Modeling of Spatial Correlations in Process, Device, and Circuit Variations

Ning Lu
IBM Semiconductor Research and Development Center, Systems and Technology Group,
Essex Junction, VT 05452 USA   lun@us.ibm.com

ABSTRACT

We present an innovative method to model the spatial correlations in semiconductor process and device variations or in VLSI circuit variations. Without using the commonly adopted PCA technique, we give a very compact expression to represent a given spatial correlations among a set of similar statistical variables/instances located at different places on a chip/die. Our compact expression is easy for implementation in a SPICE model and is efficient in circuit simulations.

Keywords: Spice modeling, statistical modeling, spatial correlations, statistical static timing analysis

1 INTRODUCTION

In semiconductor processes and devices as well as in VLSI circuits, the degree of correlation between any two intra-die instances of a process or device parameter or of a circuit decreases with increasing separation between them. Examples include CMOS field effect transistor (FET) channel length, FET channel width, diode current, diffused or poly resistor's resistance value, MIMCAP's capacitance value, interconnect resistance or capacitance values, ring oscillator's period/speed, the speed of other logic circuits (e.g., NAND, NOR, etc.), and many other device parameters. Various measured hardware data has revealed such a gradual de-correlation of spatial correlation over distance (see Fig. 1; also [1–2]). In statisitcal static timing analysis for VLSI circuits, spatial correlation modelling has been studied [1, 3–5]. However, all of these papers use the principal component analysis (PCA) technique, resulting a large correlation matrix for a set of spatially correlated instances of a process/device/ciruit parameter located across a part of a chip or the whole chip. Subsequent matrix diagonalization yields many eigenvalues and eigenvectors.

In this paper, we present an innovative method to model the spatial correlations in semiconductor process and device variations or in VLSI circuit variations. Without using the commonly adopted PCA technique, we give a very compact expression to represent a given spatial correlation among a set of similar statistical variables/instances located at different places on a chip/die. Our compact expression is easy for implementation in a SPICE model and is efficient in circuit simulations. Describing intra-die variations using spatial (i.e., distance-dependent) correlations unifies various descriptions of intra-die variations, such as mismatch, across chip variations, random uncorrelated variations, and random correlated variations, etc.

2 MODELLING ONE-DIMENSIONAL SPATIAL CORRELATIONS

Consider spatial correlation within a strip of a chip region. All instances of a device parameter characteristics have the same mean value x_0 and the same standard deviation σ, but the correlation between any two instances vary (e.g., decrease) with the separation/distance between them. We devide the strip into I sub-regions (Fig. 2), and I can be a very large integer. All instances within the same sub-region are treated as perfectly correlated, but any two instances from different sub-regions are treated as either partially correlated or have no correlation. Let c_{ij} be the correlation coefficient between a device instance in sub-region i and another device instance in sub-region j.

I. We start from a special nearest-neighbor-only correlation problem,

$$c_{ii} = 1, i = 1, 2, \cdots, I; \quad c_{i,i+1} = c_{i+1,i} = \pm\frac{1}{2}, i = 1, 2, \cdots, I-1;$$
$$c_{ij} = 0, \quad i, j = 1, 2, \cdots, I, \quad |i - j| \geq 2. \qquad (1)$$

The size (I x I) of the correlation matrix can be very larg. A set of compact solutions is simply

$$x_i = x_0 + \sigma(g_i \pm g_{i+1})/\sqrt{2}, \quad i = 1, 2, \cdots, I, \qquad (2)$$

where (and throughout the paper) each of g_1, …, g_{I+1} is an indepenent stochastic/random variable of mean zero and standard deviation one. Solution (2) uses (I + 1) indepenent stochastic variables, but each x_i has only two stochastic (i.e., g_j) terms and thus is very compact and very suitable for fast SPICE simulations. If the PCA technique were used to find a solution of (1), then I indepenent stochastic variables would be used and each x_i would depend on many stochastic terms.

II. When the degree of correlation in (1) is more general,

$$c_{i,i+1} = c_{i+1,i} = r, \quad i = 1, 2, \cdots, I-1, \quad |r| \leq 1/2, \qquad (3)$$

then the solution (2) is replaced by

$$x_i = x_0 + \sigma[a_1 g_i + a_2(r) g_{i+1}], \quad i = 1, 2, \cdots, I, \qquad (4a)$$

$$a_2(r) = \mathrm{sgn}(r)\sqrt{(1+\sqrt{1-4r^2})/2}, \quad a_1 = \sqrt{1-a_2^2}. \qquad (4b)$$

III. We next consider a long-distance partial correlation in which the degree of correlation decreases linearly with the distance between sub-regions,

$$c_{i,i+m} = c_{i+m,i} = 1 - (m/M), \; i = 1, \cdots, I, \; j = 0, 1, \cdots, M,$$

$$c_{ij} = 0\,, \quad i, j = 1, 2, \cdots, I, \quad |i - j| \geq M\,. \tag{5}$$

A set of compact solutions is found to be

$$x_i = x_0 + \frac{\sigma}{\sqrt{M}} \sum_{k=1}^{M} g_{i+k-1}\,, \quad i = 1, 2, \cdots, I\,, \tag{6}$$

which uses ($I + M - 1$) indepenent stochastic variables, but each instance x_i uses only M stochastic terms.

IV. We now consider a general spatial correlation of correlation length ($M - 1$) units,

$c_{i,i\pm m} = f(M, m),\ m = 0, 1, \ldots, M,$

$$f(M, 0) = 1,\ f(M, M) = 0, \text{ and all other } c_{ij} = 0. \tag{7}$$

We use ($I + M - 1$) indepenent stochastic variables $g_1, \ldots, g_{I+M-1}$ to represent I correlated instances of a process, device, or circuit parameter characteristics,

$$x_i = x_0 + \sigma \sum_{k=1}^{M} a_k\, g_{i+k-1}\,, \quad i = 1, 2, \cdots, I\,. \tag{8}$$

Here M weighting coefficients a_k satisfy M equations,

$$f(M, m) = \sum_{k=1}^{M-m} a_k a_{k+m}\,, \quad m = 0, 1, \cdots, M - 1\,. \tag{9}$$

$m = 0$ case is the normalization condition, $\sum_{k=1}^{M} a_k^2 = 1$.

(i). Often, it is easier to first try an expression for a_k and then find corresponding spatial correlation. An example: Let a_k vary linearly with k,

$$a_k = k\sqrt{6/[M(M+1)(2M+1)]}\,, \quad k = 1, 2, \cdots, M\,. \tag{10}$$

Then, the corresponding spatial correlation is found analytically,

$$f(M, m) = (1 - m/M)[1 - m/(M+1)][1 + m/(2M+1)]. \tag{11}$$

Analytic solutions can also be obtained when a_k is proportional to k^n with n being a positive integer.

(ii) In practice, it is straightforward to first select a family of solution a_k curves and then calculate a family of corresponding spatial correlations numerically. Figure 4(a) shows a family of solution a_k curves in the power form,

$$a_k = \beta k^p\,, \quad k = 1, 2, \cdots, M\,, \tag{12}$$

and Fig. 4(b) plots a family of corresponding spatial correlations. In Eq. (12) and Eqs. (13)–(15) in the following, p is a real value, and β is a normalization constant. All spatial correlations in this family have a sharp peak at the center $m = 0$. Changing a_k in Eq. (12) to a symmetric form about the middle k value, one has

$$a_k = \beta(k_0 - |k - k_0|)^p\,, \quad k_0 = \tfrac{1}{2}(M+1)\,. \tag{13}$$

Figures 5(a) and 5(b) show solution a_k curves in Eq. (13) and corresponding spatial correlations for several power p values, respectively. Some of spatial correlations in this family have a smoother peak at the center $m = 0$. Various other forms of a_k curves are easily constructed. Here are two additional families of a_k curves. Figures 6(a) and 6(b) show the following family of asymmetric solution a_k curves,

$$a_k = \beta[3(k/M)^2 - 2(k/M)^3]^p\,, \quad k = 1, 2, \cdots, M\,, \tag{14}$$

and corresponding spatial correlations for several power p values, respectively. All spatial correlations in this family have a sharp peak at the center $m = 0$. Figures 7(a) and 7(b) plot the following family of symmetric solution a_k curves,

$$a_k = \beta \sin^{2p}[\pi k/(M+1)]\,, \quad k = 1, 2, \cdots, M\,, \tag{15}$$

and corresponding spatial correlations for several power p values, respectively. Most of spatial correlations in this family have a smoother peak at the center $m = 0$. Last, we select one of spatial correlations that is closest to a given spatial correlation, and use the corresponding set of a_k in a SPICE model or in a statistical static timing analysis.

3 MODELLING TWO-DIMENSIONAL SPATIAL CORRELATIONS

We now consider spatial correlation in a region of a chip or in the whole chip. A chip/die is divided into ($I \times J$) sub-regions. Let $C(i, j; k, l)$ denote the correlation coefficient between a device in sub-region (i, j) and another device in sub-region (k, l) (Fig. 8).

I. We first treat a nearest-neighbor-only correlation problem: Correlation degree is r, and its range is 1 unit in either x or y direction but not in diagonal directions: $C(i, j; i \pm 1, j) = C(i, j; i, j \pm 1) = r$, but $C(i, j; i \pm 1, j \pm 1) = 0$. We use ($IJ + I + J$) independent stochastic variables to model IJ correlated instances of a device parameter characteristics,

$$x_{i,j} = x_0 + \sigma\sqrt{1 - a_2^2}\, g_{i,j} + \sigma a_2(\sqrt{2}r)(g_{i+1,j} + g_{i,j+1})/\sqrt{2},$$

with $|r| \leq \sqrt{2}/4$, where a_2 function is given in Eq. (4b).

II. We next study a general two-dimensional correlation problem of correlation length ($M - 1$) ($N - 1$) units in the x (y) direction, $C(i, j; i \pm m, j \pm n) = F(M, m; N, n)$, $m = 0, 1, 2, \ldots, M$, $n = 0, 1, 2, \ldots, N$, $F(M, M; N, n) = 0$, $F(M, m; N, N) = 0$, and all other $C(i, j; k, l) = 0$. We use ($I + M - 1$)($J + N - 1$) independent stochastic variables to represent IJ correlated process/device/circuit instances,

$$x_{ij} = x_0 + \sigma \sum_{k=1}^{M} \sum_{l=1}^{N} A_{kl}\, g_{i+k-1,\, j+l-1}\,, \ i = 1, \ldots, I,\ j = 1, \ldots, J.$$

Here, MN weighting coefficients satisfy MN relations

$$F(M, m; N, n) = \sum_{k=1}^{M-m} \sum_{l=1}^{N-n} A_{k,l} A_{k+m,l+n} ,$$

$$m = 0, 1, \ldots, M-1, n = 0, 1, \ldots, N-1. \qquad (16)$$

The relation at $m = n = 0$ is the normalization condition, $\sum_{k=1}^{M} \sum_{l=1}^{N} A_{kl}^2 = 1$. For any one-dimensional spatial correlation $f(M, m)$ and corresponding solution a_k, we can generate corresponding two-dimensional spatial correlation and solution by a decomposition method in Eq. (16): $A_{kl} = a_k a_l$, $F(M, m; N, n) = f(M, m) f(N, n)$. For example, the one-dimensional linear-decay correlation relation in Eq. (5) (Fig. 3) becomes a bilinear decay spatial correlation in two dimensions (Fig. 9).

4 SUMMARY

We have presented a compact solution to the problem of modeling spatial correlations in semiconductor process, device, and circuit variations. By seeking solutions in a higher dimensional space, we have obtained a much compact solution than the commonly used PCA technique would give. We have also presented a method to obtain a closest solution for a desired spatial correlation. Our compact expression is easy for implementation in a SPICE model and is efficient in circuit simulations. Also, our compact solution is good for statistical static timing analysis, where the spatial correlation is an important consideration.

ACKNOWLEDGMENT

The author would like to thank IBM management for suppport.

REFERENCES

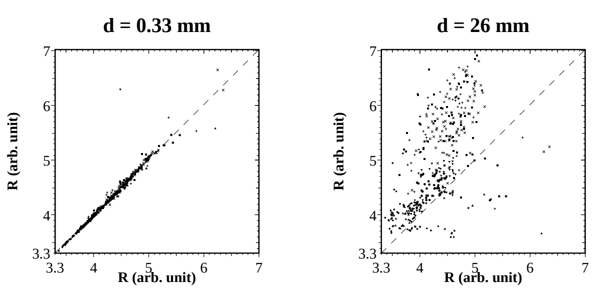
[1] H. J.-S. Doh, et al., *Proceedings of International Conference on Simulation of Semiconductor Processes and Devices*, 2005, pp. 131−134.

[2] A. Gattiker, M. Bhushan, and M. B. Ketchen, *Proc. IEEE International Test Conference,* 2006, pp. 1–10.

[3] H. Chang and S. S. Sapatnekar, *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, Vol. **24**, pp. 1467–1482 (2005).

[4] A. Agarwal, et al., *Proceedings of the Asia and South Pacific Design Automation Conference (ASP-DAC)* 2003, pp. 271–276.

[5] H. Mangassarian, M. Anis, *Proceedings of Design, Automation and Test in Europe*, 2005, pp. 132–137.



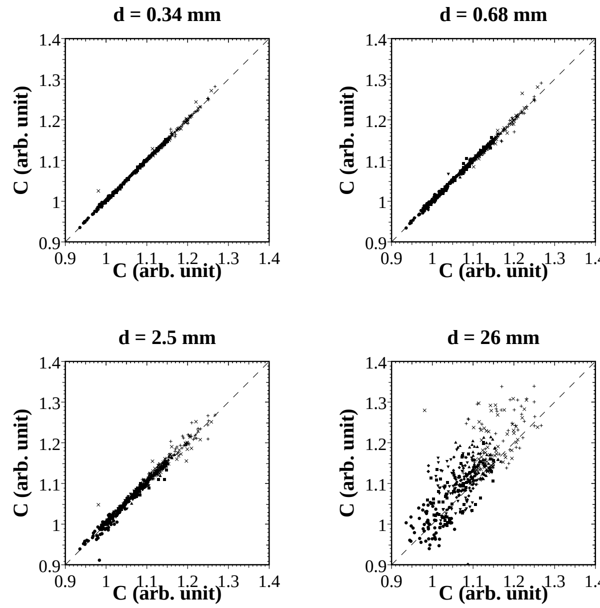


Fig. 1. Scatter plots show the correlation between measured interconnect resistance (R) and capacitance (C) of same wire width/space (but distance d apart) from an IBM 45 nm technology.

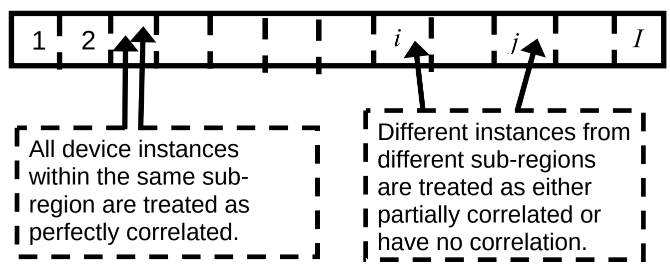


Fig. 2. One-dimensional case: A strip of a chip region is divided into I sub-regions. c_{ij} is the correlation coefficient between a device instance in sub-region i and another device instance in sub-region j.

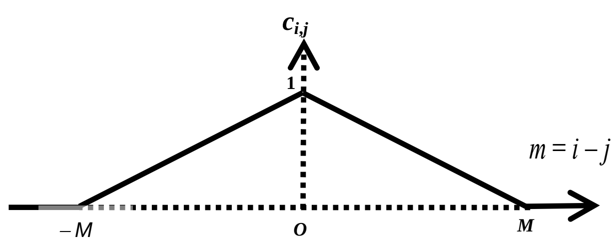


Fig. 3. One-dimensional linear-decay spatial correlation.

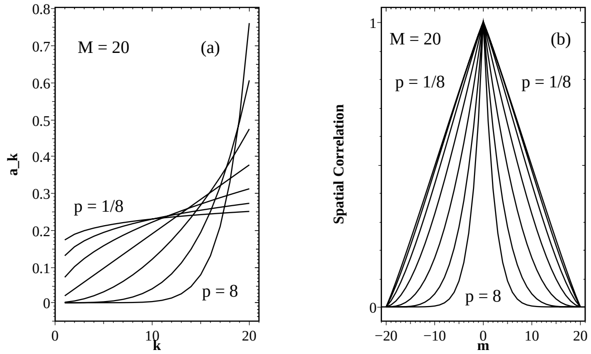


Fig. 4. (a) shows a family of asymmetric solution curves a_k, Eq. (12), and (b) shows a corresponding family of spatial correlation curves for several real power p values: p = 1/8, ¼, ½, 1, 2, 4, and 8.

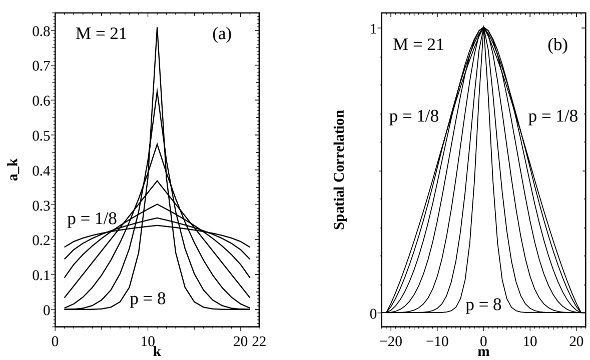


Fig. 5. (a) shows a family of symmetric solution curves a_k, Eq. (13), and (b) shows a corresponding family of spatial correlation curves for several real power p values: p = 1/8, ¼, ½, 1, 2, 4, and 8.

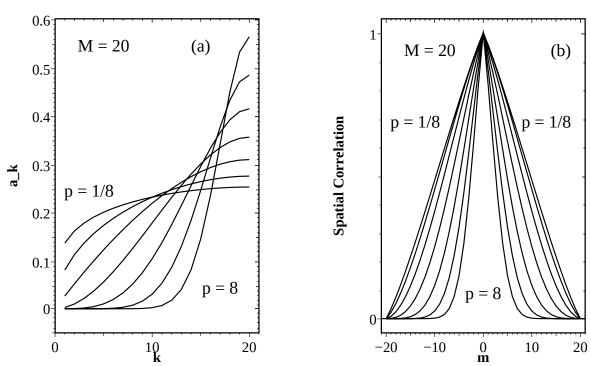


Fig. 6. (a) shows a family of asymmetric solution curves a_k, Eq. (14), and (b) shows a corresponding family of spatial correlation curves for several real power p values: p = 1/8, ¼, ½, 1, 2, 4, and 8.

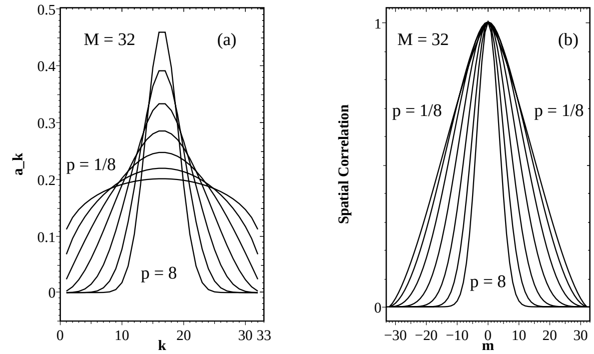


Fig. 7. (a) shows a family of symmetric solution curves a_k, Eq. (15), and (b) shows a corresponding family of spatial correlation curves for several real power p values: p = 1/8, ¼, ½, 1, 2, 4, and 8.

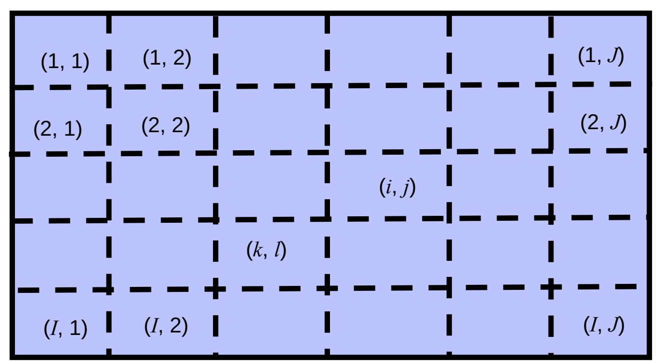


Fig. 8. Two-dimensional case: A chip/die is divided into (I x J) sub-regions. $C(i, j; k, l)$ denotes the correlation coefficient between a device in sub-region (i, j) and another device in sub-region (k, l).

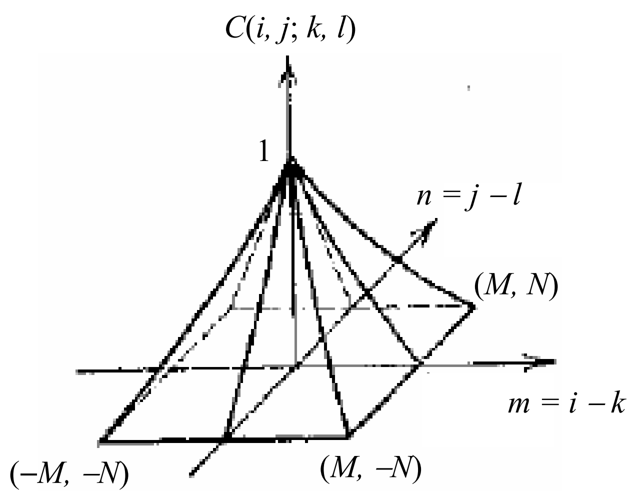


Fig. 9. Two-dimensional bilinear-decay spatial correlation.