this paper, we show that the levitation effect can be modeled more quickly and precisely by using Schwarz-Christoffel mapping (SCM). Our method is several times faster than FEA-based method because it does not descretize the subdomains and boundaries into a large number of coupled equations. We find that the equilibrium levitation position is $1.239\mu m$, which is slightly larger and more precise than previously reported values. Our improvement in accuracy and precision is most likely due to SCM's ability to treat the electrostatic fields at the corner edges exactly. So others can make use of our efforts, we have developed an online tool based on our present algorithm that is available at nanoHUB.org through a web interface and remote computation.

Keywords: Schwarz-Christoffel mapping, comb drive levitation, online simulation tool, MEMS modeling and simulation

1 INTRODUCTION

The comb drive levitation effect was first reported about two decades ago in [1]. The levitation effect is caused by an asymmetric electric field about the comb fingers when the substrate is within close proximity to the fingers. If the substrate is at the same potential as the fingers, then the surface charge density on the surface of the finger that is nearest to the substrate is lower than the surface charge density on the surface that is furthest way from the substrate. Static surface charge on a conductor produces a surface pressure that is outward and normal to the surface. This surface pressure is proportional to the surface charge density squared. Hence, the greater the surface charge, the greater the surface pressure. Therefore, the net surface pressure about the comb finger is directed away from the substrate, which forces the finger away from the substrate. This phenomenon is the so-called levitation effect. The levitation effect usually applies to devices with an underlying substrate and a relatively low out-of-plane stiffness. The effect can be substantial for microdevices that have an out-of-plane stiffness on the same order as the driving mode in-plane stiffness. For example, in [2] Painter reports that both the scale factor and the cross-axis sensitivity of their capacitive sensing gyroscope could deteriorate due to the levitation effect. For high-precision devices, an improved model of the levitation effect may be used to either reduce the effect or exploit the effect.

Lumped comb drive models found in the literature ignore the electrostatic levitation effect, which can decrease the accuracy of the model. Finite element analysis (FEA) and boundary element analysis (BEA) have been used to compute the equilibrium levitation position on a small number of comb fingers in [1], [3]. However, the computational effort required for such analyses on large systems may be prohibitive on today's personal computers. In this paper, we use Schwarz-Christoffel mapping (SCM) to quickly, accurately, and precisely model the levitation effect for a large number of comb fingers. Its increased speed mostly due to it not having to discretize the subdomains and boundries into a multitude of coupled equations. And its increased accuracy and precision is due to its ability to treat electrostatic fields exactly at the corner edges of geometries. Discritized methods are relatively inaccurate at such vertices, and their precision relies on both the refinement of the discretization, configuration, and quality of the mesh. In addition, our method accounts for the presence of a large number of neighboring comb fingers.

Our SCM-based model reduces the computational cost and can be used to determine the electrostatic levitation forces for lumped models. We have developed an easy-to-use tool based on our algorithm that is publically available at nanoHUB.org through a web interface and remote computation.

The rest of this paper is organized as follows. In Section 2 we present our SCM algorithm. In Section 3 we present our results and compare them against FEA and BEA methods. And in Section 4 we conclude what we have learned with this effort.

2 ALGORITHM

The comb drive configuration under investigation is shown in Figure 1a. It consists of an array of interdigitated fingers. Figure 1b shows a cross section of a part of this array, showing a levitated rotor finger. The domain of one pair of fingers is identified by dashed lines in Figure 1b. In Figure 1c, we show our SCM configuration which captures the levitation contributions due to the multitude of other fingers. We capture the effect of the periodic array in the model by reflecting an image of the domain to both the left and the right directions. The boundaries of the substrate and the rotor finger are grounded, and the boundaries of the stator finger have a constant voltage *V*. They have Dirichlet boundary conditions. The reflective boundaries have Neumann boundary conditions, i.e. $\partial V / \partial y = 0$.

We use a MATLAB-based [4] SCM tool by Driscoll [5] to model the domain shown in Figure 1c as a simply-connected polygon. The potential distribution, $\Phi(y,z)$, within the domain can be obtained. The electric field $\vec{E}(y,z)$ is equal to the negative gradient of the potential distribution [6],

$$\vec{E}(y,z) = -\nabla\Phi(y,z). \tag{1}$$

We use Newton's difference quotient to find the electric field component in z-direction, $\vec{E}(y,z)$, along the upper and lower boundaries of the rotor finger numerically,

$$\vec{E}(y,z) = -\frac{\Phi(y,z+\Delta h)-\Phi(y,z)}{\Delta h}, \tag{2}$$

where Δh is the distance between the rotor finger boundary and the segment $\overline{AB}$. We assume that both the rotor and the stator are good conductors so that the electrostatic pressure component in z-direction can be calculated as [6]

$$P_Z(y,z) = \frac{1}{2}\varepsilon_0 \left|\vec{E}(y,z)\right|^2, \tag{3}$$

where $\varepsilon_0 = 8.854\times10^{-12}[F/m]$ is the permittivity of free space. The electrostatic forces per unit length along the boundaries, F_{upper} and F_{lower}, can be calculated as

$$F_Z(y_0,z) = P_Z(y_0,z)\,dy, \tag{4}$$

where *dy* is the length of the small interval containing the point y_0.

The flow of the algorithm is summarized in Figure 2. We repeat the process for different levitation position of the rotor finger in order to find the equilibium levitation position where the net electrostatic pressure is zero.

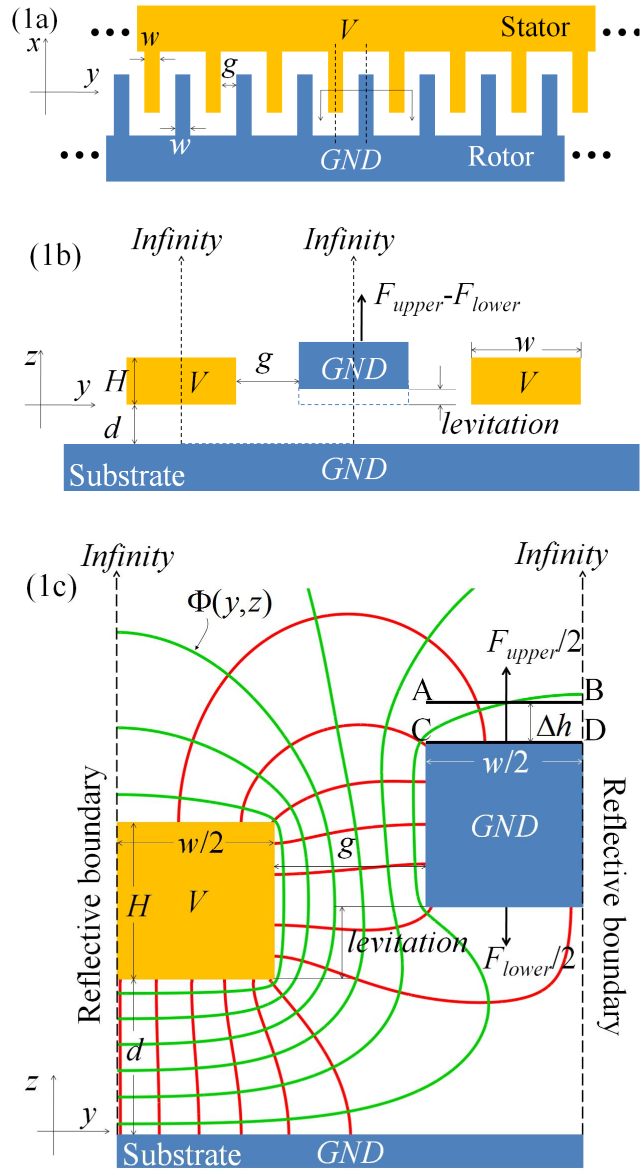

Figure 1: Comb finger configuration. In 1a-1c, the stators have a surface potential of *V*, and the rotors and substrates are grounded *GND*. In 1a we show an overhead view of a comb drive with a large number of comb fingers of width w and gap g. We consider a subset of the periodic array of comb fingers that captures the contributions of the large number other fingers through reflective boundaries. In 1b we show the levitation of the rotor, its net electrostatic force, and the substrate. The electrostatic force consists of a net distributed force normal to the upper and lower boundaries of the rotor F_{upper} and F_{lower}. In 1c the vertical dashed lines are reflective Neumann boundaries and the substrate and the boundaries of the fingers are Dirichlet boundaries. The quantities *H* and *d* are the layer thickness and initial gap between the finger and substrate. The potential distribution along the segment $\overline{AB}$ a small distance Δh above the rotor is used to find the electrostatic pressure along the upper boundary of the rotor finger by (2)-(4). The size of Δh is exaggerated in 1c for clarity.

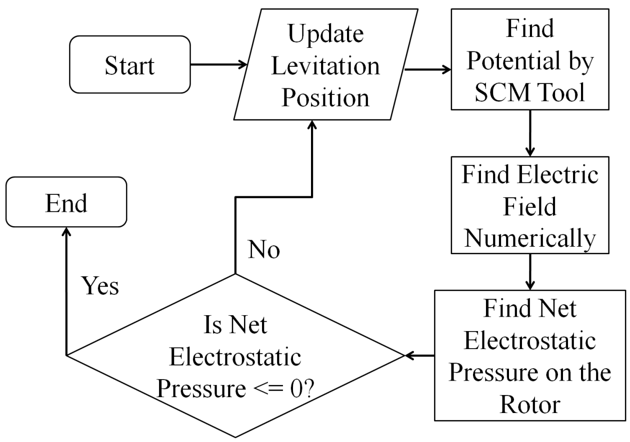

Figure 2: The flowchart of our SCM algorithm.

3 RESULTS

To compare our SCM model with the results obtained by others in [1] and [3], we use the same geometric dimensions. That is, the width, *w*, and the thickness, *H*, of the comb drive fingers are $4\mu m$ and $2\mu m$, respectively. The air gap *g* between the stator and the rotor fingers is $2\mu m$. And the initial air gap *d* between the fingers and the substrate is $2\mu m$.

3.1 Potential Distribution

To verify the potential distribution found by our SCM tool, we compare our results with an FEA-based tool called COMSOL 3.5a [7]. The domain under investigation is shown in Figure 1c. The levitation of the rotor finger is $1\mu m$. The voltage applied on the stator finger is 20*V*. The potential along the segment $\overline{AB}$ found by both methods are compared in Fig. 3b. We choose the distance between $\overline{AB}$ and the upper boundary of the rotor finger to be $\Delta h = 0.01\mu m$. The choice of this distance is discussed in Section 3.2.

We perform the mesh convergence analysis in COMSOL, as shown in Figure 3a. The interval between the evaluation points along $\overline{AB}$ is $0.01\mu m$. The maximum relative change of potential is defined as the maximum of the ratio of the potential difference at each evaluation point with current mesh condition to the potential at the same evaluation point with the previous mesh condition. Therefore, the maximum relative change with the first mesh condition is not defined, shown as N/A in Fig. 3a. We stop the mesh refinement until the maximum relative change decreases to 9.65%. The simulation of the model with further mesh refinement requires more than 4GB computer memory.

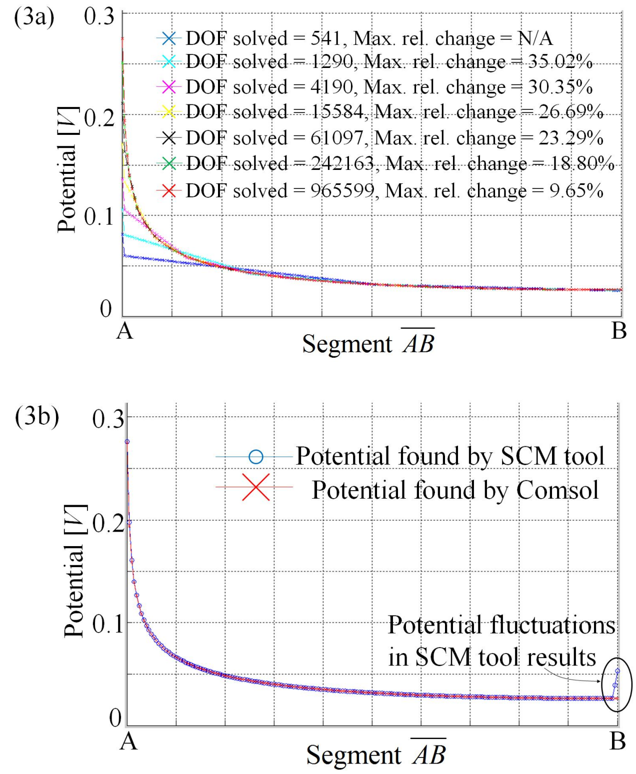

Figure 3: Potential distribution. In (3a), we show the potential distribution along $\overline{AB}$ with seven different meshing conditions. In (3b), we compare the potential distribution along $\overline{AB}$ found by SCM against the one with the finest meshing refinement in (3a).

The potential values along $\overline{AB}$ found by SCM and COMSOL are compared in Figure 3b. The maximum relative difference is 1.69% if we ignore the potential fluctuations noted with the black circle in Fig. 3b. We find that the potential fluctuations happen when $\overline{AB}$ is close enough to the rotor finger. For example, the potential distribution is smooth when Δh is $0.1\mu m$. We think the undesired fluctuations are caused by the numerical error in SCM tool, but further investigations are needed.

3.2 Electric Field, Pressure, and Levitation

Equation (2) and (3) are used to find the electric field and the electrostatic pressure along the upper and lower boundaries of the rotor finger. We set the step size Δh as $0.1\mu m$ where the SCM potential numerically fluctuates to avoid numerical instability, and as $0.01\mu m$ elsewhere along the boundaries. The electric field and the electrostatic pressure along the upper boundary of the rotor finger found by our SCM tool are compared against COMSOL results in Figures 4a and 4b, respectively. The maximum relative differences between the SCM results and the COMSOL

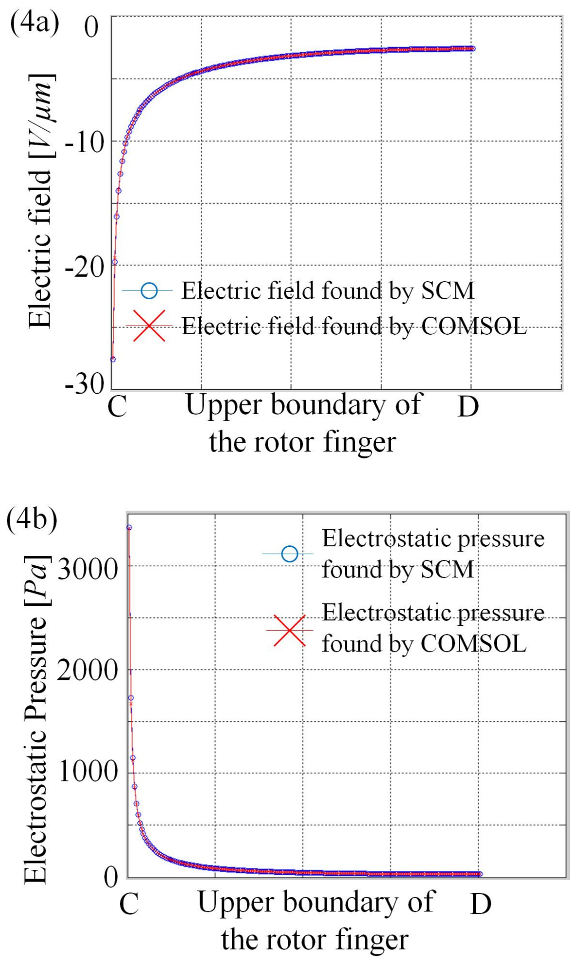

Figure 4: Electric field and electrostatic pressure along the upper boundary of the rotor finger shown in Fig. 1c. In (4a), we compare the electric field against the COMSOL results, and the relative error is 1.69%. In (4b), we compare the electrostatic pressure against the COMSOL results, and the relative error is 3.4%.

results are 1.69% for the electric field and 3.4% for the pressure. And the results are smooth along the entire boundary after choosing larger step size for the positions where the potential fluctuates.

We plot the average net electrostatic force per unit length of the rotor finger with respect to different levitation position for three applied voltages in Figure 5. The equilibrium levitation position is about 1.239μm. As expected, the equilibrium levitation position is independent of nonzero applied voltage. Our result is larger than those reported previously, 1.22μm in [1] by FEA and 1.19*um* in [3] by BEA. We think this is because our SCM configuration captures the levitation contributions due to the multitude of other fingers in the comb finger array and due to a more accurate and precise electrostatic pressure at the corner edges.

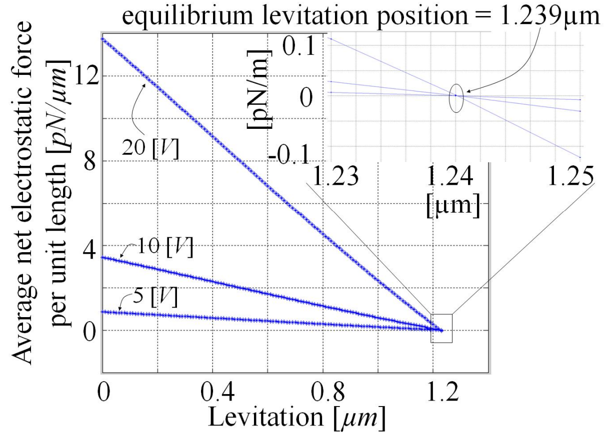

Figure 5: Average net electrostatic force per unit length of the rotor finger versus the levitation distance at three different voltages. As expected, the equilibrium levitation displacement is independent of the voltages. As shown in the inset, the equilibrium levitation position is about 1.239μm.

4 CONCLUSION

We presented an improved modeling method for the comb drive levitation effect based on SCM. Our method is faster, and more accurate and more precise than previous efforts based on discretized methods. In comparison, SCM is more computationally efficient and treats vertices exactly. We found that the equilibrium levitation position is 1.239μm, which is 1-10% larger than the values reported previously. We think this is because our SCM configuration captures the levitation contributions due to the multitude of other fingers in the comb finger array and accurately accounts for the fringing field charges at the corners of the geometry. With FEA analysis converged to 10%, our SCM method is over 3 times as fast. We have also developed an easy-to-use tool based on our SCM algorithm, publically available at nanoHUB.org.

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