The Casimir Force and quantum interaction between conducting macro-bodies at nanoscale distances

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ABSTRACT

We report on the presision measurements of the periodical mechanical displacements of the thin aluminium pellicle produced by the Casimir Force. We report both, theoretical and experimental results of the influence of the material parameters of the pellicle on the Casimir Force. The measurements were performed with a super-sensitive adaptive holographic interferometer.

Keywords: nanomechanics, Casimir Force, adaptive holography

The growing attention to the problem of the Casimir force is caused by fundamental and practical interests because of the progress in quantum electrodynamics and fast development of micro and nano-nechanical systems. This force can impose limitations on miniaturization of mechanical systems. However, on the other hand, it can be the basis for creation of new systems, such as sensors, memories, etc. The origin of the Casimir force is associated with the existence of the so-called zero-point fluctuations of electromagnetic field in vacuum. The fact is that the energy density of these fluctuations becomes spatially inhomogeneous if metallic bodies are placed in the space. For instance, if two ideal conducting plates are located at a distance Z from each other, they form a waveguide for electromagnetic waves, and the energy density of the electromagnetic field between the plates turns out to be lower than the energy density outside the plates. As a result, the force attracting these two plates to each other, i.e., the so-called the Casimir force, appears [1,2]. The goal of the work presented here was to elaborate the experimental method for investigation of the Casimir force using the principles of dynamic holography and establish the dependence of the Casimir force on the thickness of the interacting bodies. We studied the Casimir force between two conducting layers deposited on bulk dielectric substrates placed in a vacuum chamber. One of the substrates was a thin lavsan pellicle which was covered with a 150-nm thick aluminium film and the other one was a glass lens, also covered with a thin Al film. It has been found that the Casimir force depends on the thickness and dielectric permittivity of the conducting films.

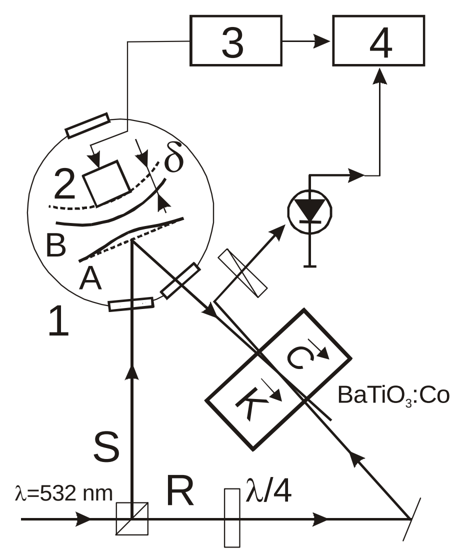

Fig.1. Experimental setup for the Casimir Force measurements. 1, vacuum chamber; 2, piezodriver; 3, sync generator; 4, lock-in amplifier; A, pellicle (“plane”); B, lens (“sphere”).

Fig.1 shows the experimental setup. The lens position was controlled by a piezoelectric driver, so the lens could move and oscillate along the axis of the system. If the distance between the lens and pellicle was set in the interval 300-600 nm, oscillations of the lens position caused the pellicle periodical displacements. The periodical pellicle displacements were detected by an adaptive holographic interferometer in which a single crystal of $BaTiO_3$:Co was used. The basic idea of this kind of interferometers is that one of the laser beams experiences phase modulation when it reflects from the pellicle, and then this phase modulation transforms into the intensity modulation of the output signal through the two-wave mixing effect provided by the photorefractive holographic material ($BaTiO_3$:Co). A detailed description of the experimental technique will be given elsewhere [3]. The experiment showed that in spite of the fact that the lens oscillations have a sinusoidal character, the periodical pellicle displacements had overtones, which means that the sinusoidal modulation of the distance between the lens and pellicle resulted in the nonlinear oscillations of the Casimir force. It is known [2] that the Casimir force between two bodies, one in the form of a sphere and the other in the form of a plate, at zero temperature is given by

$$F_c = -\frac{\pi^3 \hbar c R}{360 Z_0^3} \qquad (1)$$

where c is the speed of light, ħ is the Planck constant, Z_0 is the initial distance between the lens and pellicle, and R is the radius of the lens surface curvature. In the case the distance Z periodically oscillates as

$$Z = Z_0 + \delta \cos \Omega t \qquad (2)$$

where δ is the amplitude of distance variations, the Casimir force contains a set of overtones. We consider only the first and second harmonics of oscillations. Their amplitudes are

$$F_\Omega = -\frac{3F_c b}{\left(1-b^2\right)^{5/2}}, \qquad F_{2\Omega} = \frac{3F_c b^2}{\left(1-b^2\right)^{5/2}} \qquad (3)$$

where $b = \delta/Z_0$, and it is assumed that b<1. Fig.2 shows the experimental and theoretical data for Z_0 = 300, 450, and 600 nm and R = 1 m and 0.3 m. One can see that the theory is in a reasonable agreement with experiment. The fitting parameter here is Z_0.
The data on the amplitude of the Casimir force oscillations given above are in relative units. The absolute calibration of the experiment is rather a difficult task. To measure absolute values of the Casimir force, we used the technique for light pressure detection described in [4]. To make a calibration, the lens was removed from the chamber and the pellicle was illuminated with a relatively strong laser beam ("striking" beam), the intensity of

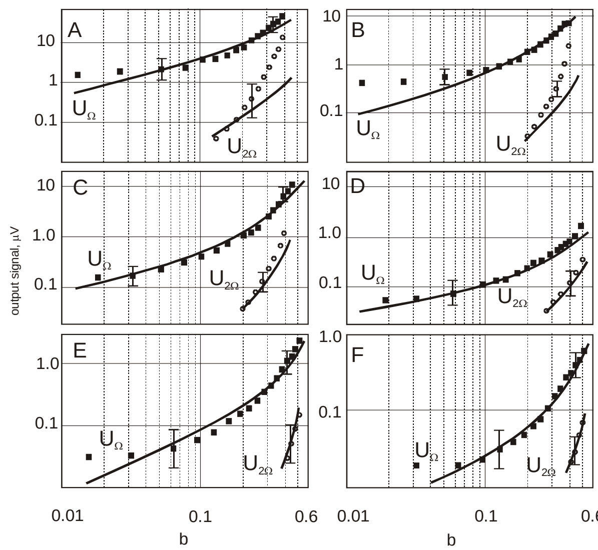

Fig.2. Experimental and theoretical results of the Casimir force measurements. The dependence of the output signals U_Ω and $U_{2\Omega}$ versus parameter b.
A: Z_0 = 300±30 nm, R = 1 m; B: Z_0 = 300±30 nm, R = 0.3 m; C: Z_0 = 400±40 nm, R = 1 m;
D: Z_0 = 400±40 nm, R = 0.3 m; E: Z_0 = 600±60 nm, R = 1 m; F: Z_0 = 600±60 nm, R = 0.3 m.
Squares and circles are experimental points; solid lines are theoretical fit by formula 3.

which was sinusoidally modulated, i.e., the intensity of "striking" light was $I_p(t) = I_p(1+\cos \Omega t)$, where I_p is the average intensity. In this case the light pressure amplitude is

$$P_{light} = \frac{I_p(1+r)}{c} \qquad (4)$$

and the amplitude of the alternating part of the light pressure force is

$$F_{light} = \frac{I_p S_{light}(1+r)}{c} \approx \frac{W(1+r)}{c} \qquad (5)$$

Here W is the average power and S_{light} is the effective cross-section of the striking beam, and r is the reflection of the striking light. The light pressure caused by this beam imitated the Casimir force pressure. Since the light pressure can be easily calculated from the measured value of W, we could find the magnitude of the Casimir force if the output signal is the same for these two experiments (oscillations of the lens position and illumination with a striking beam). Such a calibration of the absolute value of the Casimir force showed that the detected Casimir force is two orders of magnitude lower than that predicted by Eq.(3). Several factors can be offered for explanation of this discrepancy between theory and experiment. We analyzed the source of experimental errors and found that a limited accuracy of experimental measurements could not explain such a large difference between theory and experiment. So we analyzed the more fundamental physical reasons. It is known that there are several factors reducing the Casimir force. One of them is the temperature corrections, which has to be included in Eq.(1) [2]. These corrections provide a 15-20-% reduction in the Casimir force. Another important factor is plasma oscillations in conducting materials. The fact is that zero-point electromagnetic field fluctuations with the frequencies higher than the frequency of plasma oscillations of a metallic plate do not contribute to the Casimir force. The plasma frequency ω_p is one of important characteristics of metallic materials, and it shows that the dielectric permittivity is zero when the electromagnetic field frequency is equal to the plasma frequency. In most metals, this frequency is higher than the visible light frequency. It has been shown [2] that because of plasma oscillations the Casimir pressure for two parallel plates (P_c) transforms into

$$\widetilde{P}_c \cong P_c\left[1-\frac{16\lambda_p}{3Z}+120\left(\frac{\lambda_p}{Z}\right)^2+.....\right] \qquad (6)$$

where $\lambda_p=c/\omega_p$.

The next factor reducing the Casimir force is the finite thickness *(l)* of the conducting plates. We performed calculations for the parallel metallic plates using the dielectric permittivity in the form

$$\varepsilon(\omega)= 1-(\omega_p/\omega)^2 \qquad (7)$$

and found the Casimir force as a function of dimensionless parameter $\sqrt{Zl}/\lambda_p$. If this parameter is low (less than unity), the contribution of the TE electromagnetic mode into the Casimir pressure is given by

$$\widetilde{P}_{TE} \cong -\frac{\hbar c}{512\pi^2 Z^2\lambda_p^2}\left(\frac{l}{\lambda_p}\right)^2 = P_c\frac{15}{32\pi^2}\left(\frac{\sqrt{Zl}}{\lambda_P}\right)^4 \qquad (8)$$

It is assumed also that $l \ll Z, \lambda_p$. Under the same conditions, the TM mode contribution is

$$\widetilde{P}_{TM} \cong -\frac{\hbar c 15}{1024\sqrt{\pi Z}\,d^4}\left(\frac{Zl}{\lambda_p^2}\right)^{1/2} = P_c\frac{5\pi^{3/2}}{128}\frac{\sqrt{Zl}}{\lambda_p} \qquad (9)$$

As comparison of Eqs.(8) and (9) shows, in the geometry of thin plates the major contribution into the Casimir force comes from the TM mode. A decreasing Casimir force with decreasing plate thickness as compared with Eq.(6) is due to the fact that in Eq.(9) an important contribution comes from the frequencies of zero-point fluctuations lower than $\omega_p\sqrt{l/Z}$, while for thick plates the region of essential frequencies increases up to the limit equal to ω_p.

These results suggest that a significant reduction of the Casimir force can be achieved for thin films if $\sqrt{Zl}/\lambda_p \ll 1$. However, we have no reliable data on λ_p for our aluminium films, so no precise comparison of theory and experiment could be made. Nevertheless, qualitatively the obtained formulas can explain our experimental data.

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