A charge based compact flicker noise model including short channel effects

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ABSTRACT

In this work we have presented a general compact modeling methodology to calculate noise PSD for flicker noise for any arbitrary velocity field relationship. The results show that,for accurate modeling of bias dependence,the effect of field dependent mobility needs to be considered in future compact models.

1 Introduction

The low-frequency (LF) noise in MOS devices has been the subject of intensive research during past years. It is becoming a major concern for scaled devices because the LF noise increases as the inverse of the device area. Therefore accurate compact modeling of power spectral density (PSD) of flicker noise is becoming increasingly important. The modeling approaches used in this subject can be classified into two broad classes. First one is the Langevin method [1]–[4] and another is the flat band perturbation technique [5], [6]. Recently, we have shown that even for a long channel MOSFET both the methods give different result and explained why flat band perturbation technique is not correct in the presence of non zero drain bias [7]. Although some of the commercial circuit simulator uses Langevin based technique, none of them considers the effect of mobility reduction on noise PSD. Although the impact of field dependent mobility on thermal noise has received a lot of attention, almost no study has been done on the flicker noise. Therefore, it is not very clear how field dependent mobility affects flicker noise. In this work we apply our generalized noise calculation methodology developed in [4] for the flicker noise calculation.

2 Theory

Let us consider that we have a nonuniform channel (whose ends are terminated by ac grounds) with a distributed noise current source $\delta i_n(x,t)$. Our interest is to calculate the noise current at drain and gate. We begin with the fact that the current at any position x can be written as

$$I(x) = g(V, \frac{dV}{dx}) \cdot \frac{dV}{dx}, \tag{1}$$

where $g = W\mu Q_i$ and W, μ, Q_i and V are the width, mobility, inversion charge density and channel potential respectively. In presence of velocity saturation, μ starts to depend on the electric field so g will be a function of V and $\frac{dV}{dx}$. Presence of a noise current in the channel generates a perturbation of the channel potential, which then causes a change in transport current. Therefore the total current flowing at position x can be expressed as a sum of transport current (including the effect of perturbation in channel potential) and the noise current itself. Fig. 1 illustrates this situation. So the effect of adding the Langevin noise source in (1) can be written as

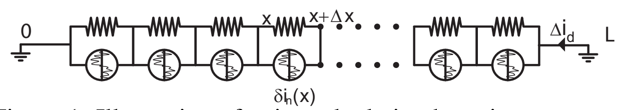

Figure 1: Illustration of noise calculation by using a generalized KP approach.

$$I(x)+i_d(x) = g\left(V+v, \frac{d(V+v)}{dx}\right)\frac{d(V+v)}{dx}+\delta i_n(x,t),$$

where $I(x)$ and V are the unperturbed current and Voltage in the channel. In the following derivation we will denote $\frac{dV}{dx}$ by E (we will take care of the sign latter). Taking a first order Taylor series expansion around (V, $\frac{dV}{dx}$) (the unperturbed point) we obtain

$$I(x) + i_d(x) = \left(g(V, \frac{dV}{dx}) + \frac{\partial g(V,E)}{\partial V}v + \frac{\partial g(V,E)}{\partial E}\frac{dv}{dx}\right)\frac{d(V+v)}{dx} + \delta i_n(x,t).$$

Using $I(x) = g(V, \frac{dV}{dx})\frac{dV}{dx}$ and keeping only the first order terms one obtains

$$i_d(x) = \left(g(V, \frac{dV}{dx}) + \frac{\partial g(V,E)}{\partial E}\frac{dV}{dx}\right)\frac{dv}{dx} + \left(\frac{\partial g(V,E)}{\partial V}\frac{dV}{dx}\right)v + \delta i_n(x,t).$$

Now as $I(x)$ is constant along the position, differentiating $I(x)$ we obtain

$$0 = \frac{dg(V,E)}{dx}E + g(V,E)\frac{dE}{dx}, \tag{2}$$

and hence $\frac{dE}{dx}$ is given by

$$\frac{dE}{dx} = -\frac{\frac{dg(V,E)}{dx}E}{g(V,E)} \tag{3}$$

We have

$$\frac{dg(V,E)}{dx} = \frac{\partial g(V,E)}{\partial V}E + \frac{\partial g(V,E)}{\partial E}\frac{dE}{dx} \quad (4)$$

Combining (3) and (4) we obtain $\frac{dg(V,E)}{dx}$ as

$$\frac{dg(V,E)}{dx} = \frac{g(V,E)}{g(V,E)+\frac{\partial g(V,E)}{\partial E}E} \cdot \frac{\partial g(V,E)}{\partial V} \cdot \frac{dV}{dx} \quad (5)$$

Combining (2) and (5) we obtain the key equation of this section

$$i_d(x) = \frac{g(V,E)+\frac{\partial g(V,E)}{\partial E}E}{g(V,E)} \cdot \frac{d}{dx}\left(g(V,E)v\right) + \delta i_n(x,t) \quad (6)$$

Up to now for convenience we have assumed $E = \frac{dV}{dx}$ but actually $E = -\frac{dV}{dx}$ so to take care of the sign we have to replace E and $\frac{\partial g(V,E)}{\partial E}$ with a minus sign. As they always appear as a product, nothing changes in the above equation. Defining $g_c(V,E) = g(V,E)^2/(g(V,E)+\frac{\partial g(V,E)}{\partial E}E)$, Eqn. (6) can be rewritten as

$$\frac{g_c(V)}{g(V)}i_d(x) = \frac{d}{dx}\left(g(V)v\right) + \frac{g_c(V)}{g(V)}\delta i_n(x,t) \quad (7)$$

Now we integrate both sides from 0 to L. Noticing that $i_d(x)$ is constant along the channel and v vanishes at the end points, we obtain the total drain current $\Delta i_d(t)$ as

$$\Delta i_d(t) = \frac{1}{L_c}\int_0^L \frac{g_c(V)}{g(V)} \cdot \delta i_n(x,t)dx, \quad (8)$$

where $L_c = L\int_0^L \frac{g_c(V)}{g(V)}dx$. We will assume that the noise sources are spatially uncorrelated. So the PSD of the local noise source $S_{\delta i_n^2}(x,x')$ can be written as

$$S_{\delta i_n^2}(x,x') = S_{\delta i_n^2}(x)\delta(x-x'). \quad (9)$$

Therefore the drain current PSD $S_{i_d^2}$ becomes

$$S_{I_D^2} = \frac{1}{L_c^2}\int_0^L \frac{g_c(V)^2}{g(V)^2} \cdot S_{\delta i_n^2}dx \quad (10)$$

We take mobility as $\mu = \mu_0/(1+(E/E_c)^p)^{1/p}$ (p=1,2) and

$$S_{\delta i_n^2} = \frac{q^2 \cdot N_T(E_f) \cdot k \cdot TI_D^2}{f \cdot W \cdot \gamma Q_i^2} \cdot \left(\frac{C_i}{C_{ox}+C_i+C_d+C_{it}}\right)^2 \quad (11)$$

where γ is the tunneling constant, $N_T(E_f)$ is the trap density at the fermi level, C_{ox}, C_i, C_d and C_{it} are the oxide, inversion, depletion and interface trap capacitance respectively. Note that in this expression we are assuming a constant trap density over energy and ignoring the effect of correlated mobility fluctuation whose effect, anyway, is proven to be small [8]. For developing a compact expression we take $p = 1$ in the mobility expression. It can be shown that for $p = 1$, $\frac{g_c(V)}{g(V)} = 1 + \frac{E}{E_c}$. Using EKV formulation [9] the charge based expression of drain current is written as

$$i = -\frac{2q+1}{1-\lambda_c\frac{dq}{d\xi}}\frac{dq}{d\xi}, \quad (12)$$

where q is the inversion change normalized by $2nU_TC_{ox}$, ξ is the normalized distance $\xi = x/L$, i is drain current normalized by the specific current $I_{spec} = 2n\mu C_{ox}(W/L)U_T^2$, λ_c is indicative of mobility reduction and given as $\lambda_c = \frac{2U_T}{E_CL}$, and n is the slope factor. From definition it flows that $E/E_c = -\lambda_c\frac{dq}{d\xi}$ and using the fact that $i = i_d$ is constant along the channel,from (12) it can be shown that

$$\frac{dq}{d\xi} = -\frac{i_d}{2q+1-\lambda_c i_d} \quad (13)$$

Now we use the fact that $C_i = Q_i/U_T$ and $n = 1+(C_{it}+C_d)/C_{ox}$ to obtain

$$\frac{C_i}{C_{ox}+C_i+C_d+C_{it}} = \frac{2q}{2q+1}. \quad (14)$$

From (12) and the fact that $\frac{g_c(V)}{g(V)} = 1+\frac{E}{E_c}$, we obtain

$$\frac{g_c(V)}{g(V)} = \frac{2q+1}{2q+1-\lambda_c i_d}. \quad (15)$$

Using (11),(15),(14) and (10) and $dx = Ld\xi$, we obtain

$$S_{I_D^2} = \frac{\mu^2W^2S_{Q_t^2}U_T^2i_d^2}{L^3(1+\lambda_c(q_s-q_d))^2} \int_0^1 \frac{1}{q^2}\frac{4q^2}{(2q+1)^2}\frac{(2q+1)^2}{(2q+1-\lambda_c i_d)^2}d\xi. \quad (16)$$

Using (13) we can convert (16) to a charge based integral as

$$S_{I_D^2} = \frac{\mu^2W^2S_{Q_t^2}U_T^2i_d}{L^3(1+\lambda_c(q_s-q_d))^2}\int_{q_d}^{q_s}\frac{4}{(2q+1-\lambda_c i_d)}dq, \quad (17)$$

where q_s and q_d are the normalized source and drain charge. Integrating (17) and using the fact that $i_d = \frac{(q_s^2+q_s)-(q_d^2+q_d)}{1+\lambda_c(q_s-q_d)}$ we obtain the charge based expression for drain noise PSD as

$$S_{I_D^2}(q_s,q_d,\lambda_c) = \frac{2\mu^2W^2S_{Q_t^2}U_T^2}{L^3} \cdot \frac{(q_s^2+q_s)-(q_d^2+q_d)}{(1+\lambda_c(q_s-q_d))^3}\log\left(\frac{q_s+\frac{1}{2}-\frac{\lambda_c\left((q_s^2+q_s)-(q_d^2+q_d)\right)}{2(1+\lambda_c(q_s-q_d))}}{q_s+\frac{1}{2}-\frac{\lambda_c\left((q_s^2+q_s)-(q_d^2+q_d)\right)}{2(1+\lambda_c(q_s-q_d))}}\right) \quad (18)$$

The channel length appearing in the expression of S_{id} is the length of the active transistor and excludes the length of the velocity saturated region. In our modeling approach the velocity saturated region does not contribute to the drain current

PSD. This is in contrast with the development presented in [2], [3] depending on which Langevin based compact models of flicker noise are developed. The reason behind this formulation is the following: noise current has to be constant along the channel, if the pinch-off voltage is known exactly then for the total current calculation it becomes irreverent what happens beyond the pinch of point.

3 Discussions

From our analysis it is clear that the mobility reduction can impact noise PSD by three different mechanism. First by changing the noise PSD expression (18) through λ. Second by reducing the effective channel length through channel length modulation (CLM) and, finally, changing the value of saturation charge at the end of the active channel. In Fig.1, we show the limitation of the flat band perturbation technique even for a long channel MOSFET which motivates the use of Langevin based technique in noise calculation. In Fig. 1 $S^{L}_{I_D^2}$ and $S^{FB}_{I_D^2}$ denote noise PSD calculated using Langevin method and FBP respectively. In order to study the impact of field dependent mobility we construct a quantity η such that $\eta(q_s, q_d, \lambda_c) = S_{I_D^2}(q_s, q_d, \lambda_c)/S_{I_D^2}(q_s, q_d, 0)$. The η enables us to consider the effect due to noise transfer function only. We do not consider the effect of CLM and saturation charge (and also effect of vertical field on λ_c) in this abstract because they are determined independently of the noise model. Fig. 2 shows the plot of η vs q_s (or equivalently the gate overdrive, because $q_s = (v_g - v_{T0})/2n$) for different values λ_c and for two different mobility model (p=1,2). η_{sat} is the value of η at saturation. The plots illustrates that the impact of the mobility degradation considerably affect the noise properties. In Fig. 3, we study the effect of mobility degradation on drain voltage dependance of noise PSD. These results suggest that, depending on the mobility model and bias condition, neglecting the effect of mobility reduction can easily overestimate the noise PSD by a factor of 2-3. Therefore, this effect needs to be considered in future compact models.

4 Conclusion

In this work we have presented a general compact modeling methodology to calculate noise PSD for flicker noise for any arbitrary velocity field relationship. Using a specific field dependent mobility that is commonly used compact models, we presented closed form charge based expression for flicker noise PSD.The results suggest that, depending on the mobility model and bias condition, neglecting the effect of mobility reduction can easily overestimate the noise PSD by a factor of 2-3. Therefore,for accurate modeling of bias dependence,this effect needs to be considered in future compact models.

Acknowledgment

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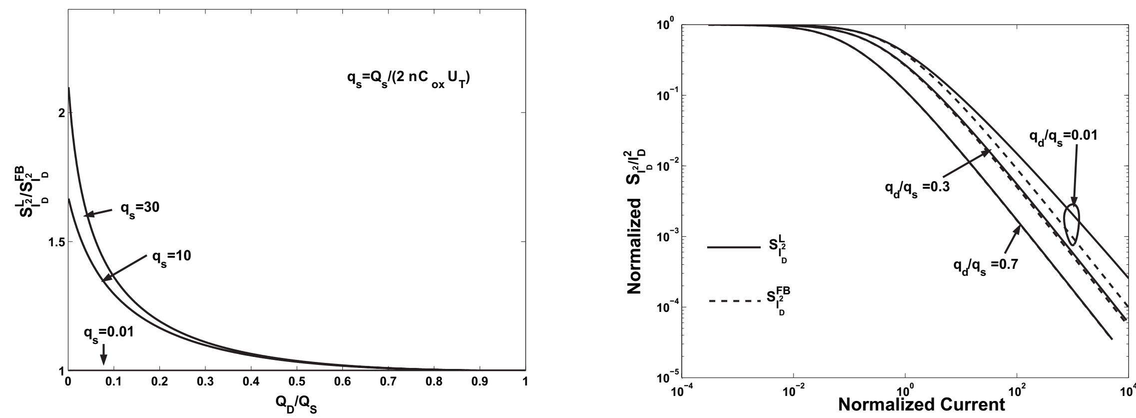
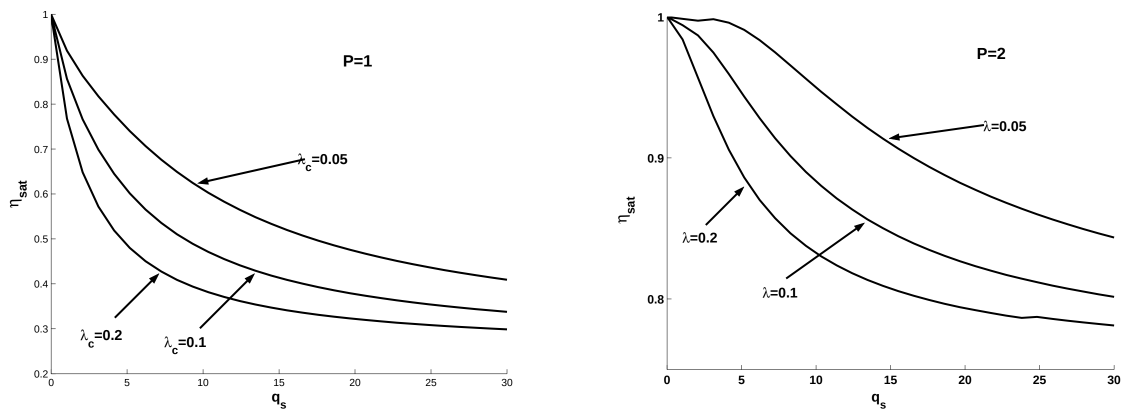


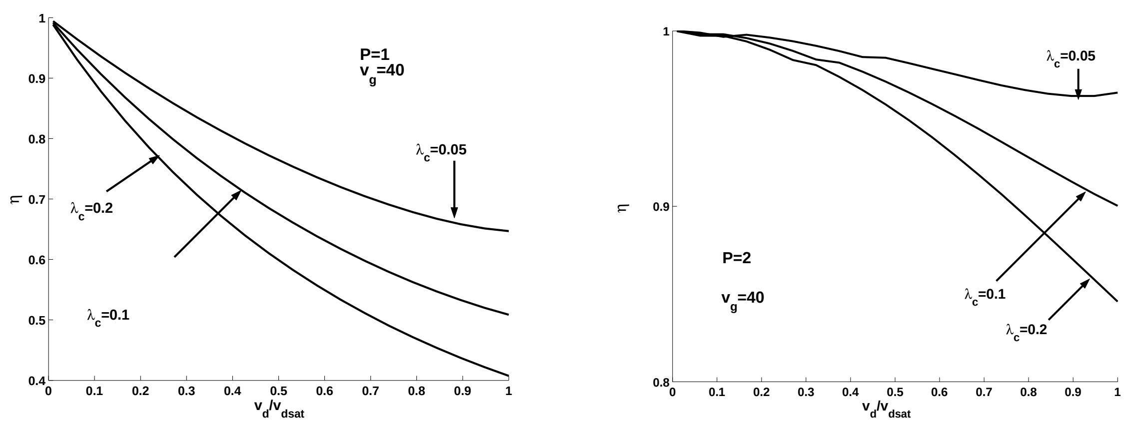
(a) (b)

Figure 2: Langevin method and FBP. (a) A plot of $S^L_{I_D^2}/S^{FB}_{I_D^2}$ vs Q_D/Q_S to illustrate the limitation of flat band perturbation method. It clearly shows in non ohmic region it underestimates the flicker noise in strong inversion. (b) A plot of normalized $S_{I_D^2}/I_D^2$ vs normalized I_D in a log-log scale. It illustrates that Langevin method will also show a good correlation with $(g_m/I_D)^2$.



(a) (b)

Figure 3: Impact of mobility reduction on saturation drain current PSD. (a) A plot of η at saturation vs normalized inversion charge source for p=1 mobility model. (b) same plot for p=2 mobility model. Both plots show that if mobility degradation is not considered then noise actually gets over estimated. As p=2 model implies a lesser extent of mobility degradation, the effect is considerably less for p=2.



(a) (b)

Figure 4: Impact of mobility degradation on drain voltage dependance. (a) A plot of η at $v_g = 40$ vs drain voltage normalized to saturation voltage for p=1 mobility model. (b) same plot for p=2 mobility model. As expected, the effect of field dependent mobility becomes increasingly important as drain voltage increases and the effect is considerably less for p=2.

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