A semi-analytical model for calculating touch-point pressure and pull-in voltage for clamped diaphragms with residual stress

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ABSTRACT

A closed form model for evaluating touch point pressure and pull-in voltage of clamped square diaphragm with residual stress is proposed. Square diaphragms are used in numerous applications. The design parameters for all these structures are pull-in voltage and/or touchpoint pressure. The materials employed for fabricating diaphragms for these structures are p+ doped silicon, polysilicon, silicon nitride, polyimide etc. All these materials have residual stress, which influences the behavior of the transducer. In addition to this, a capacitive transducer may or may not employ an intervening layer of dielectric on the fixed electrode. Closed form expressions for evaluating touch-point pressure and pull-in voltage have been derived for such a structure by means of semi-analytical model. The method proposed is less complex and less time consuming in comparison with FEM tools.

Keywords :MEMS, residual stress, diaphragm ,pull-in

1. INTRODUCTION

A MEMS capacitive sensor is basically an electrostatic transducer employing a parallel plate-structure that depends on electrical energy in terms of constant voltage (voltage drive) or constant charge storage (current drive) to facilitate monitoring of capacitance change due to an external mechanical excitation, such as force, acoustical pressure or acceleration [1]. The parallel plates comprises of one fixed electrode and the other deformable as shown in Fig 1. An intervening layer of dielectric is used over the fixed electrode in transducers, which have touchmode operation, or to avoid electric short in electrostatic actuators at pull-in. The deformable electrode is usually a clamped diaphragm and can be fabricated using different materials and different geometries, such as, circular, square and rectangular. Square diaphragms are used in numerous MEMS structures because of better area efficiency and process capability using IC lithography [2]. Besides touch-mode capacitive pressure sensors [2], square diaphragms find use in numerous applications like electrostatic valve actuator for high-pressure applications [3], polysilicon micromirrors [4], silicon capacitive microphone [5], micropumps [6] and bio-medical applications [7]. Different materials used are boron doped silicon[8], polysilicon, Si_3N_4[9] and polyimide[10]. All these materials are known to have residual stresses. The residual stress affects the device behavior by influencing its touch-point pressure and pull-in voltage. The pull-in voltages of micro test structures can be used to extract the material parameters of thin films, such as Young's moduli and residual stresses [11, 12]. Determination of the pull-in voltage is critical in the design to determine the sensitivity, instability in the operational range and the dynamics of devices. Accurate determination of the pull-in voltage is very challenging by virtue of the mechanical–electrical coupling effect and the nonlinearity of electrostatic force. Several methods like FEM(Finite Element Method), lumped model approach and solving coupled PDE's using numerical techniques are available to find the pull-in voltage[13]. Simple fast solutions are available for determination of pull-in voltage of cantilever beams, fixed-fixed beams and circular diaphragms with excellent accuracies and can determine the pull-in voltage for the mentioned structures within 1% agreement with FEM results under certain limitations [11]. However, published analytical or empirical solutions to determine the pull-in voltage for square diaphragm predict pull-in voltage that show significant error when compared with the finite element analysis results or experimentally measured values[14]. Analytical model[14] based on a linearized uniform approximation model of the electrostatic pressure and a 2-D load deflection model under uniform pressure gives the expression of pull-in voltage by assuming that the pull-in occurs at a critical displacement equal to one-third of the gap between the electrode. A method is proposed in this paper to solve the fourth order partial differential equation by using a trial solution. The closed form expression of pull-in voltage and critical distance are the outcome of the solution. Another distinct feature of the method are that the deflection versus pressure graph depicts a realistic situation as no further deflection takes place after touchpoint pressure is reached.

2. THEORY

For the plates with residual stress the governing equation is[18]

$$D\Delta\Delta w(x,y) - \sigma h \Delta w(x,y) = P \tag{1}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

w(x, y) is the deflection at any point (x, y) of the

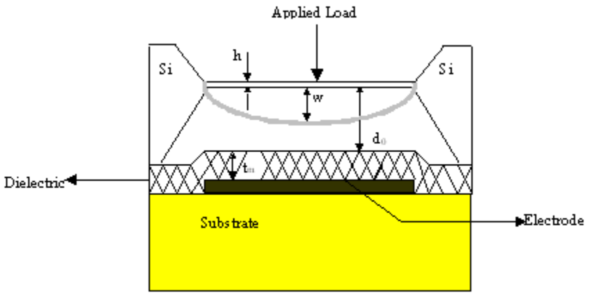

Fig. 1 Structure of a capacitive pressure sensor

diaphragm, σ is the residual stress, h is the thickness of the diaphragm and P is the distributed pressure load. D flexural rigidity is given by

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \quad \text{where,}$$

E is the Young's modulus of the diaphragm material, h is the thickness of the diaphragm and υ is the Poisson's ratio. In the presence of applied pressure and applied voltage eq. (1) is modified as

$$D\Delta\Delta w(x,y) - \sigma h \Delta w(x,y) = P + P_{el} \tag{2}$$

where, P_{el} is the electrostatic pressure and P is the mechanical pressure. The electrostatic pressure P_{el} is given as:

$$P_{el} = \frac{\varepsilon_0 \varepsilon_r}{2d_0} V^2 \tag{3}$$

where, d_0 is distance between the plates, ε_r is relative permittivity of the medium or the dielectric constant of the medium and ε_0 is permittivity of free space. The diaphragm deflection *w(x, y)* with air as dielectric is given by considering that the distance d_0 between the plates changes to $(d_0 – w)$ due to the displacement w of the diaphragm in the presence applied pressure and voltage as shown in fig. 1.Substituting eq. (3) in eq. (2) for diaphragm without the intervening layer of dielectric, the equation becomes

$$D\Delta\Delta w(x,y) - \sigma h \Delta w(x,y) = P + P_{el} \tag{4}$$

For diaphragm with intervening layer of dielectric

$$D\Delta\Delta w(x,y) - \sigma h \Delta w(x,y) = P + \frac{\varepsilon_0 \varepsilon_a \varepsilon_i^2}{2(\varepsilon_a t_m + \varepsilon_i (d_0 - w))^2} V^2 \tag{5}$$

.

where d_0 is zero pressure gap, ε_a is the dielectric constant of the air, ε_i is the dielectric constant of the insulator and t_m is the thickness of the insulator. The equations (4) and (5) can be written in a generalized form as

$$D\Delta\Delta w(x,y) - \sigma h \Delta w(x,y) = P + \frac{\varepsilon_0}{2(d_{eff} + d_0 - w)^2} V^2 \tag{6}$$

where $d_{eff} = t_m/\varepsilon_i$ (for a single layer of dielectric) and $d_{eff}=0$ for air

3. SEMI-ANALYTICAL MODEL

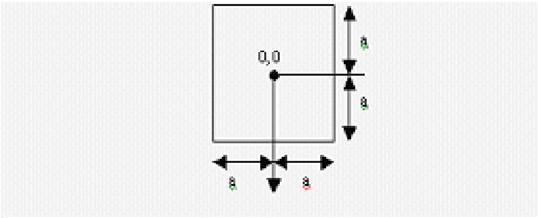

Fig. 2 Co-ordinate system and dimensions for a square diaphragm

The boundary conditions for the square diaphragm with clamped edges are as follows

$$w(x,y) = 0 \text{ at } x = \pm a, y = \pm a$$
$$\frac{\partial w}{\partial x} = 0 \text{ and } \frac{\partial w}{\partial y} = 0 \text{ at } x = 0, y = 0, x = \pm a \text{ and } y = \pm a$$

where 2a is the side length of the diaphragm . The trial solution that satisfies the above given boundary conditions is

$$w(x,y) = \lambda (x^2 - a^2)^2 (y^2 - a^2)^2 \tag{7}$$

The value of λ can be found out by the solving the following integral and equating the result to zero [15]

$$\int_{-a}^{+a}\int_{-a}^{+a} w(x,y)(D\Delta\Delta w(x,y) - \sigma h \Delta w(x,y))dxdy - \int_{-a}^{+a}\int_{-a}^{+a} w(x,y)\left(P + \frac{\varepsilon_0}{2(d_{eff} + d_0 - w)^2} V^2\right)dxdy \tag{8}$$

The above equation has no closed form analytical solution, and can only be solved numerically. Following methodology is used to find the closed form solutions. Initially, there is no deflection, therefore w=0, hence the value of λ from eq.(8) is found as

$$\lambda = \frac{147}{512(27a^4 D + 2a^6 \sigma h)} \left(2P + \frac{\varepsilon_0}{(d_{eff} + d_0)^2} V^2\right)$$

Evaluating $D\Delta\Delta w(x,y)-\sigma h\Delta w(x,y)$ at x=0, y=0, i.e., at the center of the diaphragm with this value of λ, we have

$$D\Delta\Delta w(x,y)-\sigma h\Delta w(x,y)=\frac{245}{144}((P+\frac{\varepsilon_0}{2(d_{eff}+d_0)^2}V^2) \quad (9)$$

The eq. (9) has to be modified to take into account the new electrostatic pressure every time a deflection w(x,y) takes place. Hence, the value of λ is re calculated by observing that at the center x=0, y=0, the following equation should hold good.

$$D\Delta\Delta w(x,y)-\sigma h\Delta w(x,y)=\frac{245}{144}((P+\frac{\varepsilon_0}{2(d_{eff}+(d_0-w(x,y))^2}V^2) \quad (10)$$

Equation (10) gives a third degree polynomial in λ at x=0 and y=0, as given below

$$\lambda^3-\lambda^2\frac{(245a^4P+23040(d_0+d_{eff})D+2304a^2(d_0+d_{eff})h\sigma}{1152a^8(10D+a^2\sigma h)}$$
$$+\lambda\frac{(245a^4P(d_0+d_{eff})+5760(d_0+d_{eff})^2D+576a^2(d_0+d_{eff})^2h\sigma}{576a^{16}(10D+a^2\sigma h)}$$
$$-\frac{1}{2304a^{20}(10D+a^2\sigma h)}(490P(d_0+d_{eff})+245\varepsilon_0V^2)=0 \quad (11)$$

This equation has got three roots and only one of them gives a stable value. The value of λ is substituted in eq. (7) to get the deflection at any point (x, y) in terms of applied voltage, pressure and residual stress

3.1 Pull-in Voltage

Pull-in voltage is determined by differentiating the deflection *w(x,y)* at the center, i.e., at x=0, y=0 with respect to voltage at zero mechanical pressure and equating *dV/dw* to zero. The critical distance is got by substituting the pull-in voltage for voltage *V* in expression for deflection. The closed form expression for pull-in voltage (V_{pull}) and critical distance w_{cr} are

$$V_{pull}=\frac{32\sqrt{(d_0+d_{eff})^3(10D+a^2\sigma h}}{7a^2\sqrt{15}\sqrt{\varepsilon_0}} \quad (12)$$

$$w_{cr}=\frac{(d_0+d_{eff})}{3} \quad (13)$$

The results of the pull-in obtained from this model are compared with those obtained by simulation or reported experimentally in Table I. For a diaphragm with a = 1.2 mm, h = 0.8μm, d_0 = 3.5μm , ε_0 = 8.85x10^{-12} F/m , E = 169GPa, ν=0.3, σ=20 MPa, fig 3 shows the comparison of deflection with voltage

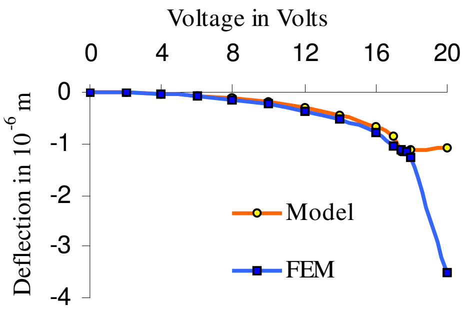

Fig 3. The pull-in voltage from the model. Pull-in voltage is represented by discontinuity in the graph

3.2 Touch-point pressure

Touch-point is defined as the pressure at which diaphragm just touches the fixed electrode and is of importance for the design of Touchmode Capacitive Pressure Sensors [2]. The touch-point pressure(P_{touch}) is found out by differentiating the deflection *w(x,y)* at the center i.e. at x=0, y=0 with respect to pressure P at zero voltage and equating *δw/δP* to zero, as after touchpoint is reached, there is no further deflection in vertical direction. The closed form expression for touch-point pressure is

$$P_{touch}=\frac{1152(d_0+d_{eff})}{245a^4}(10D+a^2h\sigma) \quad (14)$$

Table 2 compares the touch-point pressure obtained from this model with that obtained by simulation using Intellisuite®. Figure 4a and 4b compare the deflection with applied pressure as obtained from the propsed model with those simulated using Intellisuite® for a square diaphragm with a = 250 μm, h = 20μm, d_0 = 8μm , ε_0 = 8.85x10^{-12} F/m , E = 130GPa, d_{eff} = 0, ν=0.3 with and without residual stress .

4. CONCLUSIONS

A semi-analytical technique is proposed for calculating the touch-point pressure and pull-in volatge of square diaphragm with clamped edges in presence of residual stress. The deflection versus pressure studies reported in literature show that the deflection continues to increase beyond the gap d_0 between the diaphragm and fixed electrode. However, in present study, the deflection gets restricted at the gap d_0. This enables accurate determination of touchpoint pressure and gives a realistic picture of deflection. The pull-in voltage and critical distance has also been computed. The advantage of this technique lies in its simplicity and speed unlike the FEM tools, which though accurate, take a longer computational time, and requires suitable skills in deciding the mesh size and making the choice of mesh element. The results are in agreement with the simulated and experimental ones.

Table 1. Comparison of square diaphragm pull-in voltages

Reference	**Diaphragm half-side length a**	**Thickness h µm**	**Air-gap d_0 µm**	**Stress σ MPa**	**Pull-in Voltage V_{pull} Volts**	
					Present model	**Reported**
Osterberg[11]	12.7 µm	0.1	0.76	50	46.28	45.25
Bergqvist[16]	1 mm	5.1	2.3	6	9.98	10.0
Sazzadur[14]	0.6 mm	0.8	3.5	20	17.43	17.75

Table 2. Comparison of square diaphragm Touch-point pressure

Reference	**Diaphragm halfside length a**	**Thickness h µm**	**Air-gap d_0 µm**	**Stress σ MPa**	**Touch-point Pressure MPa**	
					Present model	**Simulated**
Pie G.[17]	250 µm	20.0	8.0	0	9.2	9.6
Pie G.[17]	250 µm	20.0	8.0	50	9.8	10.2

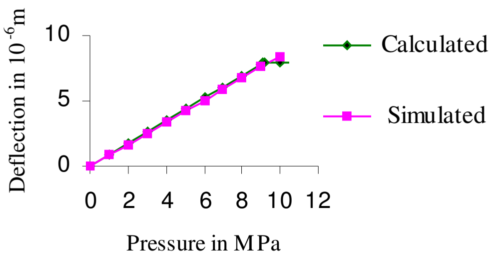

Fig 4a. Deflection Vs. Pressure residual stress=0 MPa

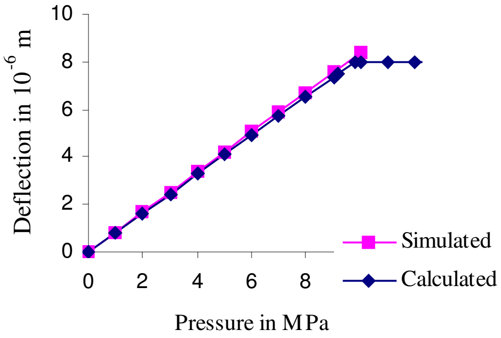

Fig 4b. Deflection vs. Pressure with residual stress= 50 MPa

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