Quantum Gates Simulator Based on DSP TI6711

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ABSTRACT

Quantum theory has found a new field of application in the information and computation fields during recent years. We developed a Quantum Gate Simulator based on the Digital Signal Processor (DSP) DSP TI6711 using the Hamiltonian in the time- dependent Schrödinger equation. The Hamiltonian describes the Quantum System by manipulating a Quantum Bit (QuBit) using unitary matrices. Gates simulated are conditional NOT operation, Controlled-NOT Gate, Multi-bit Controlled-NOT Gate or Toffoli gate, Rotation Gate or Hadamard transform and *twiddle* gate, all useful in quantum computation due to their inherently reversible characteristic. With the simulation process, we have obtained approximately 95% fidelity action of the gate on an arbitrary two and three QuBit input state. We have determined an average error probability bounded above by 0.07 ± 0.01.

1 INTRODUCTION

The basic unit of storage in a quantum computer is the *qubit.* A qubit is like a classical bit in that it can be in two states, zero or one. The qubit differs from the classical bit in that, because of the properties of quantum mechanics, it can be in both these states simultaneously [1, 2].A convenient method for representing a qubit state is the ket notation defined by Dirac [3]. In this notation the ket $|0\rangle$ denotes the zero state and the ket $|1\rangle$ represents the one state. This notation is convenient because it labels the qubit state, and therefore only those states with non zero amplitude need to be explicitly written.

A quantum computer performs operations on *qubits*, whose value can be one or zero or any *superposition* of one and zero. A quantum computer performs transformations on these qubits to implement logic gates. These quantum logic gates create correlations between qubits, referred to as *entanglement*, which allows the representation of an exponential number of states using a polynomial number of qubits. Combinations of these logic gates define quantum circuits.

All operations in a quantum computation are achieved by means of transformations on the qubits contained in quantum registers. A transformation takes an input quantum state and produces a modified output quantum state.

Typically transformations are defined at the gate level, i.e. transformations which perform logic functions. Transformations that correspond to physical processes can also be defined. These lower level transformations are then composed so that they implement gate operations.

Because of the laws of quantum mechanics each transformation of the quantum state space, other than a measurement, must leave the quantum superposition of the state intact. More specifically each transformation must be unitary.

2 METHODOLOGY

For the development of the project, we use the DSP, like the TI TMS6711, with architectural optimizations to speed up processing. This DSP can be connected on classical personal computer for transfer the data between them, and the architectural features is the next:

Program flow:

- Floating-point unit integrated directly into the data-path.
- Pipelined architecture
- Highly parallel accumulator and multiplier
- Special looping hardware. Low-overhead or Zerooverhead looping capability

Memory architecture:

- DSPs often use special memory architectures that are able to fetch multiple data and/or instructions at the same time:
- Harvard architecture
- Use of direct memory access
- Memory-address calculation unit

A quantum logic gate is a transformation which performs a logic function on the input state and produces a new output state[6]. Circuits are constructed as sequences of these gates in the same manner as is used in conventional digital circuits. The gates that perform a conditional “not operation” are useful in quantum computation because they are inherently reversible. A single bit gate performs an unconditional not operation, and multibit-gates negate the resultant bit conditionally based on the input bits.

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2.1 Controlled NOT gate

A reversible version of a conventional exclusive or gate is constructed by retaining the value of one of the inputs. This gate, called the controlled-not gate [5], is defined by the truth table shown in Table 1.

Table 1: truth table for controlled not gate

Input Qubits		**Output Qubits**	
A	B	A'	B'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

The controlled-not gate leaves the qubit unchanged and flips the value of only if is set. The controlled-not gate can be reversed by performing another controlled-not gate. The logic symbol for this gate is shown in figure 1.

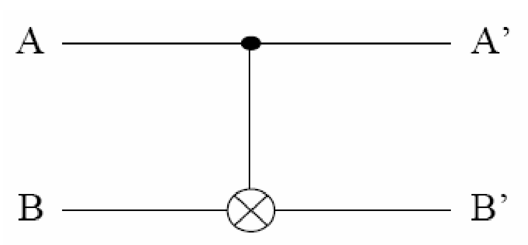

Figure 1: Logic symbol for the controlled not gate

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{X} \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix} = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{11} \\ a_{10} \end{bmatrix} \quad (1)$$

The matrix 1 show the transformation of the logic controlled not gate.

2.2 Multi-bit Controlled-not Gates

Multi-bit controlled-not gates are defined by adding additional controlled inputs. The resultant bit is only flipped if the logical AND of all the input qubits is one. These multi-bit gates are useful in the construction of logic circuits because of this AND property. Figure 2 shows the logic symbol for the three bit controlled-controlled not gate. This gate is also called the Toffoli gate after its designer [6]. It transforms the state $|1\rangle|1\rangle|0\rangle$ to $|1\rangle|1\rangle|1\rangle$ and the state $|1\rangle|1\rangle|1\rangle$ to $|1\rangle|1\rangle|0\rangle$ leaving all other states unchanged.

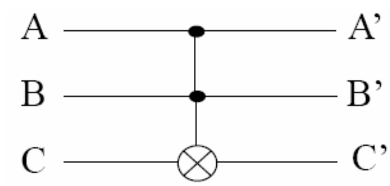

Figure 2: Logic symbol for the multi controlled not gate.

2.3 Rotation Gates

The *Hadamard* transform is a single bit rotation gate. The matrix 2 shows its definition. The logic symbol used in circuit diagrams is shown in Figure 3. The Hadamard transform applied to a qubit that is in the state $|0\rangle$ creates a state that is in the equal superposition of the $|0\rangle$ and $|1\rangle$ states. The Hadamard transform is also used in the encoding and error correction circuits.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (2)$$

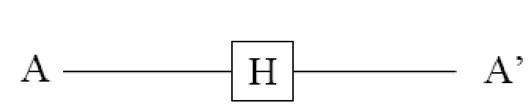

Figure 3. The Hadamard transformation

2.4 Twiddle Gates

The quantum FFT circuit requires an additional gate, the *twiddle* gate shown in the matrix 3. A twiddle is performed between two bits, denoted by the bit positions and as part of an FFT performed across L qubits.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w^m \end{bmatrix} \quad (3)$$

3 RESULTS AND DISCUSSION

In the figure 3, we show the simulation for the entropy on gate C-Not.

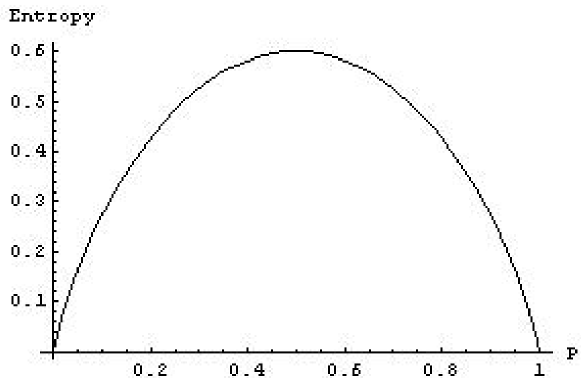

Figure 3: Entropy for C-Not gate

In the figure 4, we presents the fidelity for C-Not gate. In the matrix 4, we show the density matrix for C-Not gate, while in the matrix 5 we represent the matrix density for Hadamard gate.

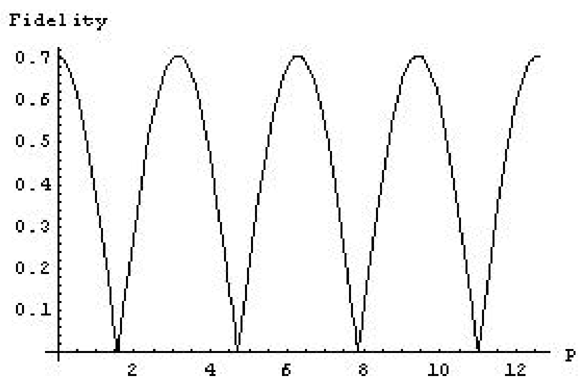

Figure 3: Fidelity for C-Not gate

Classical computation theory began for the most part when Church and Turing independently published their inquiries into the nature of computability in 1936 [7]. For our purposes, it will suffice to take as our model for classical discrete computation. So we are able to simulate the possible quantum processor using different gates and approach to the development of the Von Neuman Model[8].

The next step of this project is to integrate different gates and emulate as the classical components of the VLSI that integrated the classical semiconductor.

$$\begin{pmatrix}
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} \\
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} \\
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} \\
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \frac{1}{8}
\end{pmatrix} \quad (4)$$

The central problem that we will concern ourselves with repeatedly in these notes is the problem of universality. That is, given an arbitrarily large function f, is it possible to identify a universal set of simple functions that can be used repeatedly in sequence to simulate f on its inputs.

$$\begin{pmatrix}
\frac{1}{16}(1+(-1)^{2a})(1+(-1)^{b})^{2} & 0 & \frac{1}{16}(1+(-1)^{2a})(1+(-1)^{b})^{2} & 0 \\
0 & \frac{1}{16}(1+(-1)^{2a})(-1+(-1)^{b})^{2} & 0 & \frac{1}{16}(1+(-1)^{2a})(-1+(-1)^{b})^{2} \\
\frac{1}{16}(1+(-1)^{2a})(1+(-1)^{b})^{2} & 0 & \frac{1}{16}(1+(-1)^{2a})(1+(-1)^{b})^{2} & 0 \\
0 & \frac{1}{16}(1+(-1)^{2a})(-1+(-1)^{b})^{2} & 0 & \frac{1}{16}(1+(-1)^{2a})(-1+(-1)^{b})^{2}
\end{pmatrix} \quad (5)$$

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4 CONCLUSIONS

Hamiltonian (21) is sufficiently generic to represent most models for candidates of physical realizations of quantum computer hardware. The spin-spin term in Eq. (3) is sufficiently general to describe the most common types of interactions such as Ising, anisotropic Heisenberg, and dipolar coupling between the spins. Furthermore, if we also use spin-1/2 degrees of freedom to represent the environment then, on this level of description, the interaction between the quantum computer and its environment is included in model. In other words, the Hamiltonian (3) is sufficiently generic to cover most cases of current interest.

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