Using MEMS as Self-Calibrating Force-Displacement Transducers: A Theoretical Study

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ABSTRACT

We present a theoretical analysis of a novel microscale self-calibrating force-displacement transducer. The analysis of our novel micro electro mechanical system (MEMS) includes the typical geometric and material property variations which have prevented analysts from matching computational or analytical prediction with measurement. Applications of our proposed technology include it being used as a force-displacement measurement standard for the micro/nano-scale, or it being used to calibrate other force-displacement tools that are difficult to adequately calibrate, such as the atomic force microscopes (AFM). In this paper we propose a practical methodology that will allow researchers to employ self-calibrating MEMS as tools for force-displacement sensing and actuation applications. That is, we show that MEMS themselves can be used to determine their comb drive force and displacement characteristics without the use of large and unwieldy laboratory testing equipment such as scanning electron microscopes and the like. As a test case, we show how MEMS can be used to calibrate the displacement, force, and stiffness of an AFM cantilever.

Keywords: self-calibration, force-displacement transduction, atomic force microscope, AFM, electro micro metrology, EMM, measurement standard

1 INTRODUCTION

Accurate and precise measurements of forces and displacements are critical to the improved understanding, discovery, and prediction of microscale and nanoscale phenomena. Since geometric and material properties of micro electro mechanical systems (MEMS) vary from run to run, it is critically important to be able to easily, quickly, and precisely calibrate MEMS that are used for important sensing applications. Property variations are usually due to variations in the fabrication processes, packaging, and environmental exposure. Finding a practical and traceable method to accurately and precisely measure the minute forces and displacements in MEMS has been elusive. One reason is because the force generated by MEMS is quite often smaller than the force that can be measured by conventional force sensors. Similarly, displacements in MEMS can be as small as a fraction of the diameter of an atom, which is beyond the capabilities of standard displacement sensors. Our present analysis addresses the calibration of force and displacement for MEMS comprising comb drive sensors and actuators. Our approach for calibrating force and displacement is based on the strong and sensitive coupling between mechanical performance and electronic measurands at the microscale. That is, variations in geometry and material properties affect performance, which can be capacitively measured using on-chip or off-the-shelf capacitance meters. A novelty in our analysis is the elimination of unknown properties, which allows us to express mechanical quantities and their uncertainties solely in terms of electrical measurands. We derive analytical expressions for extracting measurements of force, displacement, stiffness, and their uncertainties by electrical probing. And we show how our method is expected to a few orders more precise than convention.

A difficulty in calibrating the AFM is primarily due to the unknown cantilever stiffness. AFM cantilever stiffnesses are often measured to about 10-15% error [1], which implies that stiffness is accurate to ~1 significant digit. Since AFM force is usually determined by multiplying a measured deflection by stiffness, then force is at most accurate to one

Table 1: Nomenclature

C , C^P	Capacitance measurement
Δ , δ	Change and uncertainty operators
Δx	Comb drive displacement
F	Comb drive force
k	Flexure stiffness
V	Applied voltage
L	Initial overlap of the comb drive fingers
N	Number of comb fingers on one side
h	Thickness of the device layer
gap	Fabricated size of the gap stop
gap_{layout}	Layout size of the gap stop
Δgap	$\Delta gap = gap_{layout} - gap$, layout to fabrication
n	Layout parameter, $n = gap_{2,layout} / gap_{1,layout} \neq 1$
g	Gap between comb drive fingers
ε	Permittivity of the medium
β	Fringing field correction factor

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significant digit as well. This is an example of a systemic lack of high-precision means of measurement. This problem was cited as one of the most significant technological bottlenecks by leading researchers from academia and industry that presented at the NSF Workshop on Control and System Integration of Micro- and Nano-Scale Systems [2]. It is well known to these researchers that predicted performance rarely matches actual performance. This is because the geometric and material properties of fabricated devices are difficult to predict, and are difficult to measure. Large relative errors have made it difficult to characterize and understand micro/nanoscale phenomena and to develop ASTM standards [3].

The rest of this paper is organized as follows. In Section 2, we present the theory behind our technology to self-calibrate stiffness, force, and displacement of MEMS. In Section 3, we propose an application of our technology to calibrate an AFM. Our conclusions are given in Section 4. The nomenclature we use in this paper is given in Table 1.

2 INTRODUCTION

In this section, we present the theoretical basis behind our method for enabling the self-calibration of MEMS. We discuss displacement, force, stiffness, and uncertainties.

Displacement. In using our present method, a microdevice should consist of opposing comb drives that are used to both actuate and capacitively sense the closure two gaps of difference sizes, where $gap_{2,Layout} = n\ gap_{1,Layout}$. Two gaps are needed to provide the information necessary to eliminate unknown material properties. We show one version of our device in its three measurement states in Figures 1 and 3. Using differential capacitive sensing, a measurement at zero-state has the form

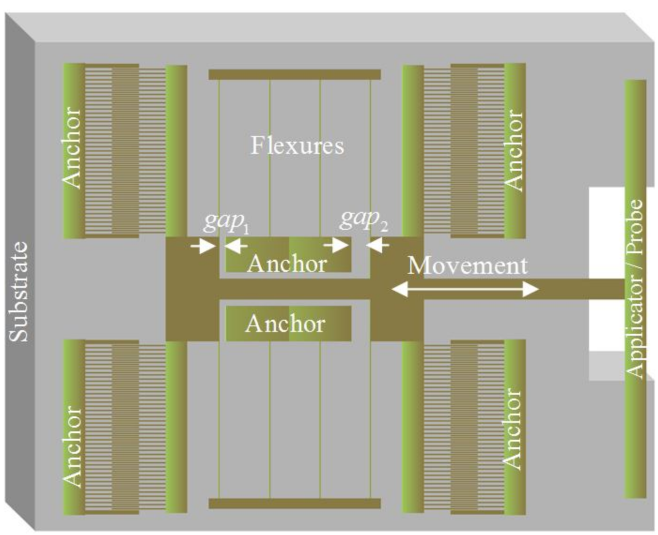

Figure 1: Self-calibrating device: Force-displacement sensor-actuator in zero state. Key components of the MEMS in this SOIMUMPs process are identified. Two different gap sizes are necessary to calibrate the change in geometry in going from layout to fabrication. Before calibration, geometric and material properties are unknown. However, it is assumed that fabrication errors are locally consistent. Gaps gap_1 and gap_2 are identified.

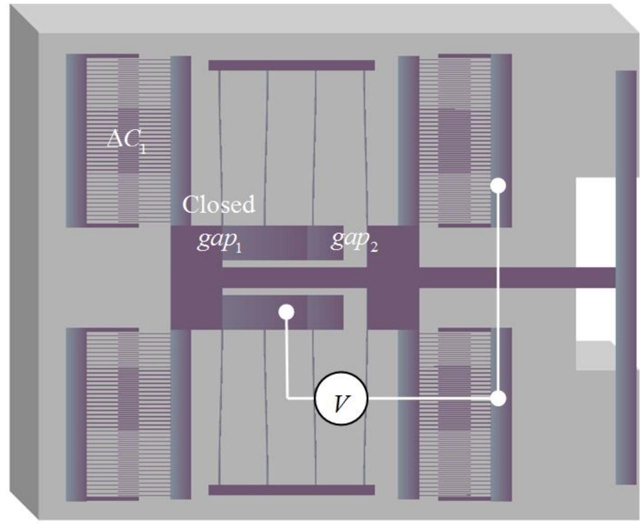

Figure 2: Calibration step 1. To calibrate the device, voltage is applied to close gap_1 while the change of capacitance is measured ΔC_1, as shown. Similarly, by closing gap_2 then ΔC_2 is measured (Figure 3). The fabricated geometry gap_1 is related to the layout geometry by $gap_1 = gap_{layout} - \Delta gap$, where Δgap is the change in going from layout to fabrication. We use Sugar for modeling, design, and simulation.

$$C_0 = \left(2N\beta \frac{\varepsilon hL}{g-\Delta gap} + C_+^P\right) - \left(2N\beta \frac{\varepsilon hL}{g-\Delta gap} + C_-^P\right), \quad (1)$$

where the quantities in this expression (and following expressions) are identified in Table 1. Upon closing gap_1 and gap_2, measures of capacitances yield

$$C_1 = \left(2N\beta \frac{\varepsilon h\left(L-\left(gap_{1,layout}-\Delta gap\right)\right)}{g-\Delta gap} + C_+^P\right) - \left(2N\beta \frac{\varepsilon h\left(L+\left(gap_{1,layout}-\Delta gap\right)\right)}{g-\Delta gap} + C_-^P\right) \quad (2)$$

and

$$C_2 = \left(2N\beta \frac{\varepsilon h\left(L+\left(n\ gap_{1,layout}-\Delta gap\right)\right)}{g-\Delta gap} + C_+^P\right) - \left(2N\beta \frac{\varepsilon h\left(L-\left(n\ gap_{1,layout}-\Delta gap\right)\right)}{g-\Delta gap} + C_-^P\right), \quad (3)$$

where we assume that the parasitic capacitances do not change during a pair capacitive measurements, and we assume that the difference in gap geometry Δgap between fabrication and layout is at least locally consistent about the device. The change in capacitance between the zero state (1) and gap-closed states (2) and (3) are

$$\Delta C_1 = C_1 - C_0 = -4N\beta \frac{\varepsilon h\left(gap_{1,layout}-\Delta gap\right)}{g-\Delta gap} < 0 \quad (4)$$

$$\Delta C_2 = C_2 - C_0 = 4N\beta \frac{\varepsilon h\left(n\ gap_{1,layout}-\Delta gap\right)}{g-\Delta gap} > 0. \quad (5)$$

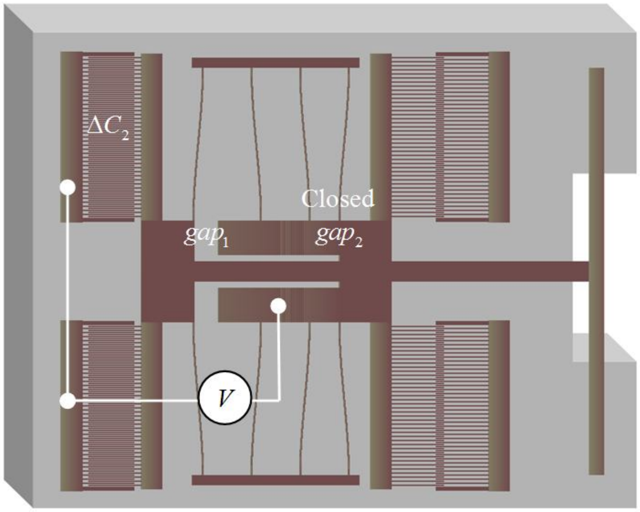

Figure 3: Calibration step 2. An opposing voltage is applied to close the gap gap_2 and its corresponding change in capacitance ΔC_2 from zero state to gap closure is measured. The relationship between the gaps is $gap_{2,layout} = n\, gap_{1,layout}$ where $n \neq 1$ is a layout parameter.

By taking the ratio of (4) to (5), we eliminate the remaining unknowns, except Δgap,

$$\frac{\Delta C_1}{\Delta C_2} = -\frac{gap_{1,layout} - \Delta gap}{n\, gap_{1,layout} - \Delta gap}. \tag{6}$$

Solving (6) for Δgap, the geometrical difference in going from layout to fabrication is

$$\Delta gap = gap_{1,layout} \frac{n\Delta C_1 + \Delta C_2}{\Delta C_1 + \Delta C_2}. \tag{7}$$

where $\Delta C_1 < 0$ and $\Delta C_2 > 0$. The salient property about (7) is that fabricated planar geometry may be measured by measuring changes in comb drive capacitances for gap closure. This measurement is independent of material properties.

After measuring Δgap using (7), we can now characterize the comb drive as

$$\Psi \equiv \frac{\Delta C_1}{gap_{1,layout} - \Delta gap} = \frac{\Delta C_1}{gap_1}, \tag{8}$$

which is the ratio of the capacitance change to gap displacement. We call (8) the comb drive constant. The comb drive constant also holds true for any correlating capacitance changes to displacement. That is, any intermediate displacement $\Delta x < gap$ that produces a change $\Delta C < \Delta C_1$ in the comb drive capacitance can be measured as

$$\Delta x = \left(\frac{gap_1}{\Delta C_1}\right)\Delta C = \frac{\Delta C}{\Psi}. \tag{9}$$

Force. In general, electrostatic force can be expressed as

$$F = \frac{1}{2}\frac{\partial C}{\partial x}V^2. \tag{10}$$

For comb drives that traverse laterally, the partial derivatives can be replaced by differences,

$$F = \frac{1}{2}\frac{\Delta C}{\Delta x}V^2 = \frac{1}{2}\Psi V^2 \tag{11}$$

where we have substituted the comb drive constant from (9). It is important to note that (11) accounts for fringing fields and allows for some nonideal geometries about the comb drive due to process variations.

Stiffness. After finding expressions for the extraction of displacement (9) and force (11) as functions electrical measurands, the ratio of these two results yield the stiffness of the device in the direction of lateral comb drive motion. We have

$$k = \frac{F}{\Delta x} = \frac{1}{2}\Psi^2 \frac{V^2}{\Delta C}, \tag{12}$$

which may not be constant for large, nonlinear deflections. That is, the ratio $V^2/\Delta C$ is usually constant for small deflections, and typically increases nonlinearly for large deflections.

Uncertainties. Uncertainties accompany all measurements. With respect to the above analyses, electrical uncertainties in the measured capacitance δC and voltage δV produce corresponding mechanical uncertainties in displacement δx, force δF, and stiffness δk. To determine such uncertainties, we first rewrite all quantities of capacitance and voltage in the above analyses as $\Delta C \to \Delta C + \delta C$ and $\Delta V \to \Delta V + \delta V$. We then identify the first order terms of their multivariate Taylor expansions as the mechanical uncertainties. That is, the uncertainty in displacement δx of a single measurement is the first order term of the Taylor expansion of (9) about δC. This gives

$$\delta x = gap_{1,layout}(1-n)\frac{\Delta C_1 + \Delta C_2 - 2\Delta C}{\left(\Delta C_1 + \Delta C_2\right)^2}\delta C \tag{13}$$

where the coefficient of δC is the sensitivity $\partial\Delta x/\partial\delta C$.

The uncertainty in force δF is the first order terms of the multivariate expansion of (10) about δC and δV. This gives

$$\delta F = \frac{V^2}{gap_{1,layout}(1-n)}\delta C + \frac{\left(\Delta C_1 + \Delta C_2\right)V}{gap_{1,layout}(n-1)}\delta V \tag{14}$$

where the coefficients of δC and δV are the sensitivities $\partial F/\partial\delta C$ and $\partial F/\partial\delta V$. Similarly, the uncertainty δk is obtained by a multivariate Taylor expansion of (12) about δC and δV,

$$\delta k = \frac{\left(\Delta C_1 + \Delta C_2\right)\left(\Delta C_1 + \Delta C_2 - 4\Delta C\right)V^2}{2(n-1)^2\Delta C^2 gap_{1,layout}^2}\delta C + \frac{\left(\Delta C_1 + \Delta C_2\right)^2 V}{(n-1)^2\Delta C gap_{1,layout}^2}\delta V \tag{15}$$

where the coefficients of δC and δV are the sensitivities $\partial k/\partial\delta C$ and $\partial k/\partial\delta V$.

The benefits of being able to express the mechanical nominal measurements and their uncertainties as presented in this section are as follows. {1} Since the parasitics vanish, the measurements are easier to repeat at different facilities using different capacitance meters and voltage sources. This benefit makes the method amenable to aqueous environments. {2} Since the change in capacitance is used instead of absolute capacitance, only the precision (not accuracy) of the capacitance meter is of concern. {3} It is not necessary to always perform a multitude of measurements to determine a measure of uncertainty. {4} Since many of the parameters used to geometrically configure the design vanish, our method can be used to compare different designs. And {5} The uncertainties can be reduced by strategically modifying the design to reduce the sensitivities or by using higher precision capacitance meters and voltage sources. Prior efforts with electrical probing for the metrology of MEMS can be found in [5]-[7].

3 AFM APPLICATION

Using our self-calibrating MEMS technology presented in Section 2, in this section we describe how our easy and practical method can be used to calibrate Atomic force microscopes (AFMs).

AFM calibration is an important application of our technology, as the AFM has been, and continues to be, a key tool used by nanotechnologists since the late 1980's. Of the problems associated with conventional AFM technology, low-quality measurements is by far the most important. This is because without precise measurements, no reliable form of science or engineering is possible [4]. There are several methods that have been used to calibrate the AFM, including the added-mass, thermal vibration, and geometric and material modeling methods [8]. However, the accuracies of such AFM calibration methods are unknown [1]. This is largely due to the coarsely-known uncertainties involved in such methods. Unfortunately, there are no ASTM measurement standards for the nanoscale. There are three ASTM measurement standards for the microscale (i.e. planar stress, strain gradient, and length [3]), none of which are for force, displacement, elastic modulus, or flexure width, which are necessary for force-displacement transduction.

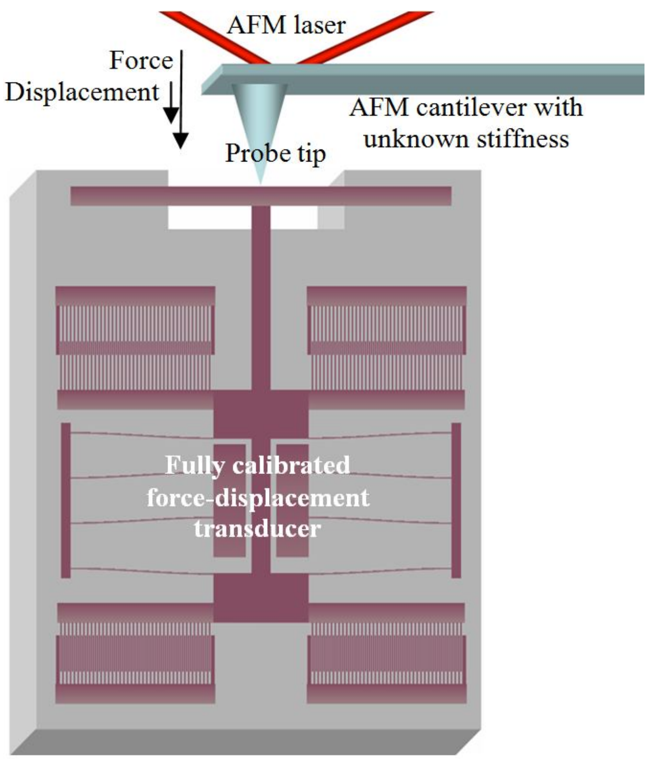

Figure 4: Proposed AFM Calibration. Once the MEMS device is calibrated for force, displacement, and stiffness, it can be used to calibrate other devices that do not have an adequate method to calibrate. A proposed application is depicted here, where the 25-micron thick SOIMUMPs foundry process by MEMPSCAP^{TM} can be used for planar force and displacement, sidewall applications. The sidewall of its applicator can also be used to weigh small objects, probe attachment, or the entire device can be attached to a manipulator for extended use.

We depict AFM calibration methodology in Figure 4. Since our force-deflection transducer operates in plane device, it is necessary that probing takes place in that plane. The 25-micron thick SOIMUMPs foundry process is applicable here. Such a thick sidewall can readily accommodate AFM probe tips, which often have a 5 nanometer radius of curvature. AFMs usually have a deflection capability of ± 4 microns. The AFM cantilever is calibrated by pressing its cantilever against our force-displacement transducer. Upon static equilibrium, the displacement is measured by the change in capacitance of the comb drives, by Equation (9). This calibrated displacement can then be related to the displacement measured by the AFM through its photodiode displacement sensor. And since the stiffness of our force-displacement transducer is known by Equation (12), its measured deflection can be used to determine the applied force due to the AFM cantilever. The ratio of the applied force to displacement measured by our calibrated transducer determines the AFM cantilever stiffness. The uncertainties of these measurements are given by Equations (13)-(15).

4 CONCLUSION

In this paper we presented a theoretical analysis of a self-calibratable force-displacement transducer that is subject to geometric and material property variations. Our analysis may be applied to microdevices with conventional comb drives and locally-consistent unknown property variations in geometry and material properties. The method is easily calibratable using on-chip or off-chip capacitance meters. Our self-calibration technology may be used on-chip post-packaged, in the field after long-term dormancy, or upon harsh environmental change. As an important application example, we propose the use of our technology to calibrate an atomic force microscope.

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