# A Continuous yet Explicit Carrier-Based Core Model for the Long Channel Undoped Surrounding-Gate MOSFETs

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## ABSTRACT

An explicit carrier-based core model for the long channel undoped surrounding-gate MOSFETs is presented in the paper. An analytic approximation solution to the carrier concentration is developed from a simplified Taylor expansion of the exact solution of Poisson's equation of the surrounding-gate MOSFETs, instead of resorting to the Newton-Raphson numerical iterative. The analytic approximation not only gives accurate dependences of the carrier concentration on the geometry structures and bias, compared with the Newton-Raphson numerical method, but also is used to develop an explicit current-voltage model of the surrounding-gate MOSFETs combined with Pao-Pah current formulation. The presented explicit model is found to be computationally more efficient than the previous numerical Newton-Raphson iterative while more accurate than the previously published explicit model.

***Keywords**:* device physics, compact model, surrounding-gate MOSFET, carrier-based model.

## 1 INTRODUCTION

The SRG-MOSFET based circuit design is contingent on the precision of the explicit transistor model involved in circuit simulation. In addition to the precise description of the SRG device characteristics, computation efficiency remains an important constraint for the SRG compact model to efficient circuit simulation. In the recent years, there were some reports investigating the compact models of the non-classical SRG MOSFET device characteristics [1-4]. D.Jimenez and his co-workers developed the $\beta$ -based model and the result matched well with the 3-D simulation [2]. Following a quite different method, Jin He's group developed a carrier-based non-charge-sheet analytic model for SRG MOSFETs directly from both Poisson equation solution and Pao-Sah current formulation [3-4]. These models provide a fundamental yet solid basement for engineers and circuit designers to understand the SRG device physics and characteristics. However, most developed SRG MOSFET models rely on numerical iteration or table-lookup to solve the fundamental nonlinear implicit equations between the input voltage and the state variables such as $\beta$ , potentials and carrier [2-4]. We noted that B. Iñíguez et al. presented an explicit charge-based compact model for SRG-MOSFET devices based on the threshold voltage concept and smooth functions in [5]. However, it is observed from the accuracy test that this explicit model is not good enough for high precise requirement of the SRG-MOSFET core model because it gives the prediction with the relative drain current error up to 32% error in the moderate inversion region which is very important for non-classical MOSFET low voltage and low power circuit design.

A SRG MOSFET compact model for circuit simulations requires accurate yet computation efficient core framework. Thus, the aim of SRG-MOSFET compact model formulation is to conflate an accurate description of device characteristics with high computational efficiency. In this brief, an explicit carrier-based SRG MOSFET current-voltage model has been presented from an accurate yet analytic approximation to the carrier concentration solution. Compared with the previous explicit model [5].

## 2 EXPLICIT MODEL DERIVATION

The undoped SRG MOSFET structure and coordinate system used in the analysis are shown in Fig.1. Following a carrier-based approach [2-4], a complete Poisson equation solution in terms of the silicon center mobile carrier concentration as a function of the gate voltage, quasi Fermi potential, and the geometry structure is derived as shown in [4].

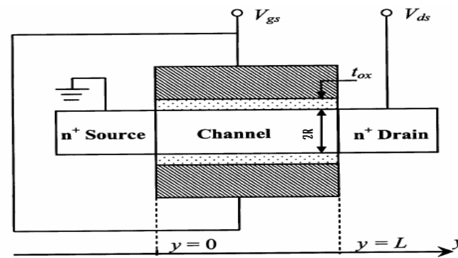


Fig.1 Schematic diagram of a SRG MOSFET

$$V_G - \Delta\phi_i - V_{ch} =$$

$$\frac{kT}{q}\ln\left(\frac{n_0}{n_i}\right) - \frac{2kT}{q}\ln\left[1-\frac{R^2}{8L_i^2}\frac{n_0}{n_i}\right] + \frac{R^2\varepsilon_{si}kT\ln\left[1+\frac{t_{ox}}{R}\right]}{2qL_i^2\varepsilon_{ox}}\frac{\frac{n_0}{n_i}}{\left[1-\frac{R^2}{8L_i^2}\frac{n_0}{n_i}\right]} \quad (1)$$

where $L_i=\sqrt{\frac{\varepsilon_{si}kT}{q^2 n_i}}$ is the Debye length of the intrinsic silicon materials, $n_i$ is the intrinsic silicon concentration with the unit of $cm^{-3}$, $\phi_s$ is the electrostatics in $Volt$ at the silicon surface. $kT/q$ and $V_{ch}$ are the thermal voltage and the quasi-Fermi-potential in $Volt$, respectively. $n_0$ is the induced electron concentration at the silicon center with the unit $cm^{-3}$. $t_{ox}$ and R are the gate oxide thickness and the SRG MOSFET silicon radius in cm, respectively.

It is evident that the mobile carrier concentration on the silicon film center can be obtained from Eq. (1) for the given gate voltage, quasi Fermi potential and structure parameters such as the oxide layer thickness and the silicon film thickness. However, Eq. (1) is a nonlinear implicit equation, which needs a very accurate solution for complete current-voltage and capacitance model development. Traditionally, this equation was solved by a Newton-Raphson iterative routine or table-lookup method in terms of some intermediate variables such as the carrier concentration and $\beta$ [1-4]. The numerical iterative and table-lookup methods, however, are not preferred for a compact model due to the need of extensive computation time and memory to store intermediate data. Thus, an accurate analytic approximate solution for the carrier concentration is desired for the explicit current-voltage model development for the SRG MOSFETs.

Here, we follow a recently developed method to derive an analytic approximation solution to the carrier concentration based on a simplified Taylor expansion formulation [6]. Firstly, we transformation Eq. (1) into a normalized formulation for simplicity

$$f(i)=g-\ln\left[i+i^2\right]-\lambda i=0 \qquad (2)$$

where

$$s=\frac{R^2}{8L_i^2},\ \lambda=\frac{4\varepsilon_{si}\ln\left[1+\frac{t_{ox}}{R}\right]}{\varepsilon_{ox}},\ i=\frac{R^2}{8L_i^2}\frac{n_0}{n_i}\left[1-\frac{R^2}{8L_i^2}\frac{n_0}{n_i}\right]^{-1} \qquad (3)$$

$$g=\frac{q\left(V_G-\Delta\phi-V_{ch}\right)}{kT}+\ln s \qquad (4)$$

In order to obtain an analytic approximation of (2), we use the perturbation method to get the correct functions for an initial guess that is accurate enough. As shown in [6, 7], the correction function can be derived out from the modified Taylor expansion of a function

$$\delta=-\frac{f(i)}{f'-\frac{f''(i)f(i)}{2f'(i)}} \qquad (5)$$

(5) will be used to build an analytical approximation solution. The exact first and second derivatives are obtained from (2)

$$f'(i)=\frac{1}{i}+\frac{1}{1+i}+\lambda \qquad (6)$$

$$f''(i)=-\frac{1}{i^2}-\frac{1}{(1+i)^2} \qquad (7)$$

And then substituting them into (5) gives

$$\delta=-\frac{f(i)}{f'+\frac{f(i)}{2i}\frac{(1+i)^2+i^2}{(1+i)^2+\lambda i(1+i)^2+i(1+i)}} \qquad (8)$$

Here, we need a continuous and accurate initial guess before the use of (8), which can be obtained from (2). Since $i$ has non maximum boundary and the term $\ln[1+i]$ has a little contribution in (2) either for the sub-threshold or the strong inversion region. Thus, we can review (2) as a W-Lambert function by neglecting the term $\ln[1+i]$ and it has a simple approximation for the principal branch

$$i_b=\frac{\ln(1+\lambda e^g)}{\lambda} \qquad (9)$$

In physics, $i_b$ presents an asymptotic approximation developed. If $\delta$ is a relative small refinement and we define

$$f_0=g-\ln\left[i_b+i_b^2\right]-\lambda i_b \qquad (10)$$

Then $\delta$ is obtained from (8). We define

$$i_0=i_b+\delta \qquad (11)$$

As a result, we have

$$i_0=i_b-\frac{f_0}{f'+\frac{f_0}{2i_b}\frac{(1+i_b)^2+i_b^2}{(1+i_b)^2+\lambda i_b(1+i_b)^2+i_b(1+i_b)}} \qquad (12)$$

Further, we define

$$f_1=g-\ln\left[i_0+i_0^2\right]-\lambda i_0 \qquad (13)$$

$$i=i_0+\alpha \qquad (14)$$

Finally, substituting (13), the first and second derivatives as a function of $i_0$ into (8) to get the symbol expression of $\alpha$, we obtain the accurate analytic approximation of the normalized carrier concentration

$$i = i_0 - \frac{f_1}{f'(i_0) + \frac{f_1}{2i_0} \frac{(1+i_0)^2 + i_0^2}{(1+i_0)^2 + \lambda i_0 (1+i_0)^2 + i_0(1+i_0)}} \qquad (15)$$

This analytic approximation requires computation of three logarithms, e.g. (9), (10) and (13) and one exponential (9), however, with no iteration. As long as $i$ is obtained, the electron concentration can be calculated based on (3).

Once the carrier concentration solution is obtained, the Poisson equation solution (1) is coupled to the Pao-Sah current formulation can result in the explicit current-voltage and capacitance-voltage model. For a given $V_g$, $n_0$ is solved from (1) as a function of $V_{ch}$. Following Pao-Sah current formulation [7], integrating $I_{ds}dy$ from the source to the drain and expressing $V_{ch}/dy$ as $(dV_{ch}/dn_0)(dn_0/dy)$, the drain current is written as

$$I_{DS} = \mu \frac{W}{L} \int_0^{V_{DS}} Q_i(V_{ch}) dV = \mu \frac{W}{L} \int_{n_{0s}}^{n_{0d}} Q_i(n_0) \frac{dV_{ch}}{dn_0} dn_0 \qquad (16)$$

where $n_{0s}$ and $n_{0d}$ are solutions of (1) corresponding to $V_{ch} = 0$ and $V_{ch} = V_{ds}$, respectively. Note that the $dV_{ch}/dn_0$ can also be expressed as a function of $n_0$ by differentiating (1). Substituting these factors into (16), integrating can be performed analytically to yield:

$$I_{ds} = \mu \frac{2\pi\varepsilon_{si}}{L} \left(\frac{2kT}{q}\right)^2 F[n_0] \Big|_{n_{0d}}^{n_{0s}} \qquad (17)$$

where

$$F[n_0] = \lambda\left(\frac{R^2}{8L_t^2}\frac{n_0}{n_i}\left[1-\frac{R^2}{8L_t^2}\frac{n_0}{n_i}\right]^{-1}\right)^2/2 + 2\left(\frac{R^2}{8L_t^2}\frac{n_0}{n_i}\left[1-\frac{R^2}{8L_t^2}\frac{n_0}{n_i}\right]^{-1}\right) - \ln\left[1+\frac{R^2}{8L_t^2}\frac{n_0}{n_i}\left[1-\frac{R^2}{8L_t^2}\frac{n_0}{n_i}\right]^{-1}\right] \qquad (18)$$

## 3 RESULTS AND DISCUSSION

The accuracy of the electron concentration calculation is one key factor for the SRG MOSFET performance prediction and circuit simulation. Comparison of the new analytic approximation (15) with the iterative numerical results of (2) for the carrier concentration calculation is shown in Fig.2 for the different silicon film radius. We note that an excellent accuracy is achieved for the whole operation region for different silicon radius, e.g. $R = 5nm$ or $R = 30nm$.

In order to further demonstrate the accuracy of the developed analytic approximation, the relative error of the electron concentration prediction with the different methods is also shown in Fig.3 for the different silicon film thickness. It is easily found that the relative error introduced by the new analytic approximation (15) is under $10^{-6}$ order. Such a precise is satisfactory enough for the

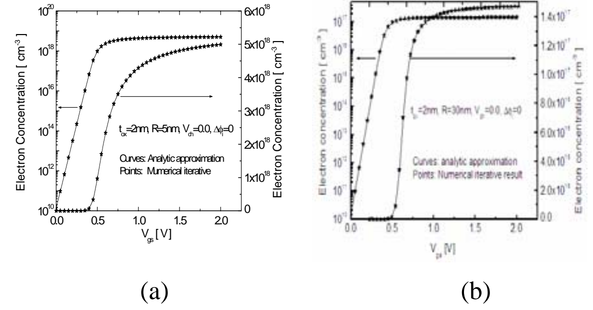


Fig.2 Comparison of the electron concentration versus the gate voltage obtained from the analytic approximation (Solid curves) and the fully numerical Newton-Raphson method (points) in undoped cylindrical surrounding-gate MOSFETs with the midgap gates for silicon radius R=5nm(a) and R=30nm(b).

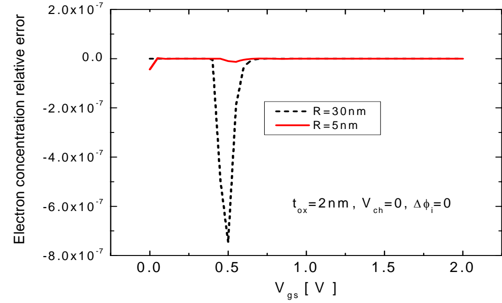


Fig.3 Relative error of the electron concentration calculation based on the analytic approximation and the Newton-Raphson iterative method for the different silicon radius in undoped cylindrical surrounding-gate MOSFETs with the midgap gates.

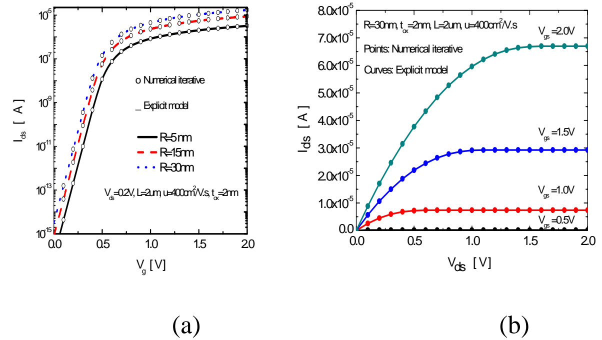


Fig.4 (a) Ids-Vgs curves and (b) Ids-Vds curve calculated from the explicit model (solid curves), compared with the numerical iterative results (points).

compact modeling development of the SRG MOSFETs.

Fig. 4(a) shows the good comparison of the drain current versus gate voltage between the explicit model and the iterative method for three silicon body radius, e.g. $R = 5nm$, $15nm$ and $30nm$. It is easily found that the explicit prediction shows in agreement with the iterative results. Again, it is found that the sub-threshold current is almost proportional to the square of the silicon body diameter because of the "volume inversion" effect. To optimize the device performance, the silicon film body radius should be reduced as much as possible, e.g., a nanowire body to be used, to suppress the off current although it is difficult in fabrication process. Fig. 4(b) is $I_{ds}$-$V_{ds}$ curves calculated from the explicit model (solid curves), compared with the numerical iterative results (dots). Both match well in both the linear and the saturation region.

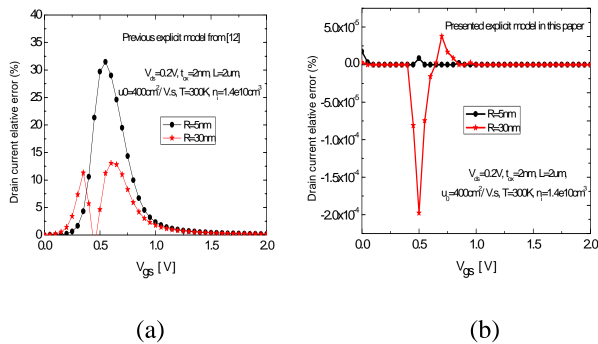


Fig.5 Predicted drain current relative error from B. Iñíguez et al's explicit model in reference [5] (a) and from the presented explicit model in this paper (b), compared with the iterative method.

We noted that B. Iñíguez et al. presented an explicit charge-based compact model for SRG-MOSFET devices based on the threshold voltage concept and smooth functions in [5]. One interesting issue is to compare the accuracy and computation efficiency of two different explicit SRG models. Such a relative drain current error comparison is shown in Fig.6 for two different silicon body radius. It is observed from Fig.5 that B. Iñíguez et al's explicit SRG-MOSFET model gives the drain current prediction with the different relative error, e.g. the maximum relative error is 13% for $R = 30nm$ while up to about 32% for $R = 5nm$.

## 4 CONCLUSIONS

An explicit carrier-based model for the undoped surrounding-gate MOSFETs has been developed in this brief by an accurate yet analytic carrier concentration approximation from Poisson equation solution of the SRG MOSFETs. The accuracy of the electron concentration calculation has also been verified compared with the result of the iterative method. It is shown that the predicted current-voltage curves are in complete agreement with the fully numerical iterative results without any fitting parameter. Compared with the previous explicit model and numerical iterative method, the presented explicit model requires no numerical iteration but more accurate and computation efficient, thus it is more suitable to implement SRG MSOFET core model into the circuit simulators for circuit design and application.

## ACKNOWLEDGEMENT

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