An Improved Analytic Solution to Surface Potential for Un-doped Surrounding-Gate MOSFET

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ABSTRACT

Compact modeling of Surrounding-Gate (SRG) MOSFETs involve the solution of an implicit equation. In this brief we propose a simple, computationally efficient and numerically robust solution to the implicit equation. By suitably conditioning the implicit equation and thereafter using appropriate correction terms we derive a simple yet highly accurate solution, without employing any smoothing function or empirical relation. We also show that the proposed model is applicable to all regions of operation, unlike previous models.

Keywords—Surrounding-Gate, Lambert function, Analytical solution, Compact modeling.

1. INTRODUCTION

CMOS scalability in recent times has been retarded due to gate tunneling current, random body doping fluctuations, low mobility and severe short channel effects. To continue with Moore's Law, the Double-Gate (DG) and the Surrounding-Gate (SRG) MOSFETs have been proposed as viable alternatives to CMOS scalability especially in the sub 50 nm range due to their ideal subthreshold swing and improved mobility.

Continuous analytic drain current and charge models for the SRG MOS have already been proposed in [1-2]. But use of these models is only possible by solving an implicit equation. A numerical solution to the implicit equation was first proposed in [1]. The scope of numerical approach is severely restricted due to its large simulation times and non-convergence problems. Later, explicit solution using charge based models was derived [3]. However, the approximation was quite crude and deviated significantly when the derivatives were computed, as noted in [4]. A different approach, using W Lambert functional form, was proposed in [5]. However, even after using suitable approximations for the W function, numerical steps were required (depending on the accuracy), to solve the equation. Another approximation, also employing the W function, by He et al [6], gave error in mV range and with no plot comparing the accuracy and continuity of the derivatives. Recently solutions have been proposed [4] which solves the transcendental equations explicitly. Although the model has been shown to be quite accurate, it uses smoothing functions and fitting parameters to join two asymptotic approximations. Use of these empirical relations can create convergence problems (especially in the fitting regions) in circuit simulators and should be avoided as much as possible. Also, it has been rightly pointed out [7] that the approximation in [4] for DG FET gives *singularity of log function* error. The same problem occurs for surrounding gate [4, eq 22] for low values of $V_{GD} - V_{FB}$ (about <-1.3 V), when the second term under the square root becomes zero and 'z_1' is computed incorrectly as 0. Here V_{GD} and V_{FB} denote the gate to drain and flat band voltages respectively.

In this paper we introduce explicit solution of the equation for the SRG MOS which is quite different from the earlier models. Our solution differs from the earlier ones in its computational efficiency and the nature of approximations completely eliminates the need for smoothing functions and empirical relations. The model is not only numerically robust and accurate in terms of the surface potential and drain current but also predicts their correct derivatives as has been extensively tested by comparing with exact numerical simulations.

This brief is organized as follows: In section 1 we highlight some of the proposed methods to solve the implicit equation and their drawbacks. In section 2 we propose our own method. In section 3 we compare the results obtained from the present method with numerical simulations and finally we conclude in section 4.

2. DEVELOPMENT OF NON-ITERATIVE SOLUTION FOR SURROUNDING-GATE MOSFET

The Poisson's equation for an un-doped body (i.e. considering only inversion charge) SRG MOS under the gradual channel approximation [3,4] takes the following form

$$\frac{d^2\psi}{d\rho^2} + \frac{1}{\rho}\frac{d\psi}{d\rho} = \frac{q}{\varepsilon_{si}} n_i e^{\frac{q(\psi - V)}{kT}}, \qquad (1)$$

where $\psi(\rho)$ is the electrostatic potential in radial direction, ρ being the radial distance away from centre, as shown in Fig.1, n_i being the intrinsic carrier concentration and V is the electron quasi-Fermi potential, which is constant in the ρ-direction but varies from source to drain.

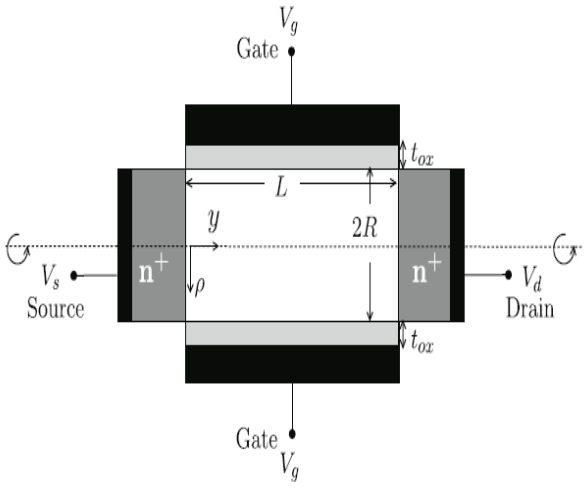

Fig.1. Surrounding-Gate MOSFET structure.

Here, we consider an n type MOSFET with qψ/kT >> 1, so that the hole concentration is negligible. The solution of (1) has been presented in previous papers [3,4] as

$$\psi(\rho) = V - \frac{2kT}{q}\ln\left[\frac{R}{2L_{Di}\beta}\left(1-\frac{\beta^2\rho^2}{R^2}\right)\right] . \quad (2)$$

Here β is a state variable denoting the extent of strong inversion and takes the values in the range (0,1). R is the radius of the cylinder. Again β has to be determined from an implicit equation, which has the final form of [3,4]

$$\ln\beta - \ln(1-\beta^2) + s\beta^2/(1-\beta^2) - G = 0 . \quad (3)$$

Here, $s = 2\varepsilon_{si}\ln(1+t_{ox}/R)/\varepsilon_{ox}$ is a structural parameter, $G = q(V_{GS} - V_{FB} - V)/kT - \ln(2L_{Di}/R)$, t_{ox} is the oxide thickness, V_{GS} the gate to source voltage and L_{Di} is the intrinsic Debye length given in [4], ε_{Si} and ε_{ox} being the permittivity of Si and SiO_2 respectively. Equation (3) has been derived by equating the normal electric fields at the Si-SiO_2 interface. To solve (3) we start by approximating it as

$$\ln\left(\frac{\beta^*}{1-(\beta^*)^2}\right) + s\left(\frac{\beta^*}{1-(\beta^*)^2}\right) - G = 0 . \quad (4)$$

Here β^* is a first approximate of β. Equation (4) yields a good approximation of (3) as for small β the 'ln' term dominates and so the approximation has no effects. In strong inversion β ≈ 1 and so the approximation is also quite good in this region. Note that (4) can be rearranged to yield a Lambert W equation. Substituting $y = \ln\left(\frac{\beta^*}{1-(\beta^*)^2}\right)$ and transforming (4) in the following steps we get

$$y + se^y = G . \quad (5)$$

$$se^G = e^{G-y}(G-y) . \quad (6)$$

Now (6) is a Lambert W function and has an exact solution in the principal branch W_0. Thus we have

$$y = G - W_0(e^G s) \quad (7)$$

Putting back the value of y in (7), we have

$$\ln\left(\frac{\beta^*}{1-(\beta^*)^2}\right) = G - W_0(e^G s) . \quad (8)$$

Rearranging (8) we get

$$\frac{\beta^*}{1-(\beta^*)^2} = \exp\left[G - W_0(e^G s)\right] = \frac{1}{k} . \quad (9)$$

where $k = e^{-(G-W_0(e^G s))}$ has been substituted. The W Lambert function has been well documented in literature [8,9] and has proven its utility in various circuit analysis. We then utilize the following closed-form approximation [10] for the Lambert-W function:

$$W_0(x) = 0.04 + 0.665[1 + 0.0195\ln(1+x)]\ln(1+x) \quad (10)$$

which for small values of x can be expanded in the following fashion

$$W_0(x) = 0.04 + 0.665[1 + 0.0195x]x . \quad (11)$$

The value of β^* can be easily computed from (9) expression as follows

$$\beta^* = \frac{-k + \sqrt{k^2+4}}{2} . \quad (12)$$

Here we neglect the negative root as it does not yield a practical solution. It is a common practice in compact models to rationalize the numerator, as when k ≈ 0, the numerator is actually a very small difference and sometimes mistakenly approximated as '0'. So we rewrite (12) as

$$\beta^* = \frac{2}{k + \sqrt{k^2+4}} . \quad (13)$$

The value of β^* given by (13) is a fairly accurate. We can increase the accuracy in all regions by introducing correction terms. The correction terms we use in this work are based on Halley's formulae [11,12]. Denoting f(β) as

$$f(\beta) = \ln\beta - \ln(1-\beta^2) + s\beta^2/(1-\beta^2) - G \quad (14)$$

we can get a better estimate of β^* as

$$\beta_1 = \beta^* - \frac{f(\beta^*)/f_1(\beta^*)}{1 - f(\beta^*)f_2(\beta^*)/2f_1^2(\beta^*)}. \quad (15)$$

where $f_1(\beta)$ and $f_2(\beta)$ denote respectively the first and second derivatives of (14) w.r.t β. While the value of β_1 obtained by this method is quite accurate, we can further increase the accuracy to get ultra accurate solutions by employing (15) a second time. Thus we have

$$\beta = \beta_1 - \frac{f(\beta_1)/f_1(\beta_1)}{1 - f(\beta_1)f_2(\beta_1)/2f_1^2(\beta_1)}. \quad (16)$$

It may be noted that although the correction methods used here are similar to that in [4], the form of the correction factor used in [4] is only an approximate of (15), which is the more accurate method. Also, while [4] has evaluated up to the 3rd order derivative in Taylor series approximation, our method involves only expansion till the second order, yet giving similar accuracy. This reduces the computation time. The computation time of the present model and [4] is enumerated in Table 1. The residue given by this method is as low as $5x10^{-13}$ and is described in section 3.

3. RESULTS AND DISCUSSION

Fig. 2 compares the surface potential obtained from the present method with that obtained by solving (3) numerically (set with an accuracy of 0.1 pV) for different values of V_{DS}. As can be seen, a very good match is obtained between the results from our approach and the numerical solution. Fig. 3 shows the residue in the surface potential value obtained by plugging the approximate values of β given by the present method in (3). The residue is as small as $5x10^{-13}$ which is quite sufficient for compact modeling and comparable to [4].

For compact models, it is important not only to achieve high accuracy but also to have continuous derivatives of the surface potential values. Such smooth derivatives are an absolute necessity for simulating fast transients in RF circuits. Fig.4 show the first derivative of the surface potential with respect to V_{GS} obtained from our model and that from exact numerical solutions. These plots do not possess any discontinuity, indicating that the method reported in this paper is extremely suitable for state of the art compact models. In order to check the accuracy of the β values from our model, it was implemented in the charge and drain current equations in [2]. The drain current versus gate voltage (I_{DS} vs V_{GS}) characteristics are compared in Fig. 5 (both in logarithmic and in linear scale) with the case where β is calculated numerically. The extremely good match shows that the model prediction is quite satisfactory.

An important benchmark test for analytical surface potential solution is the accuracy and smoothness of transcapacitances. In Fig. 6 normalized transcapacitances (C_{gg}, C_{dg} and C_{sg}) versus gate voltage V_{GS} obtained using the proposed analytical approximation are compared with the case where the surface potential is calculated numerically. These plots have been obtained using equations in [2]. A good match shows the accuracy and robustness of our model.

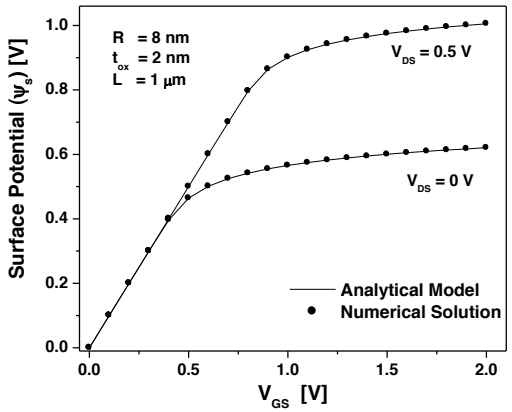

Fig.2. Comparison of surface potential as obtained from the proposed analytical solution and exact numerical method.

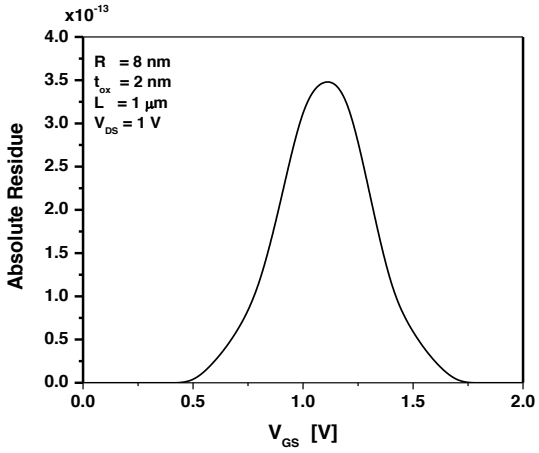

Fig.3. Plot of absolute residue in β values, obtained from plugging the results from our analytical model in (3), in [4]

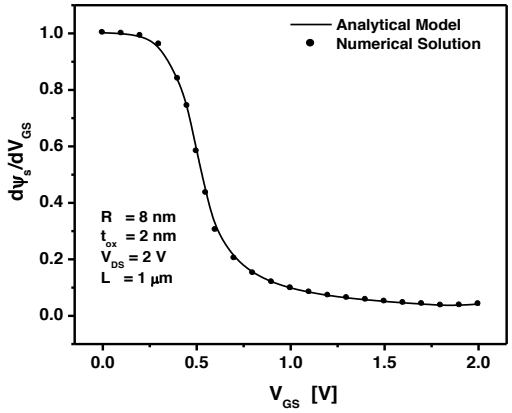

Fig.4. Comparison of first derivative of surface potential obtained from the analytical solution for different values of V_{GS}, with that from numerical solution.

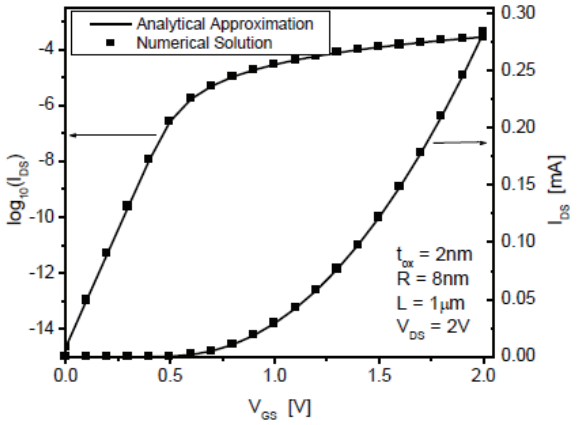

Fig.5. Comparison of I_{DS} (both log and linear scale) as obtained from surface potential values calculated numerically and from our analytical solution for different V_{GS} using equations in [2].

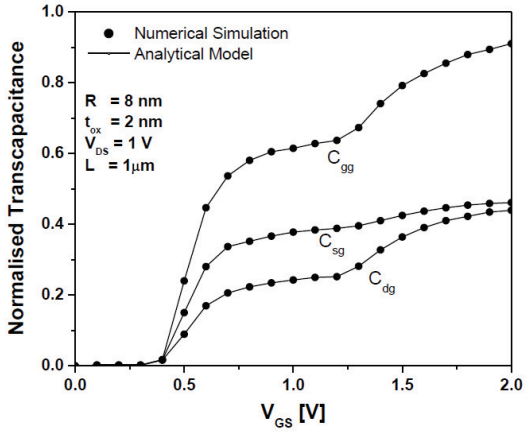

Fig.6. Comparison of normalized transcapacitances obtained from our analytical model and numerical simulations using capacitance equations in [2].

Table 1 enumerates the computational efficiency and applicability of the present model and that presented in [4]. The CPU time refers to the elapsed time for each of the algorithm, calculated in matlab.

Table 1. Comparison of models in [4] and present work

Approximation	CPU time (a.u)	Remarks
[4]	1.22	Fails if V_{GS}-V_{FB} <-1.3V
Present Work	1.212	Works in all cases

4. CONCLUSION

In this paper we have presented a simple analytical approximation to solve the implicit equation for the Surrounding-Gate MOS without the use of any empirical relation and/or smoothing functions. The solution is computationally more efficient compared to that in [4]. Also it has been found to be applicable to low V_{GD} values, thus forming the new standard for solving (3).

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