Computed Metallofullerene Yields in the X@C₇₄ and Z@C₈₂ Series

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ABSTRACT

The contribution reports computations for $Al@C_{82}$, $Sc@C_{82}$, $Y@C_{82}$ and $La@C_{82}$ based on encapsulation into the IPR (isolated pentagon rule) C_{2v} C_{82} cage and also on $Mg@C_{74}$, $Ca@C_{74}$, $Sr@C_{74}$ and $Ba@C_{74}$ based on encapsulation into the only C_{74} IPR cage. Their structural and energetic characteristics are used for evaluations of the relative production yields, employing the encapsulation Gibbs-energy terms and saturated metal pressures. It is shown that the results can be well related to the ionization potentials of the free metal atoms.

Keywords: Endohedral fullerenes; carbon-based nanotechnology; molecular electronics; molecular modeling; molecular electronic structure; ionization potentials; metallofullerene stability islands.

1 INTRODUCTION

There are several well-established families of metallofullerenes based on one common carbon cage, for example $X@C_{74}$ or $Z@C_{82}$. Although the empty C_{74} fullerene [1] is not yet available in solid form, several related endohedral species $X@C_{74}$ have been known like $Ca@C_{74}$ [2,3], $Sr@C_{74}$ [4], $Ba@C_{74}$ [5] or $La@C_{74}$ [6-8], all based on the isolated pentagon rule (IPR) D_{3h} C_{74} cage. Another common metallofullerene family, $Z@C_{82}$ is based on the IPR C_{2v} C_{82} cage - for example $Sc@C_{82}$ [9], $Y@C_{82}$ [10] and $La@C_{82}$ [6,11] (while $Al@C_{82}$ was never isolated). The present paper deals with computational evaluations of the structural, bonding and stability features in the homologous series $Z@C_{82}(Z=Al,Sc,Y,La)$, and also $X@C_{74}$ (X=Mg, Ca, Sr, Ca, Ca,

Fullerenes and metallofullerenes have represented objects of very vigorous research activities in connection with their expected promising nanoscience and nanotechnology applications, see e.g. [12-17]. In particular, various endohedral cage compounds have been suggested as possible candidate species for molecular memories and other future molecular-electronic devices. One approach is built on endohedral species with two possible location sites of the encapsulated atom [13] while another concept of quantum computing aims at a usage of spin states of $N@C_{60}$ [14] or fullerene-based molecular transistors [15]. Although there can be (three-dimensional) rotational motions of encapsulates in the

cages, the internal motions can be restricted by a cage derivatization [16] thus in principle allowing for a versatile control of the endohedral positions needed for the molecular-memory applications. However, a still deeper knowledge of various molecular aspects of the endohedral compounds is needed before their tailoring to nanotechnology applications is possible.

2 COMPUTATIONS

The full geometry optimizations were carried out using density-functional theory (DFT), namely employing Becke's three parameter functional with the non-local Lee-Yang-Parr correlation functional (B3LYP) in the combined basis set of the 3-21G basis for C atoms and the LanL2DZ basis set with the LANL2 effective core potential for the metal atoms (3-21G~la) as implemented in the Gaussian 03 program package [18]. In the optimized B3LYP/3-21G~la geometries, the harmonic vibrational analysis was then performed. Moreover, in the optimized geometries, higher-level single-point energy calculations were also carried out with the standard 6-31+G* (6-31+G*~la) basis set for C atoms, and finally also with the standard 6-311+G* basis for carbon atoms and the SDD basis with the SDD effective core potential for the metals (6-311+G*~sdd). The basis set superposition error (BSSE) was estimated by the Boys-Bernardi counterpoise method [19]. In addition to the traditional B3LYP functional, a newer MPWB1K functional suggested recently by Zhao and Truhlar [20] as the best combination for evaluations of long-range interactions has also been employed in this study.

The Gibbs energies were evaluated using the rotational-vibrational partition functions constructed from the calculated structural and vibrational data using the rigid rotator and harmonic oscillator (RRHO) approximation. Although the temperature region where fullerene or metallofullerene electric-arc synthesis takes place is not yet known, there are some arguments to expect it around or above 1500 K. Thus, the calculations here are presented for two illustrative temperatures of 1500 and 2000 K.

3 RESULTS AND DISCUSSION

The Z@C₈₂ metallofullerenes have been known [12] to be formed via metal encapsulations into the IPR C_{2v} C₈₂ cage with a strong charge transfer from the metal to the cage leaving the metal between the Z²⁺ and Z³⁺ states. For example, the Mulliken atomic charge in La@C₈₂, Y@C₈₂, and Sc@C₈₂ is at the B3LYP/3-

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21G \sim la level calculated as 2.67, 2.38, and 2.44, respectively. However, the natural population analysis, for example at the B3LYP/6-311+G* \sim sdd level, produces for La@C₈₂, Y@C₈₂, and Sc@C₈₂ charges of 2.32, 2.05, and 1.77, respectively.

We can consider an overall stoichiometry of a metallofullerene formation:

$$X(g) + C_n(g) = X@C_n(g)$$

$$\tag{1}$$

although it is not really relevant what kind of reactants is on the left side as they will in the end cancel out in our considerations. The encapsulation process is thermodynamically characterized by the standard changes of, for example, enthalpy $\Delta H^o_{X@C_n}$ or the Gibbs energy $\Delta G^o_{X@C_n}$. An illustration is given here on the reaction series Al@Cs2, Sc@Cs2, Y@Cs2 and La@Cs2 with the encapsulation potential-energy changes computed at the B3LYP/6-31+G*~la level. The original Boys-Bernardi counterpoise method was suggested [19] for dimers with a fixed geometry. Although a BSSE-respecting geometry optimization would be possible [21], it is rather practical only for simpler systems. Still, in order to reflect the cage distortion, a steric-corrected BSSE treatment is also applied here (B3LYP/6-31+G*~la & steric) which includes the difference between the energy of the carboncage geometry simply taken from Z@Cs2 and the energy of the related fully-optimized empty IPR C_{2v} Cs2 cage.

The equilibrium composition of the reaction mixture is controlled by the encapsulation equilibrium constants $K_{X@C_n,p}$:

$$K_{X@C_n,p} = \frac{p_{X@C_n}}{p_{X}p_{C_n}},$$
(2)

expressed in the terms of partial pressures of the components. The encapsulation equilibrium constant is interrelated with the the standard encapsulation Gibbs energy change $\Delta G^o_{X@C_n}$:

$$\Delta G_{X@C_n}^o = -RT ln K_{X@C_n,p}. \tag{3}$$

Temperature dependency of the encapsulation equilibrium constant $K_{X@C_n,p}$ is then described by the van't Hoff equation:

$$\frac{dlnK_{X@C_n,p}}{dT} = \frac{\Delta H_{X@C_n}^o}{RT^2} \tag{4}$$

where the $\Delta H^o_{X@C_n}$ term is typically negative so that the encapsulation equilibrium constants decrease with increasing temperature.

Let us further suppose that the metal pressure p_X is actually close to the respective saturated pressure $p_{X,sat}$. While the saturated pressures $p_{X,sat}$ for various metals are known from observations [22,23] (and belong to the essential input set of experimental information [23-24] still necessary for our computational treatment), the partial pressure of C_n is less clear as it is obviously influenced by a larger set of processes (though, p_{C_n} should exhibit a temperature maximum and then vanish). Therefore, we avoid the latter pressure C_n in our considerations at this stage. However, if we consider the combined $p_{X,sat}K_{X@C_n,p}$ term:

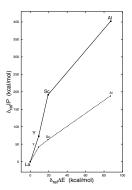


Fig. 1. The computed B3LYP/6-31+G*~la relative potential-energy changes upon encapsulation $\delta_{rel}\Delta E$ and the observed [24] relative ionization potentials (IP) of the free atoms $\delta_{rel}IP$ for Z@C₈₂ (solid line - 3-rd IP, dashed line - 2-nd IP).

$$p_{X@C_n} \sim p_{X,sat} K_{X@C_n,p},$$
 (5)

that directly controls the partial pressures of various $X@C_n$ encapsulates in an endohedral series (based on one common C_n fullerene), we get an applicable scheme. As already mentioned, the computed equilibrium constants $K_{X@C_n,p}$ themselves have to show a temperature decrease with respect to the van't Hoff equation (eq. 4) which however does not necessarily mean a yield decrease with increasing temperature. Actually, the considered $p_{X,sat}K_{X@C_n,p}$ product term can frequently (though not necessarily) be increasing with temperature. An optimal production temperature could be evaluated in a more complex model that also includes temperature development of the empty-fullerene partial pressure.

Hence, if we want to evaluate production abundances in a series of metallofullerenes like Al@C₈₂, Sc@C₈₂, Y@C₈₂ and La@C₈₂, just the product $p_{Z,sat}K_{Z@C_{82},p}$ terms can straightforwardly be used. While for Al@C₈₂ the $p_{Z,sat}K_{Z@C_{82},p}$ quotient increases with temperature, it is about constant for Y@C₈₂ for the considered temperatures, and it decreases with temperature for Sc@C₈₂ and (especially) for La@ C_{82} . The behavior results from competition between the decreasing encapsulation equilibrium constants and increasing saturated-metal pressures. As the encapsulation enthalpy $\Delta H^o_{X@C_n}$ has the most negative value for La@C₈₂, its encapsulation equilibrium constant has to exhibit the fastest temperature decrease that already cannot be overcompensated by the temperature increase of the saturated metal pressure so that the $p_{Z,sat}K_{Z@C_{82},p}$ quotient decreases relatively so fast with temperature in this case. In order to allow for cancellation of various factors introduced by the computational approximations involved, it is however better to deal with the relative quotient $\frac{p_{Z,sat}K_{Z@C_{82},p}}{R_{Z,sat}K_{Z,sat}}$. The computed production yield of the $p_{La,sat}K_{La@C_{82},p}$ (never observed) Al@ C_{82} species should be by six orders of magnitude smaller than that for Sc@ C_{82} or Y@ C_{82} , while the latter two should exhibit comparable populations, and La@C₈₂ should be the most abundant endohedral in the series. This stability picture qualitatively agrees with observed populations. In principle, an endohedral with lower value of the encapsulation equilibrium constant can still be produced in larger yields if a conve-

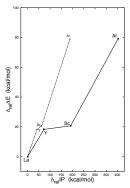


Fig. 2. The computed B3LYP/6-31+G*~la & steric relative potential-energy changes upon encapsulation $\delta_{rel}\Delta E$ and the observed [24] relative ionization potentials (IP) of the free atoms $\delta_{rel}IP$ for Z@C₈₂ (solid line - 3-rd IP, dashed line - 2-nd IP).

nient over-compensation by higher saturated metal pressure can take place.

Although the energy terms are likely still not precise enough, their errors could be comparable in the series and thus, they should cancel out in the relative term $\frac{p_{Z,sat}K_{Z@C_{82},p}}{p_{La,sat}K_{La@C_{82},p}}$. This should be the case of, for example, the higher corrections to the RRHO partition functions, including motions of the encapsulate. The motion of the endohedral atom is highly anharmonic, however, its description is yet possible only with simple potential functions. As long as we are interested in the relative production yields, the anharmonic effects should at least to some extent be canceled out in the relative quotient $p_{Z,sat}K_{Z@C_{82},p}$

 $p_{La,sat}K_{La@C_{82},p}$.

The series of metallofullerene formations with one common cage $X@C_n$ allows for yet another interesting stability conclusion. Three formal reaction steps can be considered for our illustrative series $Al@C_{82}$, $Sc@C_{82}$, $Y@C_{82}$ and $La@C_{82}$: (i) double- (or triple-) ionization of the free metal, (ii) double (or triple-) charging of the empty cage, and (iii) placing the metal di- (or tri-) cation into the di- or (tri-) anionic cage. The (ii) energy is identical for all members of the series, and the (iii) terms should be similar as they are controlled by electrostatics. The bonding situation in $Al@C_{82}$, $Sc@C_{82}$, $Y@C_{82}$ and $La@C_{82}$ can be surveyed by the highest C-Z Wiberg bond index. The very low values of the C-Z Wiberg index (at the $B3LYP/6-311+G*\sim sdd$ level: $0.04 \sim 0.21$) in $\dot{Z}@C_{82}$ indicate that instead of a covalent bond, an ionic bond is formed between the metal and cage. Moreover, the feature that the stabilization of metallofullerenes is mostly electrostatic can also be documented [25] using the topological concept of atoms in molecules (AIM) [26,27] which indeed shows that the metal-cage interactions form ionic (and not covalent) bonds. Hence, the free-metal ionization potentials should actually represent a critical yield-controlling factor - the computed relative potential-energy changes upon encapsulation $\delta_{rel}\Delta E$ and the relative observed ionization potentials of the free atoms $\delta_{rel}IP$ should according to the above three-step analysis be correlated:

$$\delta_{rel}\Delta E \sim \delta_{rel}IP.$$
 (6)

This interesting conclusion is documented in Figs. 1 and

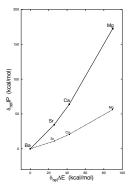


Fig. 3. The computed MPWB1K/6-31G*~la relative potential-energy changes upon encapsulation $\delta_{rel}\Delta E$ and the observed [24] relative ionization potentials of the free atoms $\delta_{rel} \Pi P$ for X@C₇₄ (solid line - 2-nd IP, dashed line - $1st \stackrel{\sim}{\text{IP}}$).

2 that use both the observed second and third ionization 2 that use both the observed second and third ionization potentials (IP) [24] for the Z atoms of the Z@C₈₂ series as the B3LYP/3-21G~la Mulliken atomic charge on the metal in Sc@C₈₂, Y@C₈₂ and La@C₈₂ is computed between 2 and 3. Fig. 1 presents the correlation for the B3LYP/6-31+G*~la relative potential-energy changes upon encapsulation $\delta_{rel}\Delta E$, Fig. 2 for the B3LYP/6-31+G*~la & steric energetics. Finally, Fig. 3 deals with the Y@C₇₄ series [28] (X = Mg. Ca. Sr. and Ba) with the $X@C_{74}$ series [28] (X = Mg, Ca, Sr, and Ba) described at the MPWB1K/6-31G*~la level. In this case, both the observed second and first ionization potentials are considered (though the second IP are more relevant to this case as the Mulliken charge on the metals is very close to 2). The scheme works even better in this case owing to a larger uniformity of the charge on the metals. All the three Figures support relationship (6). In fact, such a correlation should operate for any homologous reaction series of metal encapsulations, i.e., into any type of a common carbon nanostructure. Moreover, this type of reasoning should step by step explain the fullerene-encapsulation stability islands [29-33 known throughout the periodic system (though the under lying calculations are quite demanding).

In fact, we are dealing with a special case of clustering under saturation conditions [34-37]. The saturation regime is a useful simplification - it is well defined, however, it is not necessarily always achieved. Under some experimental arrangements, under-saturated or perhaps super-saturated metal vapors are also possible. This reservation is applicable not only to the electricarc treatment but even more likely to newly introduced ion-bombardment production technique [38,39]. Still, eqs. (2) and (5) remain valid, however, the metal pressure has to be described by the values actually relevant. For some volatile metals their critical temperature could be overcome and the saturation region thus abandoned (though practically speaking, this could come into consideration with mercury and cesium). Anyhow, the saturation regime can give a kind of upper-limit estimates

of the production yields.

ACKNOWLEDGMENTS

The reported research has been supported by a Grant-in-aid for the 21st Century COE Program, Nanotechnology Support Project, the Next Generation Super Computing Project (Nanoscience Project), and Scientific Research on Priority Area from the Ministry of Education, Culture, Sports, Science, and Technology of Japan, and by the Ministry of Education of the Czech Republic (MSM0021620857) and the Czech Science Foundation/GACR (P208/10/0179).

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