Application of Abductive Network and FEM to Predict the Stress-Strain Curve with strain hardening effect of Bulk Metals by Nanoindentation test


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ABSTRACT

The purpose of the present work was to investigate the nanoindentation process by FEM of bulk materials such as pure copper, titanium, iron and so on, considering strain hardening effect of material. In order to verify the FEM simulation results of the mechanical parameters such as Young’s modulus, yield stress and tangent modulus, the experimental data are compared with the results of the current simulation. The abductive network was then applied to synthesize the data sets obtained from the numerical simulation. The predicted results of the mechanical properties from the prediction model are consistent with the results obtained from experiment. After employing the predictive model can provide valuable references in prediction of the mechanical parameters after nanoindentation tests.

Keywords: finite element method, abductive network, nanoindentation, stress-strain curve

1 INTRODUCTION

Finite element method (FEM) has been widely used for numerical simulation of indentation tests on bulk material in order to analyze its deformation response and investigate the influence of indenter geometry, friction and material elastic and plastic properties. Pelletier et. al [1] have investigated the influence of material bilinear elastic-plastic behaviour model for numerical simulation of nanoindentation testing of various bulk metals. They employed an axisymmetric rigid cone and equal volume to the Berkovich pyramid indenter to simulate the test. The indenter and the specimen were treated as a revolution body in order to have three-dimensional situation. The numerical simulation results of loads verse displacement compared reasonable well to experimental results of nanoindentation tests of pure metals as Fe, Ni, Ti and Cu. In order to determine dimensionless functions correlating rheological factors with parameters extracted from loading and unloading curves, Pelletier [2] used a comprehensive parametric study of 48 cases was conducted; elastic modulus ranges from 70 to 430 GPa, yield stress from 100 to 600 MPa and the tangent modulus from 3 to 40 GPa. They defined two dimensionless equations which link the parameters extracted from the experimental load–displacement curve with material parameters, such as Young’s modulus, yield stress and tangent modulus. The equations have been tested with success first on load–displacement curves simulated with FEM and then on experimental curves obtained by nanoindentation testing using a Berkovich tip. Lin [3] predicts the rolling force and deformation in three-dimensional cold rolling by using the FEM and neural network. Lin and Lin [4] used the finite element method in conjunction with abductive network to investigate the geometrical shape of the deformation profile and its related fold defect during forging-extrusion process. The prediction models are established for radius ratio estimation of the barrels and fold occurrence situation judgment under various combinations of process variables.

In order to verify the FEM simulation results of the mechanical parameters such as Young’s modulus, yield stress and tangent modulus, the experimental data are compared with the results of the current simulation. The abductive network was then applied to synthesize the data sets obtained from the numerical simulation. After employing the predictive model can provide valuable references in prediction of the mechanical parameters after nanoindentation tests.

2 BASIC THEORY

2.1 Finite Element Modeling

A commercial FE code DEFORM-2D [5] is adopted to analyze the plastic deformation of the nanoindentation process. For the nanoindentation process of a elastic-plastic deformation problem, the governing equations for the solution of the mechanics in plastic deformation for materials involve equilibrium equations, yield criterion, constitutive equations and compatibility conditions. The duality of the boundary value problem and the variation problem can be seen clearly by considering the construction of the functional [6]:

\[ \pi = \int \sigma : \varepsilon \, dv - \int F_i \cdot u_i \, ds \]  

(1)

where \( \sigma \) is the effective stress, \( \varepsilon \) is the effective strain-rate, \( F_i \) represents the surface tractions and, \( u_i \) is the
velocity components. The variational form for finite-element discretization is given by:

$$\delta \pi = \left[ \int \sigma \delta \varepsilon \, dv + k \int \delta \varepsilon \dot{\varepsilon} \, dv - \int F \delta u \, ds \right] = 0 \quad (2)$$

where $\varepsilon_\pi$ is the volumetric strain rate, $\pi$ is functional of the total energy and work, and $k$, a penalty constant, is a very large positive constant. $\delta \varepsilon$ and $\delta \varepsilon_\pi$ are the variations in effective strain rate and volumetric strain rate. Eq. (1) and Eq. (2) are the basic equation for the finite element formulation.

2.2 Abductive Network Synthesis

In the abductive network, a complex system can be decomposed into smaller, simpler subsystems grouped into several layers using polynomial functional nodes. The polynomial network proposed by Ivakhnenko [7] is a group method of data handling (GMDH) techniques. Theses nodes evaluate the limited number of inputs by a polynomial function and generate an output to serve as an input to subsequent nodes of the next layer. It consists of sigma (summation) units in the hidden layer and pi (product) units in the output layer. Output of a sigma unit is a weighted sum of its inputs, and output of a pi unit is a product of its input. Let the $k$th input pattern to the network be specified by $X_k = [x_{k1}, x_{k2}, x_{k3}, \ldots, x_{kn}]$, and let the weight associated with connection from input unit $i$ to hidden unit $j$ be $w_{ij}$. Then, the output $z_{jk}$ of the $j$th sigma unit is given by

$$z_{jk} = \sum_{i=0}^{n} w_{ij} x_{ik} \quad (3)$$

and output $y_{jk}$ of the network is given by

$$y_{jk} = \prod_{j=0}^{h} z_{jk} \quad (4)$$

where $h$ is the number of hidden units in the network. Combining Eqs. (3) and (4), the general polynomial function in a polynomial functional node can be expressed as:

$$y_k = c_0 + \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{i} c_{ij} x_i x_j + \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} c_{ijk} x_i x_j x_k + \cdots$$

where $x_i, x_j, x_k$ are the inputs, $y_k$ is the output and $c_i, c_{ij}, c_{ijk}$ are the coefficients of the polynomial functional nodes. In the present study, several types of polynomial nodes are used in polynomial network for predicting the limiting drawing ratio under a suitable range of process parameters. For more detailed explanation of these polynomial functional nodes, please refer to the paper of Ivakhnenko [7].

3 RESULTS AND DISCUSSION

FEM has been widely used to simulate the nanoindentation process. The schematic diagram of the nanoindentation process and the stress-strain relationship of the elastic-linear work-hardening materials used in DEFORM-2D software are shown in Fig. 1. An axisymmetric cone with half-included angle of 70.3° in which the conical indenter has the same area function as a Berkovich tip was used in this study. The contact region of both tip and bulk material were finely meshed for good simulation accuracy. In order to confirm the feasibility of the finite element method, the results published by Pelletier et. al [1] is used to compare with the present numerical results by DEFORM-2D software. The experimental data of Pelletier et. al [1] is compared with the results of the current simulation. Figures 2, 3 and 4 show the comparison between the current simulation and the results of Pelletier et. al [1]. The loading and unloading curve predicted by current simulation show good agreement with the experimental results of Pelletier et. al [1]. Therefore, the simulation by DEFORM-2D for nanoindentation process is reasonable accurate.

![Fig. 1(a) The finite element mesh of the indentation process by current simulation (DEFORM-2D)](image)

![Fig. 1(b) a schematic representation of the bilinear constitutive law used in current simulation](image)

![Fig. 2 Comparison between the current simulation and the results of Pelletier et al [1] for loading and unloading curve (bulk material Ti and indentation depth = 100 nm)](image)

![Fig. 3 Comparison between the current simulation and the results of Pelletier et al [1] for loading and unloading curve (bulk material Fe and indentation depth = 200 nm)](image)
3.1 Database Training

The yielding stress is varied between 100-600 MPa, whereas the other material’s properties were selected by varying the Young’s modulus, the tangent modulus and the indentation depth in the ranges of 72-430 GPa, 3.6-40 GPa and 100-400 nm, respectively. There are four variables of material properties, each of which was set at three levels. Therefore, 81 (3*3*3*3) combinations of parameters of material’s property are constituted totally, and these are shown in Table 1. The load–displacement curves obtained with FEM have been analyzed with the same method as the experimental load–indentation depth curves, as shown in Fig. 5. The curves corresponding to the loading part have been fitted using a polynomial law, $P = Ah^2 + Bh$; and the unloading part can be mathematically described with a polynomial law, $P = Ch^2 + Dh + F$. The loading and unloading curve can be obtained by the FEM simulation of indentation process for the different parameters such as $E$, $Y_0$, $E_t$ and $h$. The coefficient of polynomial law such as $A$, $B$, $C$, $D$, $F$ can be obtained from the curve fitting skill. Base on the training database regarding the parameters of material’s property and coefficient of polynomial law such as $Y_0$, $E_t$ $A$, $B$, $C$, $D$, $F$, $P$ and $h$, as shown in Table 1, the abductive networks with a criterion of minimum square error can be developed for predicting the $E$, $Y_0$ and $E_t$ under a suitable range of parameters such as $A$, $B$, $C$, $D$, $F$ and $h$. Five-layer networks shown in Fig. 6 is built for prediction of the tangent modulus. The prediction of networks for the Young’s modulus and yielding stress are similar to Fig. 6.

The curves corresponding to the loading and unloading curves are fitted using a polynomial law $P = Ah^2 + Bh$ and $P = Ch^2 + Dh + F$, respectively.
3.2 Validating the Accuracy of the Prediction Model

In order to validate the accuracy of the prediction model, the experimental results of Figs. 2 and 3 are tested for the predictions of the Young’s modulus, the yielding stress and the tangent modulus. Tables 2 and 3 show a comparison of the values of E, Y₀ and E₀ between the abductive network prediction, the FEM simulation and the experimental data under various combinations of the values of A, B, C, D, F, P and h, which are around the threshold of the suitable range. The predicted results of the E, Y₀ and E₀ are in good agreement with those obtained from the FEM simulations. Therefore, the developed networks have a reasonable accuracy for predicting the value of E, Y₀ and E₀. Table 2 Prediction of E, Y₀ and E₀ between FEM [1] and abductive network for Fig. 2 (bulk material Ti and indentation depth = 100 nm) (Experiment: A=80.4853, B=3.8613, C=0.001469, D=0.03750, F=0.000389)

<table>
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<tr>
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<th>E(GPa)</th>
<th>Y₀(MPa)</th>
<th>E₀(GPa)</th>
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<tr>
<td>FEM [1]</td>
<td>130</td>
<td>600</td>
<td>36</td>
</tr>
<tr>
<td>abductive network</td>
<td>139</td>
<td>569</td>
<td>38</td>
</tr>
</tbody>
</table>

Table 3 Prediction of E, Y₀ and E₀ between FEM [1] and abductive network for Fig. 3 (bulk material Fe and indentation depth = 200 nm) (Experiment: A=49.91503, B=7.7529, C=0.004083, D=0.12378, F=0.005434)

<table>
<thead>
<tr>
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<th>E(GPa)</th>
<th>Y₀(MPa)</th>
<th>E₀(GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM [1]</td>
<td>215</td>
<td>200</td>
<td>26</td>
</tr>
<tr>
<td>abductive network</td>
<td>201</td>
<td>178</td>
<td>25</td>
</tr>
</tbody>
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Fig. 7 Comparison between the Pelletier’s experimental results [1] and DEFORM-2D in conjunction with an abductive network. (E=139GPa, Y₀ =569 MPa, E₀ = 38GPa)

Table 4 Comparison between the Pelletier’s experimental results [1] and DEFORM-2D in conjunction with an abductive network. (E=201GPa, Y₀ =178 MPa, E₀ = 25GPa)

4 CONCLUSION

In this paper, a prediction model has been established for the E, Y₀ and E₀ by using the finite element method in conjunction with an abductive network. The predicted results are consistent with the results obtained from experiment. After employing the predictive model can provide valuable references in prediction of the mechanical parameters such as E, Y₀ and E₀ after nanoindentation tests.

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