ABSTRACT
We present an innovative method to model the spatial correlations in semiconductor process and device variations or in VLSI circuit variations. Without using the commonly adopted PCA technique, we give a very compact expression to represent a given spatial correlations among a set of similar statistical variables/instances located at different places on a chip/die. Our compact expression is easy for implementation in a SPICE model and is efficient in circuit simulations.

Keywords: Spice modeling, statistical modeling, spatial correlations, statistical static timing analysis

1 INTRODUCTION
In semiconductor processes and devices as well as in VLSI circuits, the degree of correlation between any two intra-die instances of a process or device parameter or of a circuit decreases with increasing separation between them. Examples include CMOS field effect transistor (FET) channel length, FET channel width, diode current, diffused or poly resistor's resistance value, MIMCAP's capacitance value, interconnect resistance or capacitance values, ring oscillator's period/speed, the speed of other logic circuits (e.g., NAND, NOR, etc.), and many other device parameters. Various measured hardware data has revealed such a gradual de-correlation of spatial correlation over distance (see Fig. 1; also [1–2]). In statistical static timing analysis for VLSI circuits, spatial correlation modelling has been studied [1, 3–5]. However, all of these papers use the principal component analysis (PCA) technique, resulting a large correlation matrix for a set of spatially correlated instances of a process/device/circuit parameter located across a part of a chip or the whole chip. Subsequent matrix diagonalization yields many eigenvalues and eigenvectors.

In this paper, we present an innovative method to model the spatial correlations in semiconductor process and device variations or in VLSI circuit variations. Without using the commonly adopted PCA technique, we give a very compact expression to represent a given spatial correlation among a set of similar statistical variables/instances located at different places on a chip/die. Our compact expression is easy for implementation in a SPICE model and is efficient in circuit simulations. Describing intra-die variations using easy for implementation in a SPICE model and is efficient in different places on a chip/die. Our compact expression is easy for implementation in a SPICE model and is efficient in circuit simulations.

MODELLING ONE-DIMENSIONAL SPATIAL CORRELATIONS
Consider spatial correlation within a strip of a chip region. All instances of a device parameter characteristics have the same mean value \( x_0 \) and the same standard deviation \( \sigma \), but the correlation between any two instances vary (e.g., decrease) with the separation/distance between them. We devide the strip into \( I \) sub-regions (Fig. 2), and \( I \) can be a very large integer. All instances within the same sub-region are treated as perfectly correlated, but any two instances from different sub-regions are treated as either partially correlated or have no correlation. Let \( c_{ij} \) be the correlation coefficient between a device instance in sub-region \( i \) and another device instance in sub-region \( j \).

I. We start from a special nearest-neighbor-only correlation problem,

\[
c_{ii} = 1, \ i = 1, 2, \ldots, I; \quad c_{i,i+1} = c_{i+1,i} = \pm \frac{1}{2}, \ i = 1, 2, \ldots, I - 1;
\]

\[
c_{ij} = 0, \quad i, j = 1, 2, \ldots, I, \quad |i - j| \geq 2.
\]  

The size \((I \times I)\) of the correlation matrix can be very large. A set of compact solutions is simply

\[
x_i = x_0 + \frac{\sigma (g_i + g_{i+1})}{\sqrt{2}}, \quad i = 1, 2, \ldots, I,
\]  

where (and throughout the paper) each of \( g_i, g_{i+1} \) is an independent stochastic/random variable of mean zero and standard deviation one. Solution (2) uses \((I + 1)\) independent stochastic variables, but each \( x_i \) has only two stochastic (i.e., \( g_i \)) terms and thus is very compact and very suitable for fast SPICE simulations. If the PCA technique were used to find a solution of (1), then \( I \) independent stochastic variables would be used and each \( x_i \) would depend on many stochastic terms.

II. When the degree of correlation in (1) is more general,

\[
c_{i,i+1} = c_{i+1,i} = r, \quad i = 1, 2, \ldots, I - 1, \quad |r| \leq 1/2,
\]  

then the solution (2) is replaced by

\[
x_i = x_0 + \sigma [a_1 g_i + a_2(r) g_{i+1}], \quad i = 1, 2, \ldots, I,
\]  

\[
a_2(r) = \text{sgn}(r) \sqrt{(1 + \sqrt{1 - 4r^2})/2}, \quad a_1 = \sqrt{1 - a_2^2}.
\]  

III. We next consider a long-distance partial correlation in which the degree of correlation decreases linearly with the distance between sub-regions,

\[
c_{i,i+m} = c_{i+m,i} = 1 - (m/M), \quad i = 1, \ldots, I, \quad j = 0, 1, \ldots, M,
\]
We use (or circuit parameter characteristics, find corresponding spatial correlation. An example: Let center spatial correlations in this family have a sharp peak at the value, one has

\[ \beta = \frac{\sin^2 p \left( \frac{1}{M+1} \right)}{p^2}, \quad k = 1, 2, \ldots, M, \quad (13) \]

and corresponding spatial correlations for several power \( p \) values, respectively. Most of spatial correlations in this family have a sharper peak at the center \( m = 0 \). Last, we select one of spatial correlations that is closest to a given spatial correlation, and use the corresponding set of \( a_k \) in a SPICE model or in a statistical static timing analysis.

3 MODELLING TWO-DIMENSIONAL SPATIAL CORRELATIONS

We now consider spatial correlation in a region of a chip or in the whole chip. A chip/die is divided into \((I \times J)\) sub-regions. Let \( C(i, j; k, l) \) denote the correlation coefficient between a device in sub-region \((i, j)\) and another device in sub-region \((k, l)\) (Fig. 8).

I. We first treat a nearest-neighbor-only correlation problem: Correlation degree is \( r \), and its range is 1 unit in either \( x \) or \( y \) direction but not in diagonal directions: \( C(i, j; k, l) = 0 \), but \( C(i, j; k, l) = 0 \). We use \((I + J)\) independent stochastic variables to model \( IJ \) correlated instances of a device parameter characteristics, with \( |r| \leq \frac{\sqrt{2}}{4} \), where \( a_n \) function is given in Eq. (4b).

II. We next study a general two-dimensional correlation problem of correlation length \((M - 1)\) \((N - 1)\) units in the \( x \) direction, \( C(i, j; i \pm m, j \pm n) = F(M, m; N, n), m = 0, 1, 2, \ldots, M, n = 0, 1, 2, \ldots, N, F(M, m; N, n) = 0, \) and all other \( C(i, j; k, l) = 0 \). We use \((I + M - 1)(J + N - 1)\) independent stochastic variables to represent \( IJ \) correlated process/device/circuit instances,

\[ x_{ij} = x_0 + \sigma \sum_{k=1}^{M} A_{k} \sum_{l=1}^{N} A_{l} g_{i+k-l, j+l-1}, \quad i = 1, \ldots, I, j = 1, \ldots, J. \]
Here, $MN$ weighting coefficients satisfy $MN$ relations

$$F(M, m; N, n) = \sum_{k=1}^{M} \sum_{l=1}^{N} A_{k,l} A_{k+m,l+n},$$

$$m = 0, 1, \ldots, M - 1, n = 0, 1, \ldots, N - 1.$$ (16)

The relation at $m = n = 0$ is the normalization condition, $\sum_{k=1}^{M} \sum_{l=1}^{N} A_{k,l}^2 = 1$. For any one-dimensional spatial correlation $f(M, m)$ and corresponding solution $a_k$, we can generate corresponding two-dimensional spatial correlation and solution by a decomposition method in Eq. (16): $A_{kl} = a_k a_l$, $F(M, m; N, n) = f(M, m)f(N, n)$. For example, the one-dimensional linear-decay correlation relation in Eq. (5) (Fig. 3) becomes a bilinear decay spatial correlation in two dimensions (Fig. 9).

4 SUMMARY

We have presented a compact solution to the problem of modeling spatial correlations in semiconductor process, device, and circuit variations. By seeking solutions in a higher dimensional space, we have obtained a much compact solution than the commonly used PCA technique would give. We have also presented a method to obtain a closest solution for a desired spatial correlation. Our compact expression is easy for implementation in a SPICE model and is efficient in circuit simulations. Also, our compact solution is good for statistical static timing analysis, where the spatial correlation is an important consideration.

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REFERENCES


Fig. 1. Scatter plots show the correlation between measured interconnect resistance (R) and capacitance (C) of same wire width/space (but distance $d$ apart) from an IBM 45 nm technology.

Fig. 2. One-dimensional case: A strip of a chip region is divided into $I$ sub-regions. $c_{ij}$ is the correlation coefficient between a device instance in sub-region $i$ and another device instance in sub-region $j$.

Fig. 3. One-dimensional linear-decay spatial correlation.
Fig. 4. (a) shows a family of asymmetric solution curves $a_k$, Eq. (12), and (b) shows a corresponding family of spatial correlation curves for several real power $p$ values: $p = 1/8$, $1/4$, $1/2$, 1, 2, 4, and 8.

Fig. 5. (a) shows a family of symmetric solution curves $a_k$, Eq. (13), and (b) shows a corresponding family of spatial correlation curves for several real power $p$ values: $p = 1/8$, $1/4$, $1/2$, 1, 2, 4, and 8.

Fig. 6. (a) shows a family of asymmetric solution curves $a_k$, Eq. (14), and (b) shows a corresponding family of spatial correlation curves for several real power $p$ values: $p = 1/8$, $1/4$, $1/2$, 1, 2, 4, and 8.

Fig. 7. (a) shows a family of symmetric solution curves $a_k$, Eq. (15), and (b) shows a corresponding family of spatial correlation curves for several real power $p$ values: $p = 1/8$, $1/4$, $1/2$, 1, 2, 4, and 8.

Fig. 8. Two-dimensional case: A chip/die is divided into ($I \times J$) sub-regions. $C(i, j; k, l)$ denotes the correlation coefficient between a device in sub-region $(i, j)$ and another device in sub-region $(k, l)$.

$$C(i, j; k, l)$$

Fig. 9. Two-dimensional bilinear-decay spatial correlation.