A Parallel-plate-based Fishbone-Shape MEMS Tunable Capacitor with Linear Capacitance-Voltage Response

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ABSTRACT

A novel fishbone-shape parallel-plate-based capacitor with high tunability and linear *C-V* response is presented. The electrodes consist of a set of lateral cantilever beams of different lengths, attached to a longitudinal fixed-fixed beam. An insulating layer prevents contact between the electrodes. When a DC voltage is applied, the longitudinal and lateral beams undergo out-of-plane deformations changing the gap between two electrodes and the capacitance. As bias voltage increases, depending on their length and location, local pull-in for beams occurs at different voltages leading to a controlled *C-V* response. Using ANSYS® FEM simulations, a design optimization is performed to enhance the response for higher tunability and linearity. The simulation results of capacitors designed for PolyMUMPs exhibit tunability over 200%, 160% of which is highly linear. The presented design methodology, a combination of flexible structure and controlled displacements, is not limited to fishbone-shape electrodes and can be extended to different geometries to obtain higher tunability and linearity.

Keywords: MEMS tunable capacitor, fishbone-shape electrode, linear response, structural nonlinearity.

1 INTRODUCTION

MEMS parallel-plate tunable capacitors with electrostatic actuation have applications in different areas such as sensors, actuators and tunable filters and resonators. In communication engineering, such capacitors are desired for their quick responses, low energy consumptions and small sizes [1,2]. The device usually has two electrodes, one fixed to the substrate and the other one suspended by supporting beams modeled as linear mechanical springs [3]. The capacitance is obtained from:

$$C = \frac{\varepsilon_0 A}{d} \tag{1}$$

where A is the area of the electrodes, d is the gap in between, and ε_0 is the permittivity of air. When bias voltage, V, is applied, the electrostatic force obtained from

$$F_e = \frac{\varepsilon_0 A V^2}{2d^2} \tag{2}$$

deforms the supporting beams and reduces the air gap. The gap is then obtained by solving the equation of static equilibrium:

$$\frac{\varepsilon_0 A V^2}{2d^2} - k_{eq}(d_0 - d) = 0 \tag{3}$$

where k_{eq} represents the stiffness of all supporting beams.

Nonlinearity of the coupled electrostatic-structural equation (3) results in a nonlinear *C-V* response shown in Figure 1. As the voltage increases, the electrostatic force grows faster than springs resistive force and causes structural instability and pull-in at $d = 2/3d_0$, where d_0 is the initial gap between electrodes. Due to pull-in effect, electrostatically actuated parallel-plate capacitors have limited tunability of 50%, $C_{max} = 1.5C_0$, which is one of the main drawbacks of such devices.

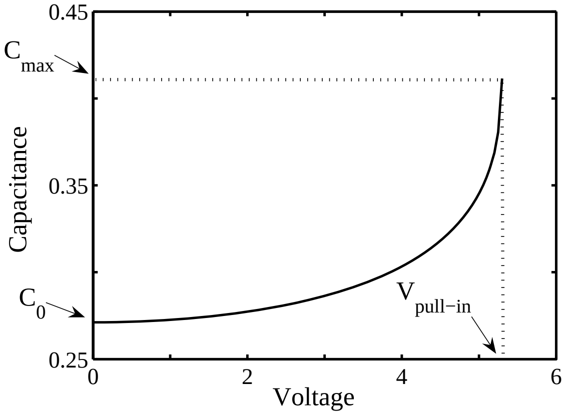


Figure 1: *C-V* response of a conventional parallel-plate tunable capacitor.

Different designs introduced in the literature offer higher tunabilities. For example, Gray *et al.* [5] and Bakri-Kassem and Mansour [4] proposed designs with structural enhancements, where higher nonlinear stiffness increases the maximum tunability of the capacitor to 95% and 410%, respectively. Using separation of actuation and sense electrodes, a wide range of tunabilities up to 1700% have been reported [6,7]. The *C-V* responses in such designs are generally divided into two distinct tuning and switching regions. The switching part is characterized by a jump in capacitance due to a small change in voltage, making the large portion of the *C-V* response unusable.

Comparing to several highly tunable capacitors presented in the literature, there are few research works focusing on the linearization of capacitance-voltage curve. Seok *et al*. [8] and Tsai *et al*. [9] reported comb-drive capacitors with 50 and 118% tunabilities, respectively. Shavezipur *et al*. [3] introduced a parallel-plate tunable capacitor with structural nonlinearity and tunability up to 150%, where 50% of this tunability is linear. Shavezipur *et al*. [10] also developed a parallel-plate-based butterfly-shape capacitor with flexible moving electrode and linear *C-V* response.

In this paper, a fishbone-shape capacitor is introduced that can provide high tunability and linearity. The electrodes have a set of lateral cantilever beams connected to a longitudinal beam which is anchored to the substrate at both ends. As will be discussed later in the paper, the novel electrode shape, structural flexibility and insulation between electrodes are key design concepts developed in this research to attain higher tunability and linearity. The FEM-based design optimization results display 160% linear tunability, where the maximum tunability exceeds 200%.

2 FISHBONE-SHAPE CAPACITORS

A fishbone-shape capacitor fabricated using PolyMUMPs [11] is shown in Figure 2. The moving electrode has a longitudinal beam, anchored at both ends. A set of lateral cantilever beams are connected to the longitudinal beam at both sides. When a bias voltage is applied, distributed electrostatic force (Figure 3-a) deforms the structurally flexible longitudinal and lateral beams and the air gap varies through the length of the beams. The displacement of each beam is therefore the result of a rigid-body displacement due to deformation of longitudinal beam and deformations of the beam due to electrostatic force. The gap for i^{th} beam is obtained from:

$$d_i(x) = \overline{D}_i - z_i(x) \tag{4}$$

where $\overline{D}_i$ is resulted in from the rigid-body displacement of the i^{th} beam and $z_i(x)$ is the beam's out-of-plane deformation under distributed electrostatic force, $f_i(x)$, (see Figure 3-b):

$$EI_i z_i^{IV}(x) = f_i(x) \tag{5}$$

$$f_i(x) = \frac{\varepsilon_0 w_i V^2}{2d_i(x)^2} \tag{6}$$

where w_i is the width of i^{th} cantilever beam. Nonlinear equations (4)-(6) don't have closed-from solutions and should be solved numerically to obtain the air gap at each point. Once the gap for all beams is known, the capacitance is calculated by integration over the area of the electrode.

When the bias voltage increases, the gap between two electrodes will decrease resulting in a higher capacitance. Since the structural rigidity and electrostatic force vary from one beam to another, local pull-in for each beam then occurs at a different voltage. The insulating layer between electrodes prevents the contact when a local pull-in happens. This is the core concept of this design that eliminates big jumps in the *C-V* curve, as will be demonstrated later in this paper. To study the behavior of a fishbone-shape capacitor, an ANSYS® FEM simulation is performed. The governing coupled electrostatic-structural equations are solved using ESSOLV command, which iteratively solves the nonlinear electrostatic and structural equations to converge to the static equilibrium.

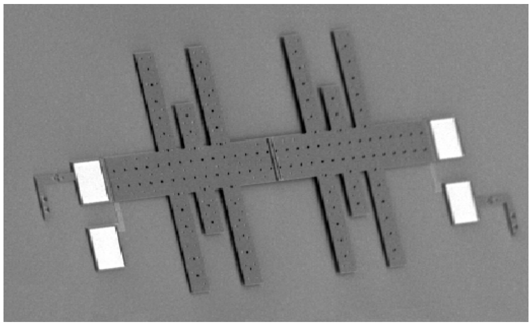
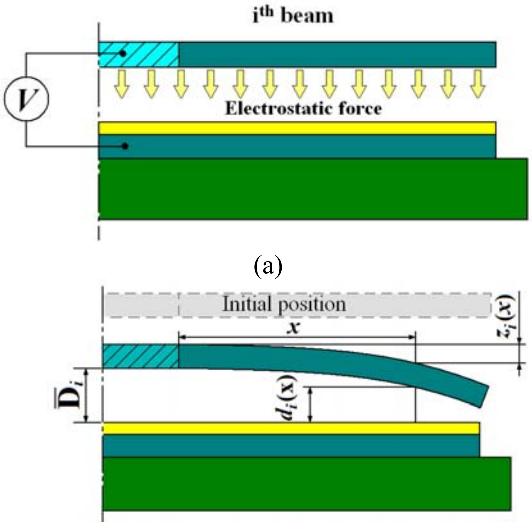


Figure 2: A fishbone-shape tunable capacitor.



(a)

(b)

Figure 3: i^{th} lateral beam and its displacement;
(a) Simplified model; (b) Beam deformations.

The capacitor studied in this section is designed for PolyMUMPs, where the fixed and moving electrodes are made of Poly0 and Poly2 layers, respectively. The initial gap between electrodes is 2.75 μm. The separation of two electrodes for lateral beams at local pull-ins is provided by DIMPLE mask, as shown in Figure 4. This technique limits the minimum distance between two electrodes without using a dielectric layer as shown in Figure 4, equivalent to the case of a virtual insulator with the same permittivity of air, $\varepsilon_r = 1$.

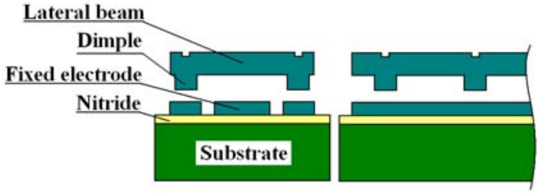


Figure 4: The virtual insulator between two electrodes using DIMPLAE mask in PolyMUMPs.

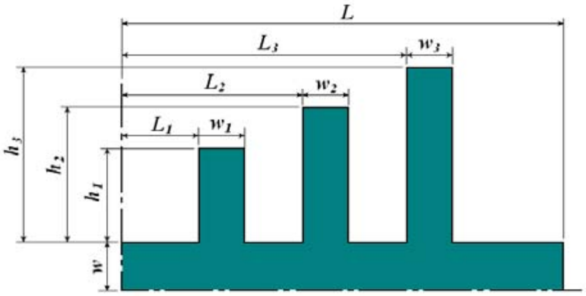


Figure 5: A ¼ model of a symmetric fishbone shape capacitor with three independent beams.

L	w	L_1	L_2	L_3	w_1	w_2	w_3	h_1	h_2	h_3
460	40	90	180	270	50	50	50	110	175	155

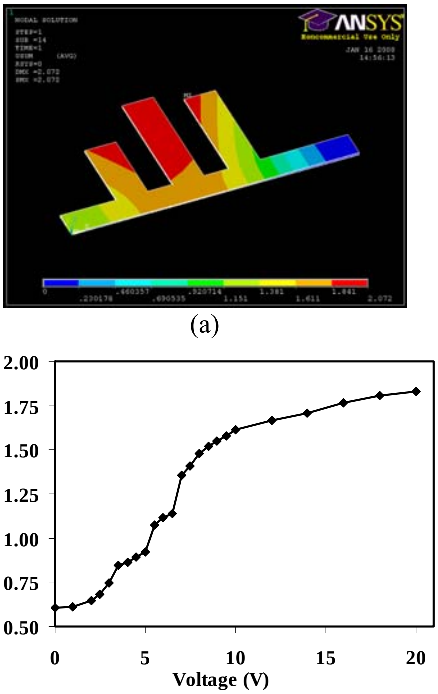
Table 1: The dimensions of a symmetric fishbone-shape capacitor (all dimensions in µm)

Figure 5 and Table 1 represent the dimensions of a ¼ symmetric model with three uneven lateral beams. It is possible to increase the number of lateral beams; however, since each extra lateral beam creates a set of contact pairs in structural model, it will drastically increase the number of iterations and computation time.

The results of ANSYS® simulations are presented in Figures 6-a and 6-b. For this capacitor, tunability exceeds 200% at $V = 20$ V. It also eliminates the pull-in as portrayed in Figure 6-b. Furthermore, using several lateral beams with different lengths breaks down the *C-V* response into small sub-sections; each of them is the result of a local pull-in for one of the beams which considerably improves the linearity of the response.

To quantitatively express the level of linearization, the coefficient of linear correlation between capacitance and voltage is considered as the linearity factor, *LF* [4,11]:

$$LF = \frac{n\sum C_i V_i - \sum C_i \sum V_i}{\sqrt{[n\sum C_i^2 - (\sum C_i)^2][n\sum V_i^2 - (\sum V_i)^2]}} \tag{7}$$



(a)

(b)

Figure 6: ANSYS simulation results for the ¼ model; (a) electrode deformations, (b) *C-V* response.

LF for the *C-V* response of a conventional parallel-plate capacitor (Figure 1) is 0.865 and as the response approaches a line, *LF* approaches one. For the response presented in Figure 6-b, $LF = 0.988$ for voltage interval $2\text{ V} < V < 9\text{ V}$, and the corresponding so-called “linear tunability” is 140%.

3 DESIGN OPTIMIZATION

Parallel-plate tunable capacitors with fishbone-shape electrodes exhibit higher tunability and linearity comparing to conventional designs. It is possible to further improve the linearity of the response by optimizing the size and location of lateral beams. The optimization problem is defined in the following form:

$$\begin{aligned} &\max\ LF(L_i, h_i, w, L) \\ &\qquad \text{subject to:} \\ &\qquad\qquad h_i < h_0 \\ &\qquad\qquad L < L_0 \end{aligned} \tag{8}$$

where h_0 and L_0 are dimensional constrains. To obtain an optimum *C-V* response with highest linear tunability, different sets of design variables (see Table 2) are examined using ANSYS simulations. As shown in Table 2, Cap IV provides the best results and the tunability in linear region, $2\text{ V} < V < 10\text{ V}$, reaches 159% with $LF = 0.998$ and its *C-V* response, displayed in Figure 6-b, demonstrates a high linearity. For all capacitors presented in Table 2, the maximum tunability exceeds 200%. It is possible to define higher tunability or a combination of tunability and linearity as the objective of the optimization problem (8). One can also add the process limitation as the optimization constraints.

The design technique proposed in this paper is not limited to fishbone-shape and based on design requirements and process capability can be extended to other flexible geometries. If a dielectric layer is deposited on the top of fixed electrode, higher tunabilities are expected to be obtained.

Design parameter (μm)	Cap I	Cap II	Cap III	Cap IV
L	460	460	460	460
w	40	40	40	40
L_1	90	100	100	110
L_2	180	180	180	180
L_3	270	260	260	250
w_1	50	50	50	50
w_2	50	50	50	50
w_3	50	50	50	50
h_1	110	90	180	180
h_2	175	190	100	80
h_3	155	160	160	180
Linear tunability (%)	140	146	148	159
LF	0.988	0.994	0.996	0.998

Table 2: Linear tunability and *LF* for fishbone-shape capacitors with different dimensions.

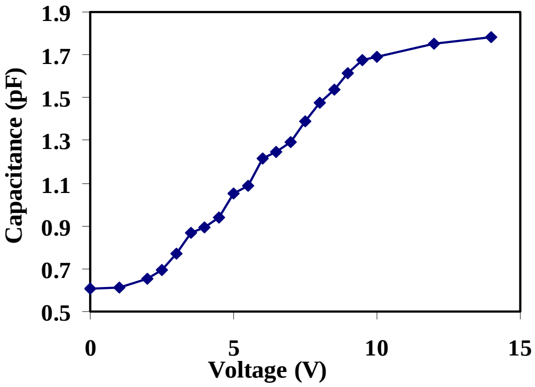


Figure 5: *C-V* response of the optimized design, Cap IV.

4 CONCLUDING REMARKS

A MEMS tunable capacitor with novel structure was introduced. The fishbone-shape electrodes provide controllable displacements that can be advantageously used to improve the tunability and linearity of the *C-V* response. As bias voltage increases, each lateral cantilever beam faces a local pull-in at a different voltage and the electrodes are separated by an insulating layer to prevent short circuit at local pull-ins. The FEM simulations demonstrate notable improvement in linearity and tunability of optimized designs. The proposed capacitor has a simple structure and can be fabricated using a two-conductive-layer process with a dielectric on the top of fixed electrode. The design methodology presented in this paper can be extended to other flexible structures and fabrication processes.

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