Theory of source-drain partitioning in MOSFET

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Abstract

The Ward-Dutton (WD) partitioning scheme [1] is used extensively to develop transient and high frequency advanced compact models in MOSFET analysis. Recently it has been shown that WD partitioning fails in the case of field dependent mobility [2] or laterally asymmetrical doping [3]. In this work we present a generalization of WD partitioning and discuss the implication of the failure of WD partitioning.

1 introduction

All of the advanced compact MOSFET models (PSP, MM11, HISIM, BSIM, EKV etc., see [4] and the references therein) dedicated to even sub 100 nm devices are entirely built on the Ward-Dutton (WD) partitioning [1], which was originally derived for long-channel MOST. However, it remained an open question whether this scheme can be used for field dependent mobility that is enhanced in state of the art submicron technologies. Recently we showed that in a velocity saturated device, in general, it may not be always possible to define a partitioning scheme. However, for small-signal operation one can always define a partitioning scheme but this scheme is different from the original WD and depends on the mobility model used. A similar situation occurs in the presence of lateral asymmetry in a MOSFET. Recently in the pioneering work by Aarts et. al. [3] it has been recognized that the WD charge partitioning [1] is not applicable to LA MOSFET and it has been concluded that "no terminal charges exist from which the capacitances can be derived". Therefore, [3] is forced to solve the small-signal continuity equation under quasi-static assumption for both real and imaginary part of the perturbed channel potential and calculate the capacitances from them. It is also possible to extend the idea of small-signal partitioning, which was developed in [2], to a LA MOSFET [5]. Existence of the small-signal partitioning implies that one can always define a partitioning of small-signal/perturbed charges to calculate the terminal capacitances. This development has two main advantages over the existing approach. First, it eliminates the need of solving the continuity equation for the imaginary part of perturbed channel potential which results in a saving of one level of numerical integration compared to [3]. Second, impact of mobility degradation can be treated in a very efficient way through the small-signal partition function, whereas, it is not clear how the existing methodology [3] can be extended to include an exact treatment of field dependent mobility.

2 Partitioning Scheme

We begin with the fact that the current at any position x can be written as

$$I(x) = g(V, \frac{dV}{dx}, x) \cdot \frac{dV}{dx}, \tag{1}$$

where $g = W\mu Q_i$ and W, μ, Q_i and V are the width, mobility, inversion charge density and channel potential respectively. In presence of velocity saturation, μ starts to depend on the electric field so g will be a function of V and $\frac{dV}{dx}$. The explicit dependence on x implies that the doping may also vary along the channel, which is the case for a lateral asymmetric MOSFET. We will assume the following general form of continuity equation

$$\frac{dI}{dx} = \widehat{s}. \tag{2}$$

Please note that this $\widehat{s}$ expression of source term is completely general. It can be a term like $W\frac{dQ}{dt}$ (rate of change of charge per unit length) in case of a transient or a term like $i_g(x)$ (gate current per unit length) in case of a gate tunneling current. Existence of a partitioning scheme means that it is possible to decouple the current when $\widehat{s} = 0$ (we will call this current as I_0) and the effect of source term. In other words, the terminal current can be expressed as a sum of I_0 and a weighted integral of $\widehat{s}$. We will start by presenting a novel derivation of the WD partitioning scheme. We first multiply (2) by x and integrate from 0 to L,

$$\int_0^L x \cdot \frac{dI}{dx} dx = \int_0^L x\widehat{s} dx. \tag{3}$$

and integrating by parts yields

$$xI|_0^L - \int_0^L I(x)dx = \int_0^L x\widehat{s}dx. \tag{4}$$

Substituting (1) in the above equation, we obtain

$$I(L) = \frac{1}{L}\int_{V_s}^{V_d} g(V, \frac{dV}{dx}, x)dV + \int_0^L \frac{x}{L}\widehat{s}dx. \tag{5}$$

When g is a function of V only (the classical constant mobility case), the first integral depends only on the value of V

at the boundaries. The point is, even if the function $V(x)$ (the channel potential distribution) changes because of the $\widehat{s}$ term, the value of the integral does not change and remains equal to the case when $\widehat{s} = 0$. In the case when $\widehat{s} = 0$, we can easily identify the integral as the DC current I_0 and we get back the WD partitioning scheme. But when g starts to depend also on $\frac{dV}{dx}$ or x, the situation is different. As $V(x)$ changes due to the presence of the source term in the continuity equation, the value of the integral starts to depend on the profile of $V(x)$. Therefore, $\frac{1}{L}\int_{V_s}^{V_d} g(V, \frac{dV}{dx}, x)dV$ is different in presence of the source term. So in the presence of mobility degradation or lateral asymmetry,

$$I(L) \neq I_0 + \int_0^L \frac{x}{L}\widehat{s}dx. \tag{6}$$

To extend the concept of partitioning, we define a quantity $\widetilde{I}(x) = I(x) - I_0$. It trivially follows that $\widetilde{I}(x)$ satisfies

$$\frac{d\widetilde{I}(x)}{dx} = \widehat{s}. \tag{7}$$

Let us now consider an arbitrary function $F(x) = \int_0^x f(x)dx$ and multiply (7) by $F(x)$ and integrate by parts from 0 to L to obtain

$$I(L) = I_0 + \int_0^L \frac{F(x)}{F(L)}\widehat{s}dx + \frac{1}{F(L)}\int_0^L f(x)(I(x) - I_0)dx \tag{8}$$

So in order to have a partitioning scheme, we need the last term to vanish or in other words, existence of a function $f(x)$ such that for any given $\widehat{s}$, the following holds

$$\int_0^L f(x)g(V, \frac{dV}{dx}, x)\cdot\frac{dV}{dx}dx = \int_0^L f(x)g(V_0, \frac{dV_0}{dx}, x)\cdot\frac{dV_0}{dx}dx \tag{9}$$

Please note that when g does not depend on x, $f(x) = 1$ trivially satisfies the above criterion and we get back the original WD charge partitioning. In general we can not guarantee the existence of the function $f(x)$ and hence the charge partitioning scheme. But we will prove that when $\widehat{s}$ can be considered as a small perturbation we can always find out the function $f(x)$ for any general form of g.

3 Generalized small-signal partitioning function

In this section we will unify and generalize the concept of small-signal partitioning by simultaneously accounting for both field dependent mobility and lateral asymmetry. From (8), we see that we can define a partitioning scheme when it is possible to guaranty the existence of a function $f(x)$ such that

$$\int_0^L f(x)(I(x) - I_0)dx = 0. \tag{10}$$

As we are considering the case when $\widehat{s}$ can be treated as a small perturbation, we will start with a perturbation analysis of $I(x)$. In the subsequent analysis subscript '0' will be used to denote the situation when $\widehat{s} = 0$. We will also use E to denote dV/dx. The perturbed current $i(x)$ can be obtained as

$$\begin{aligned} I(x) =& I_0 + i(x) = g\left(x, V_0 + v, \frac{d(V_0+v)}{dx}\right)\frac{d(V_0+v)}{dx} \\ =& \left(g\left(x, V_0, \frac{dV_0}{dx}\right) + \frac{\partial g_0}{\partial V_0}v + \frac{\partial g_0}{\partial E_0}\frac{dv}{dx}\right)\left(\frac{dV_0}{dx} + \frac{dv}{dx}\right) \end{aligned} \tag{11}$$

Neglecting second order term we obtain

$$i(x) = g\left(x, V_0, \frac{dV_0}{dx}\right)\frac{dv}{dx} + \frac{\partial g_0}{\partial V_0}\frac{dV_0}{dx}v + \frac{\partial g_0}{\partial E_0}E_0\frac{dv}{dx}. \tag{12}$$

Rearranging leads to

$$i(x) = \frac{\partial g_0}{\partial V_0}\frac{dV_0}{dx}v + \left(g_0 + \frac{\partial g_0}{\partial E_0}E_0\right)\frac{dv}{dx} \tag{13}$$

In the absence of velocity saturation, the situation is rather simple. Since the mobility is independent of E, the term $\frac{\partial g_0}{\partial E_0}$ vanishes and $\frac{\partial g_0}{\partial V_0}\frac{dV_0}{dx}$ simply becomes $\frac{dg_0}{dx} - \frac{\partial g_0}{\partial x}$ [5]. But when the mobility degradation is present, we not only have a non vanishing contribution from $\frac{\partial g_0}{\partial E_0}$ but also $\frac{\partial g_0}{\partial V_0}\frac{dV_0}{dx}$ does not equate to $\frac{dg_0}{dx} - \frac{\partial g_0}{\partial x}$ anymore. One of the important steps of the problem is to recognize this fact and to find out $\frac{dg_0}{dx}$ in the presence of mobility degradation. To proceed further, we note that

$$\frac{dg_0}{dx} = \frac{\partial g_0}{\partial x} + \frac{\partial g_0}{\partial V_0}\frac{dV_0}{dx} + \frac{\partial g_0}{\partial E_0}\frac{dE_0}{dx}. \tag{14}$$

At this point it is important to recognize that we have additional constraint on $\frac{dE_0}{dx}$ from the fact that $\frac{dI_0}{dx} = 0$. Therefore,

$$\frac{dI_0}{dx} = \frac{dg_0}{dx}E_0 + g_0\frac{dE_0}{dx} = 0 \tag{15}$$

which implies

$$\frac{dE_0}{dx} = -\frac{\frac{dg_0}{dx}E_0}{g_0} \tag{16}$$

Substituting (16) in to the expression of $\frac{dg_0}{dx}$ (i. e. (14)), one obtains

$$\frac{dg_0}{dx} = \frac{g_0}{g_0 + \frac{\partial g_0}{\partial E_0}E_0}\left(\frac{\partial g_0}{\partial x} + \frac{\partial g_0}{\partial V_0}\frac{dV_0}{dx}\right) \tag{17}$$

This reveals the impact of velocity saturation on the chain rule. From this we obtain

$$\frac{\partial g_0}{\partial V_0}\frac{dV_0}{dx} = \frac{g_0 + \frac{\partial g_0}{\partial E_0}E_0}{g_0}\frac{dg_0}{dx} - \frac{\partial g_0}{\partial x} \tag{18}$$

At this point we introduce g_{eff} as

$$g_{eff} = (g_0 + \frac{\partial g_0}{\partial E_0} E_0) \tag{19}$$

Substituting (18) in (13) and using definition of g_{eff} we obtain

$$\begin{aligned} i(x) &= \frac{g_{eff}}{g_0}\left(\frac{dg_0}{dx}v + g_0\frac{dv}{dx}\right) - \frac{\partial g_0}{\partial x}v \\ &= \frac{g_{eff}}{g_0}\left(\frac{d(g_0 v)}{dx} - \frac{g_0}{g_{eff}}\frac{\partial g_0}{\partial x}v\right) \end{aligned} \tag{20}$$

As $i(x) = I(x) - I_0$, we seek a function $f(x)$ such that

$$\int_0^L f(x)i(x)dx = 0 \tag{21}$$

For convenience, we write $f(x)$ as

$$f(x) = \frac{g_0}{g_{eff}}\lambda(x). \tag{22}$$

It immediately follows from (20) and (21) that $\lambda(x)$ satisfies the following integral equation

$$\int_0^L \lambda(x)\frac{d(g_0 v)}{dx}dx = \int_0^L \lambda(x)\frac{g_0}{g_{eff}}\frac{\partial g_0}{\partial x}v dx \tag{23}$$

Now we arrive at the most important part of the derivation. Eq. (23) does not directly give any means to calculate $\lambda(x)$. In order to get something meaningful out of (23), the trick we apply is to integrate LHS by part and use the fact that v vanishes at the boundary to obtain

$$\int_0^L \left(\frac{d\lambda(x)}{dx}g_0 + \lambda(x)\frac{g_0}{g_{eff}}\frac{\partial g_0}{\partial x}\right)v dx = 0 \tag{24}$$

for all v. This is only possible if the term inside the parenthesis vanishes. This means that the λ satisfies the differential equation given as

$$\frac{d\lambda(x)}{dx}g_0 = -\frac{g_0}{g_{eff}}\frac{\partial g_0}{\partial x}\lambda(x) \tag{25}$$

which can be solved at once as

$$\lambda = \exp\left(-\int_0^x \frac{1}{g_0}\left(\frac{g_0}{g_{eff}}\right)\frac{\partial g_0}{\partial x}dx\right) \tag{26}$$

Therefore, $f(x)$ becomes

$$f(x) = \frac{g_0}{g_{eff}}\exp\left(-\int_0^x \frac{1}{g_0}\left(\frac{g_0}{g_{eff}}\right)\frac{\partial g_0}{\partial x}dx\right) \tag{27}$$

It is easy to check when there is no asymmetry, $\partial g_0/\partial x$ vanishes and $f(x)$ becomes $\frac{g_0}{g_0 + \frac{\partial g_0}{\partial E_0}E_0}$ which is our previous result [2]. It is important to note that as the effect of mobility degradation (g_0/g_{eff} term) appears inside an exponential, the partitioning and capacitance behavior of asymmetric MOSFET will be sensitive to the mobility model. Therefore, from (8), the final expression of the terminal current then becomes

$$I(L) = I_0 + \int_0^L (F(x)/F(L))\,\widehat{s}dx, \tag{28}$$

where $F(x)$ is the integral of $f(x)$, and can be expressed as

$$F(x) = \int_0^x \left(\frac{g_0}{g_{eff}}\exp\left(-\int_0^x \frac{1}{g_0}\left(\frac{g_0}{g_{eff}}\right)\frac{\partial g_0}{\partial x}dx\right)\right). \tag{29}$$

It should be noted that when there is no lateral asymmetry, $\partial g_0/\partial x$ vanishes and when there is also no field dependent mobility, g_0/g_{eff} becomes unity. Therefore, for a classical MOSFET, $F(x)$ becomes equal to x and we get back the WD scheme. But in general, the partitioning is quite different from WD, and depends on the doping profile.

4 Discussion

All of the compact MOSFET models use a terminal charge based description to calculate transient current and the capacitances. The question is, after finding out that WD partitioning is not valid, can we still use the concept of terminal charge? To answer the question first notice that as F is obtained solving (9), it can contain x, V_0 , V, $\frac{dV_0}{dx}$ and $\frac{dV}{dx}$ *explicitly*. For transient analysis we have $\widehat{s} = WdQ/dt$, therefore the terminal current $I_D(t)$ becomes

$$I_D(t) = I_0 + \frac{W}{F(L)}\int_0^L F\left(x, V_0, \frac{dV_0}{dx}, V, \frac{dV}{dx}\right)\frac{dQ}{dt}dx \tag{30}$$

A terminal charge based description requires that we should be able to express the terminal current as

$$I_D(t) = I_0 + \frac{dQ_D}{dt} \tag{31}$$

It will be possible only when one can take the time derivative in (30) outside the integral. Now if F depends on bias (i.e. contains any of the V_0 , V, $\frac{dV_0}{dx}$ terms) , then it becomes time dependent and the time derivative can not be taken outside the integral. Therefore, a terminal charge based description is possible if F does not contain any bias dependence. In case of WD partitioning we have $F(x) = x$ which is obviously independent of bias, hence the concept of terminal charge holds. An interesting question is if there can be any description of terminal charge which is not WD. To see a non trivial example, let us consider the case when $g(x, V)$ is separable as $g(x, V) = g_x(x)g_v(V)$. It follows that $f(x) = 1/g_x(x)$ satisfies (9) and we get a bias independent partitioning function $F(x) = \int_0^x 1/g_x(x)$. But in general, asymmetries are not separable, therefore the partition

function will depend on bias and the terminal charge based description will break down.

Next best thing one can hope for is to find a QS capacitance description. Expanding $\frac{dQ}{dt}$ we obtain

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \sum_k \frac{\partial Q}{\partial V_k}\frac{dV_k}{dt}, \tag{32}$$

where k is a device terminal. Under QS description we ignore the $\partial Q/\partial t$ term and substitute dQ/dt term in (30) we obtain

$$I_D(t) = I_0 + \sum_k \frac{W}{F(L)} \int_0^L F\left(x, V_0, \frac{dV_0}{dx}, V, \frac{dV}{dx}\right) \frac{\partial Q}{\partial V_k}\frac{dV_k}{dt} dx. \tag{33}$$

Now a QS capacitance description requires current to be expressed as

$$I_D(t) = I_0 + \sum_k C_{dk}\frac{dV_k}{dt}, \tag{34}$$

were the capacitances C_{dk} need to be uniquely determined by the terminal voltages. Comparing (34) with (33) we obtain

$$C_{dk} = \frac{W}{F(L)} \int_0^L F\left(x, V_0, \frac{dV_0}{dx}, V, \frac{dV}{dx}\right) \frac{\partial Q}{\partial V_k} dx. \tag{35}$$

Now if C_{dk}-s are to be uniquely determined by the terminal voltages, then F can not contain V or dV/dx. We already showed that when $\widehat{s}$ is small, F does not contain V or dV/dx. It means that capacitance description can safely be used for relatively slow transient. It is interesting to note that for sharp transient (i.e. high dQ/dt) capacitance description may break down even if the profile can be described quasi-statically.

We will conclude this section by pointing out two difficult challenges that future compact models will face because of the bias dependence of the partitioning function. Note that when the transient is relatively slow, a QS capacitance model can provide an accurate description. But capacitance based description has its own problems. Because of the finite time step of circuit simulator, it will lead to the charge conservation problem [6] which was the main reason for adapting to a charge based description. Next, almost all compact modeling approaches for large signal non-quasi-static (NQS) effect use WD partitioning, including the most recent one [7] whose formulation is aimed specifically at capturing the field dependent mobility. As the concept of partitioning itself breaks down in the presence of field dependent mobility, next generation NQS modeling activity should be directed to capture the slope of the profile at the drain and the source end, instead of capturing the profile itself.

5 conclusion

In this work we discussed the limitation of WD partitioning and presented alternate schemes for mobility degraded and lateral asymmetric device. As failure of WD scheme makes the concept of terminal charge invalid, the future compact modeling methodologies may undergo major changes.

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