Numerical Simulation on Nonlinear Response of Two-Dimensional Electron Plasmas in the Field Effect Transistor Structures

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ABSTRACT
A new numerical method is proposed to study nonlinear response of two-dimensional electron plasmas in the channel of field effect transistor structures. The numerical simulations are not only in accordance with the existing theory, but also illustrate how phase shift influences the plasma wave transmission in the conduction channels of field effect transistor structures. The numerical simulations contain the nonlinear effects when the boundary conditions are asymmetric, i.e., when a frequency, amplitude, or phase shift exists between ac signals applied at drain and source sides of the field effect transistor (FET). The plasma wave transmission under nonresonant and resonant conditions is simulated by changing certain structure parameters with our developed simulator. These results demonstrate that the developed simulator is a useful tool in THz detector design and optimization.
Keywords: nonlinear response, field effect transistor, phase shift

1 INTRODUCTION
The channel of the field effect transistor (FET) with well defined dimensions is used as a resonator for plasma wave propagating in a two-dimensional (2D) electron gas [1]. The fundamental frequency of plasma waves of this resonator reaching terahertz (THz) range depends on its gate lengths with nanometer size L and typical carrier densities n [2]. This characteristic plasma wave frequency of the transistor is increased by increasing the carrier density and decreasing the channel length of the channel. A short channel High Electron Mobility Transistor (HEMT) has a resonant response to electromagnetic radiation at the plasma oscillation frequencies of the two dimensional electrons in the device while a long channel HEMT has a nonresonant response [2]. For their perfect high-frequency electrical response and benefits in many fields such as communication engineering and image detection, the physics of FET structures has been paid lots of attention to for many years.

Plasma waves propagate in a FET channel with a linear dispersion law $w(k) = sk$. Here s is the plasma wave velocity which depends on carrier density, and k is the wave vector. A FET biased by the gate-to-source voltage and subjected to electromagnetic radiation can develop a constant drain-to-source voltage. Dyakonov and Shur studied plasma response properties in a short FET and proposed new electronic devices operating in the terahertz range [1-2, 3].

A hydrodynamic treatment of FET structures successfully depicts the dynamics of two-dimensional (2D) plasma in the channel of FET in the terahertz range. Due to the typical electron densities on the order of $10^{16} cm^{-3}$, the electron-electron collision time in FETs with short channel is much smaller than the collision time with impurities and phonons [1]. Under this condition, the hydrodynamic description provides a rational approximation for electron density variation and velocity profiles in the channel. Hydrodynamic analysis shows that the linear plasma oscillations may become unstable when the electron density variations in the channel are not small.

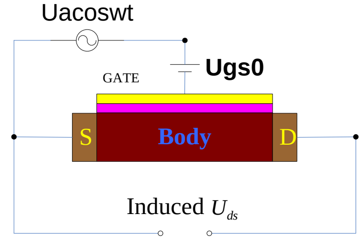

Fig.1 The equivalent circuit for terahertz detection induced by radiations between source-and-gate electrodes of device

Our group has established a simulator based on the hydrodynamic model shown in Fig.1 to simulate the plasma wave transmission in field effect transistors [4-5]. As is shown in Fig.1, the FET is biased only by the gate-to-source voltage U_0, defined as the difference between gate-to-channel voltage U_g and the threshold voltage V_{th}. $U_a \cos wt$ is an external ac voltage induced between the gate and source by the incoming electromagnetic radiation, and $j = CU_0 v$ is the electron current per unit width, here C is the gate capacitance per unit area and v is the local electron velocity.

Nonlinear response of two-dimensional electron plasma in the conduction channels of FET structures was studied in

1999[6]. The essential nonlinearity in the microwave response induces the linear hydrodynamic effects in certain device parameter ranges; However, the influence of phase shift on the plasma wave transmission has not been discussed in their article. On top of that, they adopted the Lax-Wendroff method [9] to build a simulator to testify their theory, however, when viscosity effects are not included in the momentum balance equation, shallow rapid oscillations appear behind the shock front in numerical simulations. These oscillations are numerical artifacts. In this paper, a new numerical method is developed to simulate this phenomenon by adopting Finite Difference Method. The equivalent circuit for detecting the response of FET to harmonic signals applied at source and drain sides is presented by Fig.2. The nonlinear plasma oscillation is observed under the asymmetric boundary conditions when there is a frequency, amplitude, or phase shift exists between ac signals applied at the drain and source terminals of FET. Our numerical simulation method is used both under nonresonant and resonant conditions by changing certain parameters of the FET structure. Thus, our numerical method is demonstrated to be meaningful in detector design and optimization.

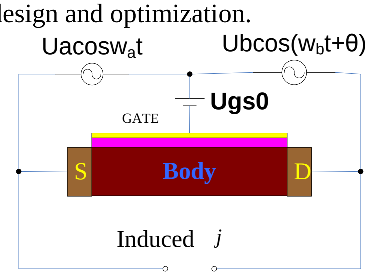

Fig.2 Equivalent circuit for measuring response of FET device to harmonic signals applied at source and drain sides

2 THEORY AND NUMERICAL METHOD

The hydrodynamic equations are derived from mass, momentum, and energy balance equations. As introduced by Ref. [6], the hydrodynamic equations which govern the THz signal transport in a field effect MOS transistor detector are described as

$$\frac{\partial n_s}{\partial t}+\frac{\partial}{\partial x}(n_s v)=0 \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{v}{\tau_m}+\frac{s_0^2}{n_0}\frac{\partial n_s}{\partial x}=0 \tag{2}$$

Here Eq. (1) is the general continuity equation and Eq. (2) is the Euler equation. In Eq. (2), n_s is the electron density in the channel, τ_m is the electronic relaxation time. n_0 is the equilibrium density of electrons in the conduction which is proportional to the average value of the gate-to-channel voltage U_0. s_0 is the velocity of small amplitude waves which is also determined by U_0, $s_0=(eU_0/m)^{1/2}$, where e is the electronic charge and m is the electron effective mass. The strength of the friction term is characterized by the dimensionless friction parameter $\gamma \equiv L/(s_0\tau_m)$, where L is the gate length. The harmonic signals are applied at the drain and source terminals of the transistor. The boundary conditions are defined as

$$n_s(0,t)=n_0+n_a\cos(w_a t) \tag{3}$$

$$n_s(L,t)=n_0+n_b\cos(w_b t+\theta) \tag{4}$$

Where n_a, n_b is calculated by Cu_a/e, Cu_b/e respectively, w_a, w_b are the values of the circular frequency w at the source and drain terminals, θ is a phase shift between ac signals applied at the drain and source terminals of a FET. With the normalization of $n=n_s/n_{s0}$, $v=V/s_t$, $\eta=x/L$, $\tau=ts_t/L$, $\tau_m^{'}=\tau_m s_t/L$, where V_t is the thermal voltage and $s_t=\sqrt{eV_t/m}$, the equations (1) and (2) becomes

$$\begin{aligned}&\frac{\partial n}{\partial \tau}+\frac{\partial (nv)}{\partial \eta}=0\\&\frac{\partial v}{\partial \tau}+v\frac{\partial v}{\partial \eta}=\frac{s_0^2}{s_t^2}\frac{\partial n}{\partial \eta}-\frac{v}{\tau_m^{'}}\end{aligned} \tag{5}$$

The two variables used in the numerical method are defined as

$$q_1=n, q_2=nv$$

The following matrixes are expressed as

$F_1(q_1,q_2)=\begin{bmatrix} q_1 \\ \dfrac{q_2}{q_1} \end{bmatrix}, F_2(q_1,q_2)=\begin{bmatrix} q_2 \\ \dfrac{q_2^2}{2q_1^2}-\dfrac{s_0^2}{s_t^2}q_1 \end{bmatrix}, G(q_1,q_2)=\begin{bmatrix} 0 \\ \dfrac{q_2}{\tau_m q_1} \end{bmatrix}$, By using the forward-difference method temporally and central-difference method spatially, the difference equation is expressed as

$$\frac{(F_1)_k^{n+1}-(F_1)_k^{n}}{\Delta\tau}+\frac{(F_2)_{k+1}^{n+1}-(F_2)_{k-1}^{n+1}}{2\Delta\eta}+(G)_k^{n+1}=0 \tag{6}$$

Where $\Delta\tau$ and $\Delta\eta$ are the time step and space step, respectively. First, the carrier density induced by radiation at the source end is fixed at $n_0+n_a\cos w_a t$, therefore, $(q_1)_1^{n+1}$ is obtained. The left boundary is described using the first of the equation (5)

$$\frac{(F_{1,2})_1^{n+1}-(F_{1,2})_1^{n}}{\Delta\tau}+\frac{(F_{2,2})_2^{n+1}-(F_{2,2})_1^{n+1}}{\Delta\eta}+(G_2)_1^{n+1}=0 \tag{7}$$

Use the same method, and let the carrier density at the drain fixed at $n_0+n_b\cos(w_b t+\theta)$, $(q_1)_{N+1}^{n+1}$ is obtained. The right boundary condition is described as

$$\frac{(F_{1,1})_{N+1}^{n+1}-(F_{1,1})_{N+1}^{n}}{\Delta\tau}+\frac{(F_{2,1})_{N+1}^{n+1}-(F_{2,1})_{N}^{n+1}}{\Delta\eta}=0 \tag{8}$$

By using similar numerical method presented in Ref [4], the carrier density and carrier velocity at any location of the channel is obtained. Besides, the response of the 2D electron gas is characterized by the induced currents at the source and drain terminals. We consider the time in dependence of the source current density.

3 RESULT AND DISCUSSIONS

In our simulation, parameters used in the simulation are listed below:

$$N_{sub} = 5\times10^{16} cm^{-3}, T_{ox} = 2nm, \mathrm{T} = 300\mathrm{K}$$

Here N_{sub} is the doping concentration of the substrate, T_{ox} is the thickness of the gate oxide and T is the temperature.

First, we define $\mu = 2.6067 m^2/Vs$ and $L = 100nm$. The frequencies of the ac signals at the source and drain terminals are $f_a = w_a / 2\pi = 1.07THz$ and $f_b = 3.21THz$, the source current density defined as the current per unit width of the channel is shown in Fig.3. The blue curve in Fig.3 is obtained under the condition of $\theta = 0$ and the red curve represents the condition $\theta = \pi/2$. When $\theta = 0$, it is clear to see that the current becomes periodic in time with a period determined by the smaller one of the terminal frequencies f_a after a short transitional interval. This phenomenon is in accordance with the theory proposed by Ref [6]. However, when $\theta = \pi/2$, there is a delay between the blue curve and the red curve and two curves are similar in shape. The value of this delay is measured as $\delta\tau = 0.05$. In Fig.4, nine different phase shifts θ are chosen to investigate the relationship of the delay $\delta\tau$ and phase shift. It is shown that the delay is proportional to the phase shift.

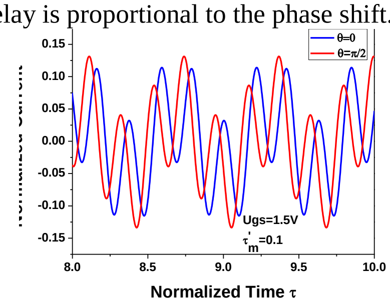

Fig.3 Source current density as a function of time. The value of the friction parameter is $\gamma = 2.02$. The gate-to-channel voltage is $U_g = 1.5V$, the ac amplitudes of signals at the source and drain are $u_a = u_b = 0.01V$, ac frequencies are given by $f_a = 1.07THz$ and $f_b = 3.21THz$. The solid curve is obtained from the numerical solution of the nonlinear equations.

In the above example, the friction is neglected, this causes a shock wave to appear and propagate as a compression wave in the channel [7]. For the parameter values of the preceding example, the local density profile does not contain propagating waves. Propagating plasma waves appear at larger values of frequency or amplitudes, and smaller values of the friction parameter γ. In order to show the propagating of plasma waves, different friction parameters are used in the example, such as γ : 0.202, 0.0202, 0.00405, and 0.00202. The resulting current density at the source is shown in Fig. 5 as a function of time. It is periodic function of the time t with a period determined by the smaller one of the terminal frequencies f_a after a short transitional interval. In Fig.5, it is clear to see the harmonics other than f_a and f_b from the shape of curves. When the value of the friction parameter is increased by an order of magnitude, the shock waves are getting smaller. The electron density variation is defined as $\delta n(x,t) = n(x,t) - n_0$. Simulation results are in accordance with the theory proposed by Ref [6]. Fig.6 illustrates that it is not a steady state and oscillates with time but it does not contain plasma waves at this time.

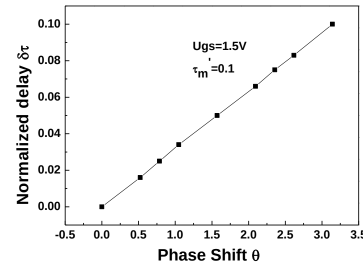

Fig.4 The delay as a function of phase shift θ, the value of the friction parameter is $\gamma = 2.02$.The gate-to-channel voltage is $U_g = 1.5V$, the source and drain ac amplitudes are $u_a = u_b = 0.01V$, ac frequencies are given by $f_a = 1.07THz$ and $f_b = 3.21THz$.

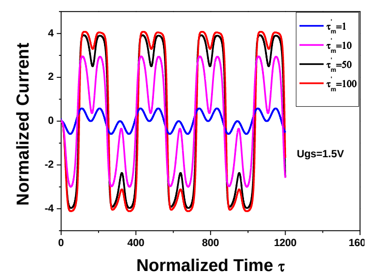

Fig.5 Source current density as a function of time with different friction parameters γ : 0.202, 0.0202, 0.00405, 0.00202. The gate voltage swing is $U_g = 1.5V$, the source and drain ac amplitudes are $u_a = u_b = 0.01V$, ac frequencies are given by $f_a = 13.49GHz$ and $f_a = 40.47GHz$.

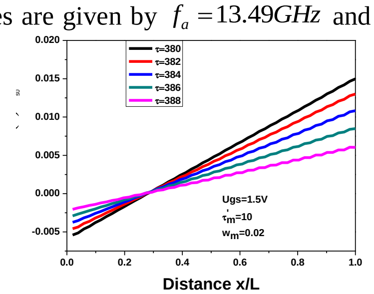

Fig.6 Electron density variation corresponding to the case of $\gamma = 0.0202$, $f_a = 13.49GHz$ and $f_a = 40.47GHz$.

When $w\tau_m >> 1$, the resonant case is realized. Under this condition, while we neglect the influence of the external friction force $f = -mnv / \tau_m$, the plasma wave propagates in the channel of field effect transistors. Fig.7 (a) presents the electron density variation in the channel at different time: 148.25, 148.26, 148.27, 148.28, 148.29 and Fig.7 (b) shows the velocity variation in the channel at different time: 148.19, 148.20, 148.21, 148.22, and 148.23. Parameters used here are $\tau_m^{'}=10, f_a=1.07THz, f_b=3.021THz$. The oscillation of electron density variation and velocity induced by radiation signals in the channel are observed clearly.

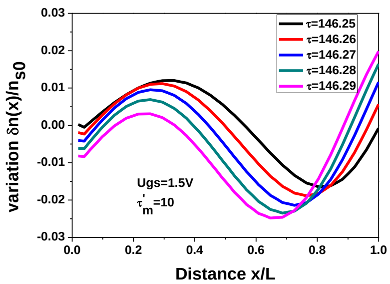

Fig.7 (a).The electron density variation is plotted at different times as a function of x/L, where L is channel length. The values of the normalized time are 148.25, 148.26, 148.27, 148.28, and 148.29. Parameters used in the simulation are $\tau_m^{'}=10, f_a=1.07THz, f_b=3.021THz$

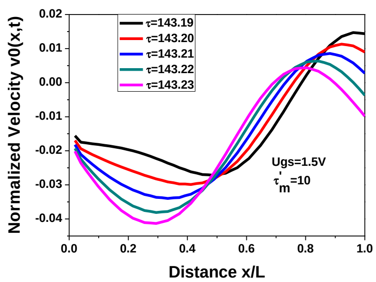

Fig.7 (b).Velocity at different time as a function of x/L, where L is channel length. The values of normalized time are 148.25, 148.26, 148.27, 148.28, and 148.29. Some parameters used here are $\tau_m^{'}=10, f_a=1.07THz, f_b=3.021THz$

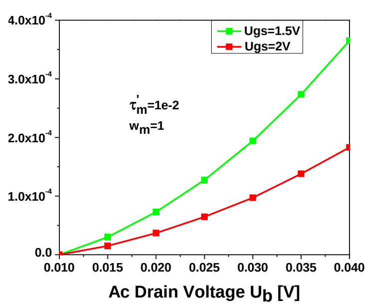

Fig.8 Induced dc component of the source current density as a function of the ac amplitude of the applied drain voltage in the case of equal frequencies of the source and drain potentials. The green line is simulated by $Ugs=1.5V$ and the red curve is simulated by $Ugs=2V$. Some parameters used here are $\tau_m^{'}=0.01, w_m=1$.

The investigation of the nonlinear effects on the device response such as drain current density is obtained from the time average $<j>$. It is similar to the dc source-to-drain voltage called photoresponse proposed by Dyakonov-Shur [1]. The induced dc component of the source current density is shown in Fig.8 as a function of the ac amplitude u_b at the drain terminal, for the equal frequencies at the source and drain terminals, the green line is obtained under the condition of $Ugs=1.5V$ and the red curve is simulated under $Ugs=2V$. Some parameters used here are $\tau_m^{'}=0.01, w_m=1$. The results are in accordance with the theory proposed by Ref [8].

4 CONCLUSION

In this paper, the nonlinear response of two-dimensional electron plasmas in the conduction of the field effect transistor structures is studied. A new numerical method which calculates the terahertz signal transport in field effect MOS transistors is developed based on the hydrodynamic equations. The simulation results are in accordance with the existing theory quantitatively. The simulator can be used to study the influence of phase shift on nonlinear response and the nonlinear response in resonant condition. Our numerical method is demonstrated to be meaningful in detector design and optimization.

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REFERENCES

[1]. Michael Dyakonov and Michael Shur, Physical Review Letters, 1993, Vol. 71, No. 15.

[2]. Mikhail Dyakonov and Michael Shur, IEEE Transactions on Electron Devices, 1996, Vol. 43,380(1996) .

[3]. M. Dykonov and M. Shur, IEEE Trans. Electron Devices 43,1640(1996).

[4]. X. Mou, Y. Chen, C. Ma, Y. Che , J. He, 2008, IEEE ICSICT, pp. 396-399 .

[5]. Yinglei Wang, Yu Chen, Frank He, Xuehao Mou and Chenyue Ma, IEEE ELECTRO-2009, India, 2009.

[6]. Sergey Rudin, Guram Samsonidze, Frank Crowne, Journal of Applied Physics, Vol 86,No.4, 1999.

[7]. S. Rudin and G. Samsonidze, Phys. Rev. B 58, 16369(1998).

[8]. D. B. Veksler, A. V. Muravjov, V. Yu. Kachorovskii, et al. Solid-State Electronics 53 571-573, (2009).

[9]. P. Lax and B. Wendroff, Commun. Pure Appl. Math. 13, 217 ,1960.

[10]. D.Veksler, F.Teppe, A.P.Dmitriev, et. al, 2006, Physical Review B 73, 125328.