Numerical Study of Carrier Velocity for P-type Strained Silicon MOSFET

Yau Wei Heong, Mohammad Taghi Ahmadi, Jatmiko Endro Suseno,
Razali Ismail

Faculty of Electrical Engineering
University Technology Malaysia
81310 Skudai, Johor

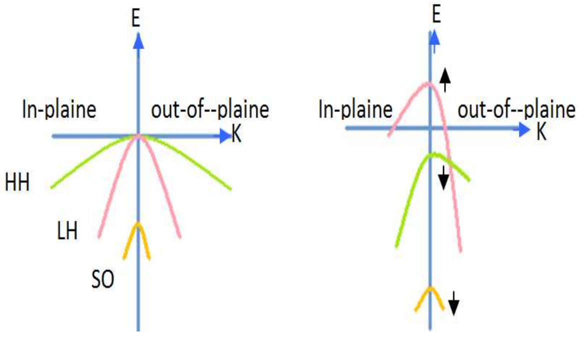
ABSTRACT

Strained induced in the silicon channel layer provides lower effective mass and suppresses intervalley scattering. In this paper, a numerical study of carrier concentration for P-type strained Silicon MOS is presented. Density of state proportion of Fermi-Dirac integral that covers the carrier statistics to all degenerate level is studied and its limits are obtained. In the nondegenerate regime the results replicate Boltzmann statistic and its result is vary in degenerate regime. The Fermi energy with respect to the transformed band edge is a function of carrier concentration for quasi two dimensional strained Silicon PMOS. Based on the Fermi – Dirac statistic, density of state the carrier concentration is obtained. Fermi energy is a function of temperature that independent of the carrier concentration in the nondegenrate regime. In the other strongly degenerate, the Fermi energy is a function of carrier concentration appropriate for given dimensionality, but is independent of temperature. The limitations on carrier drift due to high-field streamlining of otherwise randomly velocity vector in equilibrium is reported. The results are based on asymmetrical distribution function that converts randomness in zero-field to streamlined one in a very high electric field. The ultimate drift velocity is found to be appropriate thermal velocity for a given dimensionality for non- degenerately doped nanostructure. However, the ultimate drift velocity is the Fermi velocity for degenerately doped nanostructures.

Keywords: numerical study, carrier velocity, strained silicon

1 INTRODUCTION

With the purpose of increasing capacity in an integrated circuit, transistors have been scaled down with the factor of 2 in almost every 2 years. In order to longer the lifetime of silicon, strained Silicon MOSFET is a very promising device which is able to enhance the mobility and current drive without scaling down the transistor geometrically [1]-[2]. The combination of lower carrier effective mass and reduced intervalley scattering provides higher mobility. For biaxial strained Silicon PMOS, with high Ge fraction in $Si_{1-x}Ge_x$ layer, the tensile strain splits the light hole and heavy hole degeneracy of the valence band at the Γ-point and the effective masses become highly anisotropic with strain. In in-plane direction, higher-energy (ground state) valence band in biaxial strained Silicon is the light hole band, and the lower-energy band is the heavy hole band. Meanwhile, for out-of-plane direction, HH is in the higher energy band as shown in Fig.1. Thus, the light hole effective mass will be taken into account to contribute to hole mobility in in-plane direction.



(a) Unstrained (b) Biaxial Tension

Figure 1: Valence band of (a) bulk Si and (b) Si under biaxial tension [3]

In two-dimensional hole gas, when the thickness of strained Silicon, h_{si} is lesser than De – Broglie wave length, which is 10nm [4], the hole will have quantized energy level in z direction, but is free move in x and y direction as shown in Fig. 2.

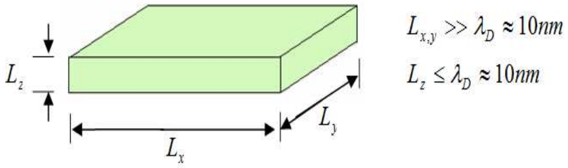


Figure 2: Scheme of Quasi-two- dimensional device quantum limit.

2 HOLE CONCENTRATION IN VALENCE BAND VALENCE

In general, the energy spectrum for strained Silicon can be expressed as a simple form [5]:

$$E_v = \Delta E_v + \sum_i \frac{\hbar^2 k_i^2}{2m_i} \tag{2.1}$$

Where ΔE_v is the band edge shift and m_i is the hole effective mass. In one dimensional quantum confinement, one of the three Cartesian directions is less than De-Broglie wave length (Z), which is digitized. Meanwhile, x and y direction are classical.

$$E = E_{vo} - \frac{\hbar^2}{2m_{lh}^*}(k_x^2 + k_y^2) + \in_{oze} \tag{2.2}$$

Where $\in_{oze(h)} = \frac{n^2\pi^2\hbar^2}{2m_{hh}^* L_z^2}$ which is digitized energy in z direction. Ttotal carrier concentration in a valence band, can be obtained by integrating the Fermi-Dirac distribution function over energy band, where the number of electrons $/cm^3$ and holes $/cm^3$ with energies between E and E+dE has been established to be $D(E)f(E)dE$ and $D(E)[1-f(E)]dE$ [6]. Therefore, the expression for the hole concentration is:

$$p = \int_{E_{bottom}}^{E_V} D(E)[1-f(E)]dE \tag{2.3}$$

The distribution function of the energy E_k is given by:

$$f(E_k) = \frac{1}{e^{\frac{E_k - E_{F2}}{k_B T}} + 1} \tag{2.4}$$

Where E_{F2} is the Fermi energy at which the probability of occupation is half, T is the ambient temperature. In nondegenerately doped semiconductors the '1' in the denominator is negligible compared to exponential factor, the distribution is then Maxwellian. In the other extreme for strongly degenerate carriers, the probability of occupation is 1 where $E_k < E_F$ and it is zero if $E_k > E_F$. Substituting the density of state $D(E)$ and Fermi function $f(E)$ expressions into above we have:

$$p = N_{V_2}\Im_0(\eta_F) \tag{2.5}$$

Where $N_{V_2} = -\frac{m_{lh}^* k_B T}{\pi\hbar^2}$, $\Im_0(\eta_F)$ is the Fermi-Dirac integral and $\eta_F = E_F - E_{vd}/k_B T$ and Fermi-Dirac integral of order i is defined as:

$$\Im_i(\eta_F) = \frac{1}{\Gamma(i+1)}\int_0^\infty \frac{x^i}{e^{(x-\eta_d)} + 1}dx \tag{2.6}$$

Under nondegenrate condition, we have

$$\Im_0(\eta_F) = e^{\eta_F} = e^{\frac{E_F - E_v}{k_B T}} \tag{2.7}$$

Thus, hole concentration in non- degenerate regime is:

$$p_2 = N_{V_2} e^{\eta_F} \tag{2.8}$$

Which depends to the temperature. Above simplified distribution function is only true for nondegenerately-doped semiconductor. However, current nano scaled transistor devices designed mainly in degenerate regime. Hence any design based on the Maxwellian distribution is not strictly correct and often leads to errors in our interpretation of the experimental results. For degenerate regime,

$$\Im_0(\eta_F) = \frac{1}{\Gamma(0+1)} \cdot \frac{\eta^{0+1}}{0+1} = \eta = \frac{E_F - E_v}{k_B T} \tag{2.9}$$

Thus, for the hole concentration in degenerate regime we can use $p = N_{V_2}.\eta$, this equation shows that hole concentration is dependent of carrier concentration. In quasi two dimensional semiconductors such as biaxial strained Silicon, the Fermi-Dirac integral proportion of exponential of e^{η} in nondegenerate approximation, and proportion of η in degenerate approximation.

3 CARRIER VELOCITY

Arora [7] modified the equilibrium distribution function of Eq. (2.4) by replacing E_{F3} (the chemical potential) with the electrochemical potential $E_{F3} + q\vec{\varepsilon}.\vec{\ell}$. Here $\vec{\varepsilon}$ is the applied electric field, q is the electronic charge and $\vec{\ell}$ the mean free path during which carriers are collision free (ballistic). Arora's distribution function is thus given by:

$$f(E_k) = \frac{1}{e^{\frac{E_k - E_{F3} + q\vec{\varepsilon}.\vec{\ell}}{k_B T}} + 1} \tag{3.1}$$

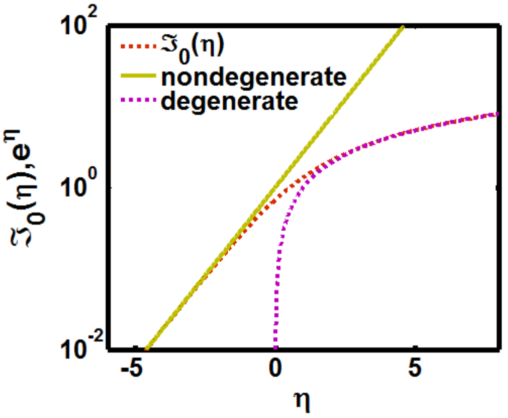


Figure 3: Comparison of the Fermi-Dirac integral, e^η and η

This distribution has simpler interpretation as given in the tilted band diagram of Fig.4.

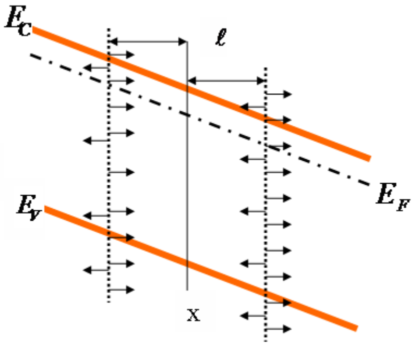


Figure 4. Partial streamlined of electron motion on a tilted band diagram in an electric-field.

The carriers at a point x arrive from left or right a mean-free-path ℓ from either side. It can be series that the Fermi level on left $E_{F3} + q\vec{\varepsilon}.\vec{\ell}$ and that on the right $E_{F3} - q\vec{\varepsilon}.\vec{\ell}$ are the two quasi Fermi levels with E_F that at x. The current flow is due to the gradient of Fermi energy $E_{F3}(x)$ is not constant when electric field is applied. The ballistic motion in a mean-free path can be interrupted by the onset of a quantum emission of energy $\hbar\omega_0$ [8].The average velocity per holes is:

$$v_i = \frac{\int v.D_2(E).(1 - f(E)).dE}{\int D_2(E).(1 - f(E)).dE} \tag{3.2}$$

From Eq. (3.2), by using Fermi integral notation the average velocity for biaxial strained Silicon is obtained as below:

$$v_i = \frac{\frac{\pi^{\frac{1}{2}}}{2}(v_{th}).\Im_{\frac{1}{2}}(\eta_F)}{.\Im_0(\eta_F)} \tag{3.3}$$

Where parameters v_{th} is thermal velocity, $\Im_{\frac{1}{2}}(\eta_F)$ and $\Im_0(\eta_F)$ are Fermi Integral order ½ and order zero v_{th} parameter is defined as below:

$$v_{th} = \sqrt{\frac{2k_B T}{m^*}} \tag{3.4}$$

The maximum average velocity per hole $v_{\max}$ is a function of temperature and doping concentration [9]:

$$v_{\max} = \sqrt{\frac{8k_B T}{\pi m^*}} \times \frac{N_{v2}}{p_2} \Im_1(\eta_{F2}) \tag{3.5}$$

Where N_{v2} is effective carrier density

$$N_{v2} = 2\left(\frac{m^* k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} \tag{3.6}$$

By replacing Fermi integral with degenerate approximation, the ultimate drift velocity is the Fermi velocity for degenerately doped nanostructures. In this approximation ultimate drift velocity only depends on carrier concentration and independent of temperature, see Fig 5.

$$v_{m3} = \frac{3\hbar}{4m^*}\left[\frac{3n}{8\pi}\right]^{\frac{1}{3}} \tag{3.7}$$

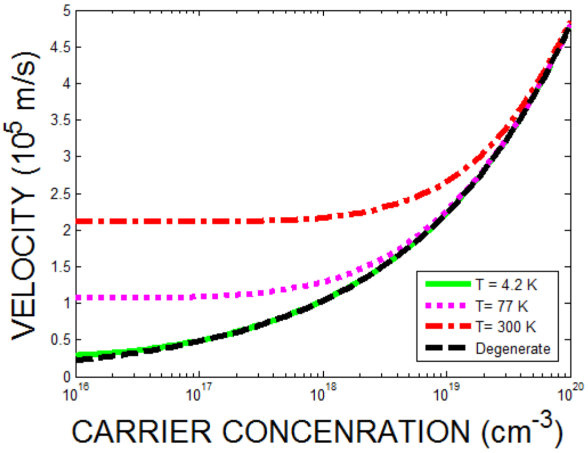


Figure 5: Velocity verses doping concentration.

4 CONCLUSION

Using this model, we have investigated biaxial strained Silicon. The distribution function transfers random velocity vectors into the streamlined in a very high electric field. The ultimate drift velocity is found to be appropriate thermal velocity for a given dimensionality for non-degenerately doped nanostructure. However, the ultimate drift velocity is the Fermi velocity for degenerately doped nanostructures.

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