

An Approximate Explicit Solution to General Diode Equation

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ABSTRACT

This paper presents a new analytic approximation to the general diode equation with the presence of a series resistance. Using exact first and second-order derivatives and a modified Newton-Rahson formula, a concise analytic approximate solution is derived for the diode equation with the significant improvement on the accuracy and computation efficiency, thus it is very useful for users to implement the diode model and the inversion charge models in advanced MOSFET compact models such as ACM, EKV and BSIM5 into the circuit simulators, e.g. SPICE for circuit simulation and analysis.

Keywords: Diode; Compact model; Analytic solution; Circuit simulation

1 INTRODUCTION

In integrated circuit technology, the P-N diode is not only one of the most elementary semiconductor devices, but also a monitoring tool for process characterization and junction quality [1]. The dopant concentration, junction depth, the thermal annealing, and the quality of contact can be estimated from the P-N junction characteristics. Thus robust and accurate diode equation solution is not only important to the device characterization and applications but also needed for the fabrication process characterization and calibration.

However, the combination characteristics of both the exponent and the linear make the desired concise solution for the general diode equation only be expressed in terms of the Lambert-W function. This Lambert-W function is a super function, non-analytic expression can be constructed from just the common elementary function for this solution so far although it is well known and widely used in various physics and engineering application [2-4]. In order to meet practical application requirement, some commercial software tools such as the Maple and Mathematica developed executed programs to precisely calculate the Lambert-W function. These softwares, however, cannot directly be used in any circuit simulators because both are the commercial software programs with different objects. On the same time, the embedded Lambert-W function programs cannot be implemented into circuit simulators due

to the copyright protection and the invisible of the executable codes.

The recent progress in advanced compact model development demonstrated that the inversion charge-bias relation also follows a like-diode equation in the MOSFETs as shown in ACM, EKV and BSIM5 [5]. How to get the exact inversion charge, thus, becomes an important yet interesting research as in the diode equation case. In engineering, various analytic approximations and improved iterative methods had to seek for applications where device and circuit simulators are used. However, an analytic expression is high desirable in view of computation efficiency and code readable. A.Ortiz-Conde made a first analytical approximation trial for the general diode equation as early as 1992, although its accuracy is not good enough [4]. Later, Fjeldly, et al. presented a Taylor's series solution in the second and cubic function forms [6]. However, this analytic solution has a non-continuous disadvantage for whole diode operation region [7]. Further, the accuracy is also not good in high forward bias condition. J.M.Pimbley and J.Huang He team provided different iterative solutions for the engineering application [8-9]. These iterative solutions, on the other hand, are not preferred in the circuit simulator due to the expensive computation and CPU memory need.

In this paper, we present a very simple and concise analytic approximate solution to this engineering problem, so to make the diode model and inversion charge equation in MOSFETs implementation easy and efficient in VLSI circuit simulators. With the exact first and second derivatives, a modified Newton-Rahson method is used to derive the diode equation solution. The result comparison with the rigorous numerical iteration demonstrated that this new analytic approximation has the significant improvement on the accuracy and computation efficiency.

2 DESCRIPTION OF NEW APPROXIMATION

The flow current through a diode driven by a voltage source through a series resistance is governed by the *KVL* and diode junction equation following semiconductor device physics. For a non-ideal diode with the series resistance R_s as shown by the diode circuit model in Fig.1, the voltage *KVL* equation is written as

$$V = V_{diode} + IR_s \quad (1)$$

Where R_s is the parasitic series resistance, V_{diode} and IR voltage drop in the intrinsic diode and the resistance R_s , respectively.

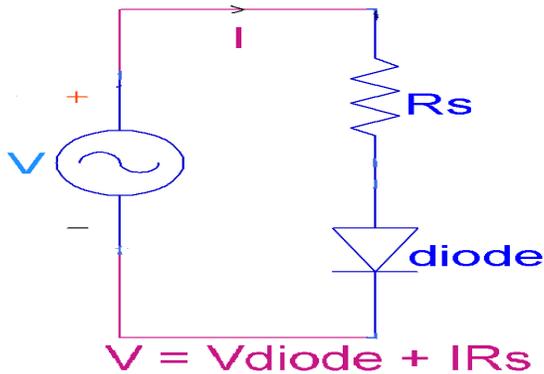


Fig.1: Diagram of the general diode circuit model with the series resistance.

The current equation of this diode circuit model with a non-unit exponential factor n follows the simple Shockley law

$$I = I_s \left[\exp\left(\frac{qV_{diode}}{nkT}\right) - 1 \right] \quad (2)$$

Where I_s is the reverse saturation current generated by the body region minority carrier induced by the thermal effect, n the diode-ideal factor to consider the different current mechanism such as the Generation-Recombination and high-level injection, kT/q the thermal voltage.

Combining (2) with (1), the current I of the diode in series with a resistance can be written as a function of the applied voltage V by the following generalized diode equation

$$I = I_s \left[\exp\left(\frac{q(V - IR)}{nkT}\right) - 1 \right] \quad (3)$$

Following Fjeldly, et al's normalized method [4], we write (3) in terms of the normalized variables into (9) gives

$$i = qR_s(I + I_s)/nkT \quad \text{and} \\ v = q(V + I_sR_s)/nkT + \ln[qI_sR_s/nkT] \quad \text{in a simple form} \\ i = \exp(v - i) \quad (4)$$

We make use of a mathematic transformation to make (4) be a familiar formulation and name the result a function, $f(i)$

$$f(i) = \ln i + i - v = 0 \quad (5)$$

(5) appear in many physics and engineering applications where the linear and exponential response is accompanied. Obvious examples are diode, solar cells and photodetectors used as circuit elements. Additional problems, not related to diodes, involving expressions of this form, include the solution of the inversion charge in the advanced compact MOSFET models such as ACM, EKV and BSIM5 [5,10,11].

In all these cases, one needs to express i explicitly as a function v , without having to resort to iterative or other numerical routines to construct analytic model and perform the parameter extraction.

We first demonstrate the basic principle of our method. Algebraically, the method derives from the familiar Taylor series expansion of the function in the neighborhood of a point

$$f(i + \delta) = f(i) + f'(i)\delta + \frac{f''(i)}{2}\delta^2 + \dots \quad (6)$$

The approximated expression of $f(i)$ can be obtained from this Taylor expansion. For small enough values of δ , and for well-behaved functions, the terms beyond linear, however, are unimportant. Hence following the Newton-Rahson method to have $f(i + \delta) = 0$ and neglecting beyond first derivative implies

$$\delta = -\frac{f(i)}{df(i)} \quad (7)$$

At the same time, we can obtain from (6) with second derivative

$$\delta = -\frac{f(i)}{1 + \frac{d^2 f(i)}{2} \delta} \quad (8)$$

Substituting (7) into (8) results in

$$\delta = -\frac{f(i)}{1 - \frac{d^2 f(i)f(i)}{2df(i)}} \quad (9)$$

The general formula of (9) would be used for construction of our analytical approximation solution of the general diode. The exact first and second derivatives obtained from (5) are substituted

$$\delta = -\frac{if(i)}{1 + i + \frac{f(i)}{2(1+i)}} \quad (10)$$

To simplified the description, we use an initial value following Fjedly method [6]

$$i_b = \ln[1 + e^v] \quad (11)$$

In mathematic meaning, i_b presents an asymptotic approximation developed. If δ is a relative small refinement and

$$f = i_b + \ln i_b - v \quad (12)$$

$$i_0 = i_b + \delta \quad (13)$$

Then, δ is obtained from (10). As a result, we have

$$i_0 = i_b \left[1 - \frac{f}{1 + i_b + \frac{f}{2(1 + i_b)}} \right] \quad (14)$$

Further, we define

$$f_1 = i_0 + \ln i_0 - v \quad (15)$$

$$i = i_0 + \alpha \quad (16)$$

Finally, substituting (15) into (10) to get the symbol expression of a new refinement α , we obtain the new analytic approximation of the generous diode equation from (16)

$$i = i_0 \left[1 - \frac{f_1}{1 + i_0 + \frac{f_1}{2(1 + i_0)}} \right] \quad (17)$$

This algorithm requires computation of three logarithms, e.g. (11), (12) and (15) and one exponential in (11), however, with no iteration.

3 RESULT AND DISCUSSION

Both the accuracy and the computation efficiency are two main concerns for the circuit simulation. Comparison of the new analytic approximation (17) with the iterative numerical results of (5) is shown in Fig.2. It is easily found that excellent accuracy is achieved for the whole operation region. The obtained absolute error is also shown in Fig.3. It is evident that the absolute error introduced by the new analytic approximation (17) is under 10^{-6} . The further tests on the previous analytical approximations in [4,6-7] demonstrate that the presented analytic approximation solution to the generalized diode is the most accurate so far.

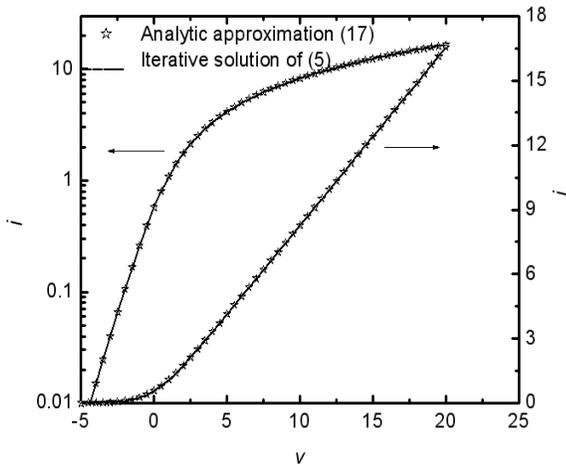


Fig.2: Diode calculated by iterative solution of (5) and analytic approximation (17) in a large operation range.

As compared to the most commonly used iterative method [6-7], the CPU time of the new analytic approximation required is reduced by approximately a factor of 1.5 for the normalized variable v in the tested range from -5 to $+25$. Of interest is a comparison of the diode current computation time between the new analytic solution and the commercial software such as Maple and Mathematica for the same Lambert-W function solution to further test the computation efficiency. The calculation recycles with eight thousand times are designed in a SUN Blade workstation coming from the SUN Microsystems Co. for both methods the recorded time demonstrated that the optimized code program of the new analytic approximation takes almost same time with the executed program of the Maple while being a bit faster than that of the Mathematical by a factor 1.3 for the same normalized variable v range from -5 to $+25$ with the step 0.05. We would like to point out that the new approximation solution is almost equivalent to the Maple and Mathematica in the accuracy. Nevertheless, the new analytic approximation has an evident advantage over the software tools, its code and algorithm are easily implanted or written into any circuit program and simulator while the codes of the Maple and Mathematica being fixed in commercial tools protected by the copyrights. Thus, the new analytic approximation will be useful for compact model developers to implement various physics effects into the circuit simulators.

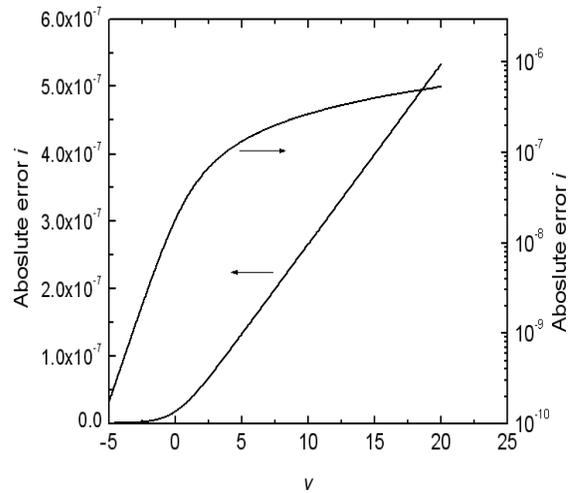


Fig.3: Absolute error induced by new analytic approximation (17) compared with iterative result of (5).

The accuracy of the normalized diode current computation is a necessary rather than a sufficient condition for the utility of the analytic approximation. In practice, the derivative of the diode current with respect to the terminal bias is a more stringent test for compact model development. Such a comparison is shown in Fig.4. At the same time, Fig.5 demonstrates the error of the derivative of

the normalized diode current induced by the analytic approximation (17). Fig.4 and Fig.5 indicate that the new analytic approximation is accurate not only in terms of the normalized diode current but also in terms of its derivative.

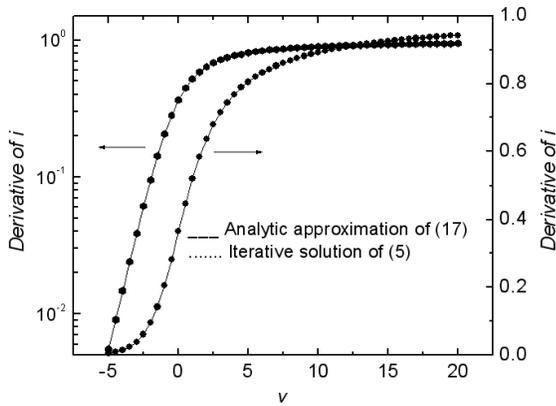


Fig.4: Derivative comparison of the diode current between the analytic approximation (17) and the iterative solution of (5).

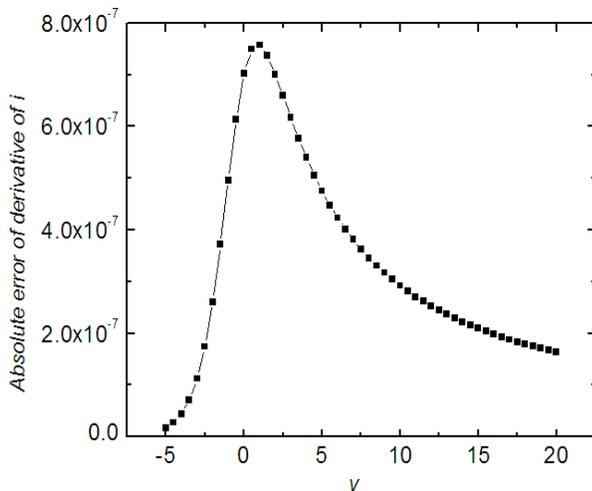


Fig.5: Comparison of the derivative error obtained from the analytic approximation (17) and the iterative solution of (5).

4 CONCLUSIONS

In summary, a new analytic approximate solution for a general diode has been developed in this paper. The accuracy of the diode current calculation and its derivative prediction has also been verified compared with the result of the iterative method. Compared with the previous others, this new approximate solution requires no numerical iteration but more accurate and computation efficient, thus it is more suitable to implement into the circuit simulators for engineering application and design.

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