Modeling and Design of Electrostatic Voltage Sensors Based on Micro Machined Torsional Actuators

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ABSTRACT

The optimization of design parameters and methods for electrostatic voltage sensors based on torsional actuators is presented. The analytical software model used for this optimization process is discussed and compared to measurement results. Voltage excitation leads to an electrostatic attractive force, inducing a deflection of the actuator which in turn is detected using capacitance measurement. The conventional measurement principle for RMS voltages is based on power dissipation by ohmic resistance. We look at devices with parallel plate capacitances in a rotational design, which allows direct, substitution and compensation measurement of RF voltages. An analytical and a numerical model for this type of sensor are presented, focusing on absolute and relative sensitivity and precision for metrology purposes. The main design parameters are extracted from this model and variation of each parameter is discussed. The influence of fabrication tolerances on the dimensions and subsequently on the measuring results is investigated and an upper limit is given. Using this model, sensor designs are developed which fulfill the requirements for electrostatic voltage sensing for different sensitivity and range requirements. The final designs are checked against fabrication process limitations and an estimate of the maximum total error is given. The results of this work will be verified with devices designed with this theoretical understanding and built in a micromechanical silicon process.

Keywords: RF-MEMS, design, modeling, electrostatic, torsional actuator, RMS-Voltage

1 INTRODUCTION

In this paper we present the theoretical foundation of electrostatic voltage sensors based on torsional actuators. The conventional method for traceable high-frequency voltage metrology is based on power dissipation measurement of ohmic resistances allowing RMS voltage conversion by the square power law. Employing the principle of electrostatic force is an alternative method to solve this problem.

Actuator based voltage sensors rely on the principle of electrostatic force. Two electrodes with opposite charges attract each other. The charge is linked to the voltage by the capacitance of the structure. By elastically suspending one of the electrodes, an equilibrium position between the repelling spring force of the actuator and attracting electrostatic force exists. Due to the quadratically decreasing force as a function of distance, it is necessary to utilize micromachining for the fabrication of this sensor. Comb-like structures are often used in sensors and actuators for this task, allowing surface micromachining, but showing unwelcome parasitic effects. For metrology applications it is preferable to have a well calculable structure with few fringing effects. Therefore, a parallel plate capacitor is chosen for actuation and sensing. Building a torsional actuator with multiple electrodes on each side makes it suitable for voltage compensation and substitution measurements. Voltage excitation on one side leads to a deflection of the actuator which in turn is detected using capacitance measurement, but may also be detected by optical detection.

In this publication, we present at first an overview of the geometrical dimensions and present an analytical steady state solution for this sensor. Subsequently, we build a model for time-domain simulations using Matlab/Simulink with special emphasis on the damping behavior for different damping models. Using these models, the influence of the physical dimension tolerances of the sensor on different measurement principles is discussed. Finally measurement results of real sensors are presented and compared to the model. Similar research has been presented in [1]. In comparison, this publication presents a more general case, where the electrodes do not necessarily have to occupy the full plate area.

The results of this research are based on practical sensors, already presented in [2-4]. Other sensors based on the principle of electrostatic charge have been presented in [5] and [6] for other frequencies and voltage ranges.

2 THEORY

The principle sensor geometry is shown in Figure 1. The sensor consists of a plate with a length of *2L* and width of *B*, which is suspended at the center axis with two beams of length *l* and width *w*. The plate and beam are made of one material, silicon in this case, with a thickness *t*. They form a common electric potential. The plate is mounted at an initial distance h_0 above the opposing electrodes. It is assumed throughout the analysis that the plate only moves in torsional direction and exhibits no translational movements. This assumption has been confirmed by the fact that the spring constant orthogonal to the plate is at least two orders of magnitude higher than the rotational spring constant. This has also been verified using finite element simulation. As mentioned in the previous section, the electrodes do not cover the full plate area but only reach from $\alpha_1 L$ to $\alpha_2 L$, with $\alpha_1 < \alpha_2$, $\alpha_1 >= 0$ and $\alpha_2 <= 1$, while still covering the full width of the plate B. The default dimensions for all calculations in this paper are summarized in Table 1.

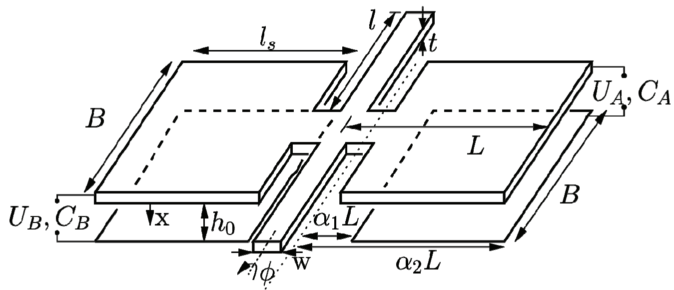

Figure 1: Geometrical model of the torsional actuator showing the important dimensions of the actuator and of the opposing electrodes.

Dimension	Symbol	Value	Unit
Plate length	L	2000	μm
Plate width	B	1000	μm
Gap	h_0	5	μm
Plate thickness	t	20	μm
Length of torsion beam	l	1000	μm
Width of torsion beam	w	20	μm
Start of electrode	$\alpha_1 \cdot L$	1000	μm
End of electrode	$\alpha_2 \cdot L$	2000	μm
Young's modulus, Silicon	E	160	GPa
Poisson's ratio, Silicon	υ	0.2	
Density, Silicon	ρ	2700	kg/m^3
Air viscosity, room temp	μ	$1.79 \cdot 10^{-5}$	Pa s

Table 1: Default dimensions used for simulations and calculations if not noted otherwise.

The following analysis is for the special case of a design with one electrode on each side of the beam, but can also easily be extended for multiple electrodes on each side of the beam. The capacitance and voltage on the side of the plate in positive direction of the angle ϕ are marked with a subscript A (C_A, U_A), the other side with a subscript B (C_B, U_B). In the following sections, we present a fully analytic steady-state model of the sensor, a numerical time-domain simulation and present first results.

The principle sensor design allows three different modes of voltage measurement. Direct measurement takes the deflection angle to calculate the applied voltage. Substitution measurement relies on a known (DC) voltage for which the deflection angle is measured. If the unknown voltage leads to the same deflection, it is known that both voltages are equal. The obvious advantage being that geometrical variances do not influence the result as they stay the same between two measurement cycles. The same advantage applies for compensation measurements, though it requires a third electrode or optical instruments for the detection of the deflection angle.

2.1 Analytical steady-state model

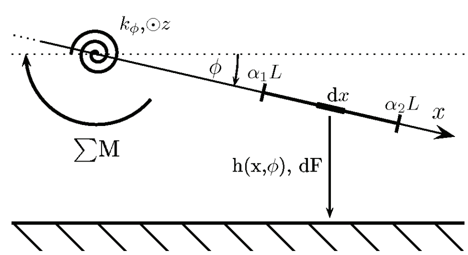

Figure 2: Geometrical model for the derivation of the analytical steady-state model.

For the derivation of the analytical model, we consider a simple 2D model of the geometry as shown in Figure 2. The local distance h along the beam, twisted with an angle ϕ can be calculated with

$$h(\phi, x) = h_0 - x \sin\phi \,. \tag{1}$$

Assuming a parallel field distribution between the electrodes, which is obviously true because $L >> h_0$, the

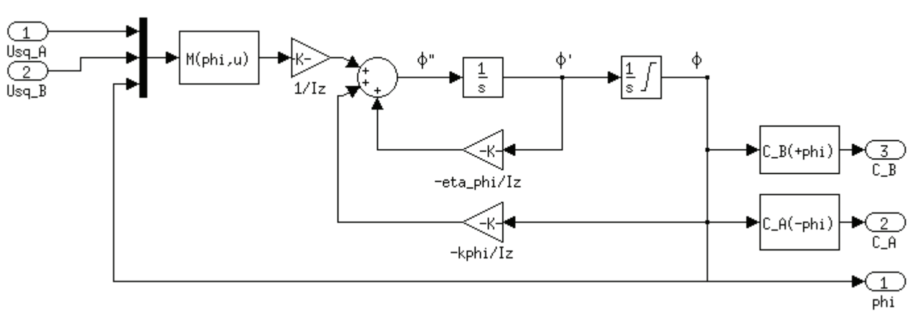

Figure 3: Matlab/Simulink time-domain model of the torsional actuator. Voltage squared inputs on the left and the capacitance and angle output on the right

force dF acting on a short section of the plate, depending on the voltage U, angle ϕ and position along the beam x, can then be derived from the parallel plate capacitance equation as

$$dF(x,\phi,U) = \frac{\varepsilon \cdot B \cdot U^2}{2 \cdot h(\phi,x)^2}. \quad (2)$$

Integrating the force along the loaded part of the beam from $\alpha_1 L$ to $\alpha_2 L$ gives an expression for the moment $M_U(U,\phi)$ around the rotating axis

$$M_U(U,\phi) = \int_{\alpha_1 L}^{\alpha_2 L} x dF(x,\phi,U) dx. \quad (3)$$

This can be analytically solved, employing the substitution $\psi = \sin(\phi)\, h_0 / L$ to

$$M_U(U,\psi) = \frac{BL^2 \varepsilon U^2}{2h_0 \psi^2}\left[\frac{1}{1-\psi\alpha_2} - \frac{1}{1-\psi\alpha_1} - \ln\frac{1-\psi\alpha_1}{1-\psi\alpha_2}\right] (4).$$

The restoring moment $M_k(\phi)$ from the torsional beam is expressed by

$$M_k(\phi) = \phi \cdot k_\phi, \quad (5)$$

with k_ϕ:calculated from

$$k_\phi = \frac{E \cdot \beta \cdot t \cdot w^3}{l(1+\nu)}. \quad (6)$$

E, υ are the Young's modulus and Poissons's ratio of the material respectively, and β is a numerical constant depending on the ratio t/w.

Figure 4 shows the generated moment for three distinct voltages over the turning angle and the corresponding restoring spring moment. It can be seen, that only for 1 V and 2 V intersections exist with the restoring moment graph. For 4 V there are no intersections and thus the actuator gets pulled-in up to the mechanical limit. Also, for 2 V clearly two stable positions exist. From this fact is may be concluded that the sensor has a hysteretic behavior.

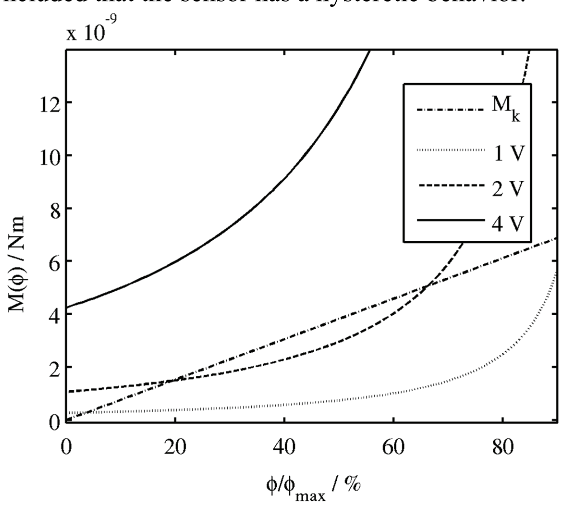

Figure 4: Acting and restoring moments on the actuator for different voltages 1 V, 2 V and 4 V

Solving $M_k = M_U$ for a function $\phi(U)$ is not analytically possible in the general case. Hence a numeric zero crossing detection is used. This gives the solution of the stable position of the actuator depending only on geometrical quantities and the applied voltage U. The solution for varying beam width, with capacitive feedback, can be found in Figure 5. For the highest sensitivity of the system, a low beam width and thus a low spring constant is required. For a high voltage range, on the other hand, a stiffer system is preferable. Depending on the application, different sensor designs have to be used. For compensation measurements, a high sensitivity is necessary, while for direct and substitution measurements the pull-in voltage has to be higher than the maximum voltage to be measured.

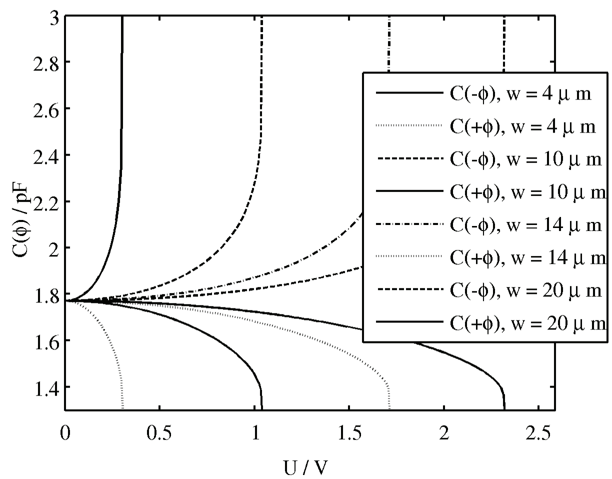

Figure 5: Equilibrium positions of the actuator with capacitive feedback, with the sensing capacitance on the same side (upper part of the figure) and opposite side (lower part of the figure) for varying beam width w.

2.2 Numerical time-domain model

For a time-domain analysis, the analytical model from section 2.1 is not adequate. It is thus extended to incorporate damping and inertia. Equation 7 gives the differential equation with the damping coefficient η and the moment of inertia I_z for the torsional actuator

$$I_z \cdot \ddot{\phi} + \eta \cdot \dot{\phi} + k_\phi \cdot \phi = M_U(U(t),\phi) \quad (7)$$

The damping coefficient can be analytically derived [7] as can be I_z using a model of a rigid plate rotating around its center. The inertia of the torsional beam can be disregarded, as it has a much lower mass and volume than the plate. Using this model, a Matlab/Simulink simulation has been implemented (Figure 3). Keeping the geometrical dimensions from Table 1, the step response of the system is recorded (Figure 6). For varying damping coefficients it can be seen that a voltage step can result in the excitation of an oscillation or a slow movement to the final equilibrium position with practically no overshoot. It has to be noted that if the overshoot is too big, it is possible that the second equilibrium position (cf. sect. 2.1) may be attained. The damping coefficient can be varied over a big range by having holes of different sizes and geometries in the plate. The theoretical value of the damping coefficient for the presented geometry is $5 \cdot 10^{-7}$ Nms [7]. For electrical voltage metrology purposes, this is fully adequate as an excitation of an oscillation is undesired and the second equilibrium position has to be avoided for valid results.

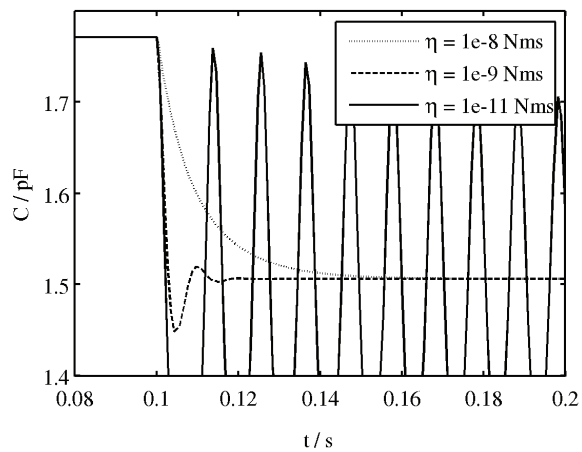

Figure 6: Time-domain step response of the system for different damping coefficients

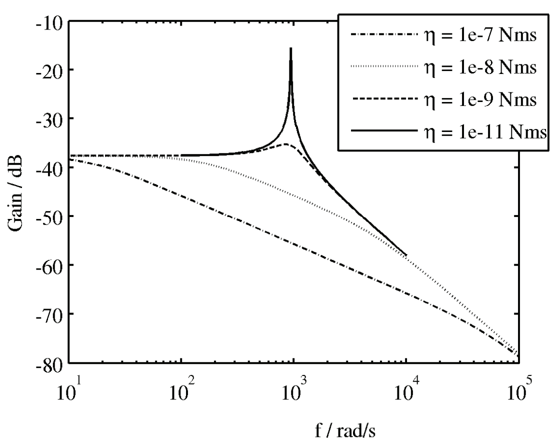

Figure 7: Bode plot for different damping coefficients η, showing a resonant behavior at f = 10^3 rad/s

The Matlab/Simulink model also allows an easy determination of the behavior of the model in the frequency domain (Figure 7). After linearizing the model at its operating point, the Bode plot shows that low damping coefficients lead to resonant behavior at the mechanical resonant frequency of the device. This can ultimatively lead to the destruction of the device. Thus, as mentioned before, a high damping coefficient is desirable for the sensor.

3 CONCLUSION

We presented a theoretical overview of electrostatic voltage measurement based on a torsional actuator. After an introduction to the basic setup of the measurement, an analytical steady-state model is presented. This model is afterwards extended for numerical time-domain simulations in Matlab/Simulink allowing the determination of resonant and transient behavior. The model will allow for designing specialized sensors for metrology applications for distinct voltage ranges and measurement applications. Future work will aim at the validation of the simulation results using real sensors. Furthermore, the electrical behavior of the device in the scope of high frequency voltages will have to be modeled as close as possible to investigate the effects with regard to metrology applications.

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