Experimental Results for a Parametrically Excited Micro-ring Resonator

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ABSTRACT

This paper reports experimental results for a parametrically excited micro-ring resonator. Actuation and sensing are performed electrostatically. The equation of motion for the electrostatically actuated MEMS ring resonator is shown to contain a stiffness modulating term which, when modulated at a frequency near twice a natural frequency of the resonator, results in parametric resonance. Frequency sweeps, centered on approximately twice the measured resonant frequency of the device, were performed at various voltages and the parametric resonance was observed electrically at half the excitation frequency. This data was used to map the 'boundary curve', demarcating the regions of stability and instability, and was compared with theoretical predictions. Ultimately, the parametric excitation will be combined with harmonic forcing in order to increase the Q-factor of the ring resonator by at least two orders of magnitude.

Keywords: parametric, resonance, electrostatic, harmonic balance, mems

1 INTRODUCTION

MEMS resonators are being developed today for a variety of applications. Some applications include inertial sensors, sensors for biological sensing, filters for RF applications and oscillators for timing applications.

First order parametric resonance occurs when a system parameter is modulated at a frequency very close to twice the natural frequency of the system.The phenomenon of parametric resonance has been observed and investigated extensively over the past century[1,2].Parametric resonances, owing to their unstable nature were previously treated as unwanted vibrations. Recently, the parametric instability phenomenon has been exploited for several applications. Parametric resonance for MEMS applications has been recently reported in [3, 4, 5, 6 and 7].

In this work, electrostatic transduction has been employed for both actuation and sensing of the MEMS resonator, thus making the excitation scheme suitable for many resonant MEMS sensors. One of the major issues facing MEMS sensors employing electrostatic transduction for both actuation and sensing is electrical feed-through. This corrupts the sense signal and limits the sensitivity of the sensor. A combined harmonic forcing and parametric excitation scheme increases the “effective” Q-factor thus permitting reduced forcing levels. Reducing the forcing level reduces the electrical feed-through at the forcing frequency by the same order. A control strategy for the combined harmonic forcing and parametric excitation scheme with the main aim of reducing feed-through for a MEMS gyroscope sensor has been proposed in [5]. However, this paper is focussed on experimental results obtained by the parametric-only excitation of the MEMS ring resonator.

Here, the MEMS ring resonator has been used as a vehicle to demonstrate the principle of parametric resonance. The ‘boundary curve’, which shows the relationship between the parametric excitation voltage and frequency, demarcating regions of stable and unstable vibration, was obtained experimentally. The equation of motion is shown to be of the form of an inhomogeneous Hill’s equation. The method of harmonic balance was used to analyse the equation and predict the ‘boundary curve’. It is shown that the theoretical predictions and experimental results demonstrate significant similarity.

2 THE MICRO-RING RESONATOR

Figure 1 shows the ring supported by a suspension. The ring has a radius a, width b and thickness d .The ring and the electrode encompassing an angle 2α are separated by a small air gap h_0. The ring is held at a fixed potential. Since $a >> h_0$ the capacitive plates formed between the biased ring and electrode may be considered flat. When a periodic voltage is applied to the electrode, a periodic electrostatic force results between the electrode and the ring and forces the ring into vibration. Table 1 shows the dimensions of the ring resonator.

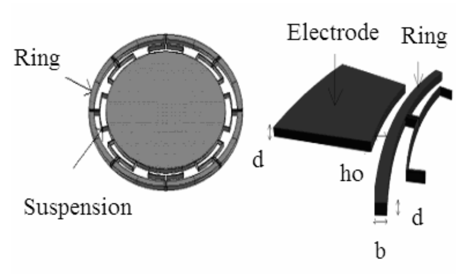

Figure 1: The ring resonator

a (mm)	4
b (μ m)	175
d (μ m)	100
h_0 (μ m)	5
α (rad)	$\pi/16$

Table 1: Ring resonator dimensions

The electrodes for actuation are chosen to excite the *n*=2 in-plane flexural mode of vibration of a thin circular ring. A detailed description of the electrode configuration for the micro-ring resonator can be found in [5, 6]. A photograph of the micro-ring resonator with the electrodes used for the drive and sense (differential sensing) is shown in Figure 2.

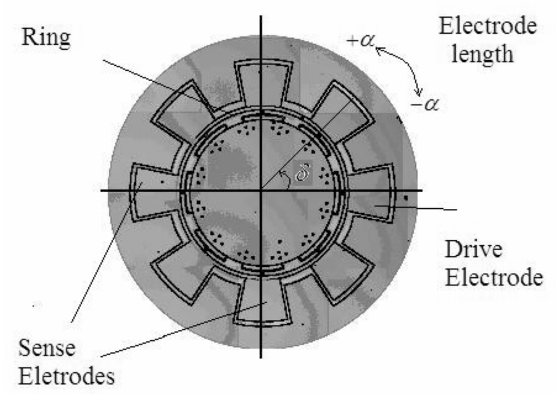

Figure 2: The drive and sense electrodes

Figure 3 shows the capacitive gap of the device between the ring and the electrode. The gap size is not constant. This may be due to a non-optimised etch process.

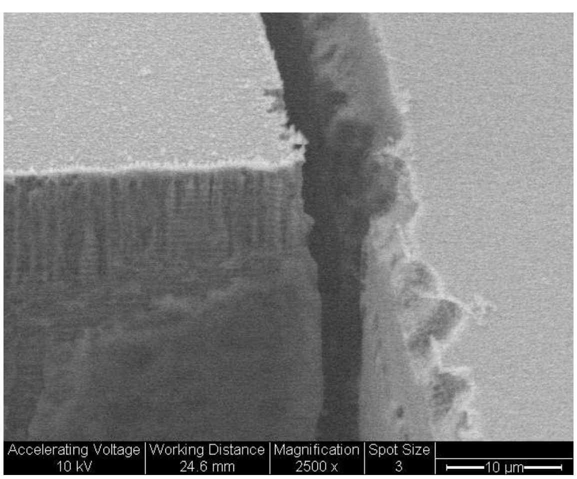

Figure 3: Gap between ring and electrode

3 EXPERIMENT

The MEMS ring resonator along with the associated sense circuitry was housed in a vacuum chamber. The pressure was maintained at 1*10^-3 mBar. Modal analysis of the resonator was performed electrically to determine the natural frequencies and the Q-factors. A *Data Physics* Dynamic Signal Analyzer was used for electrical characterization purposes. The values of the resonant frequency and the Q-factor of the *n*=2 flexural mode of vibration of the thin ring for an input excitation of amplitude 1Vpk were noted to be 18285 Hz and 1143 respectively. Figure 4 shows the frequency response of the MEMS resonator for an input excitation of 1Vpk.

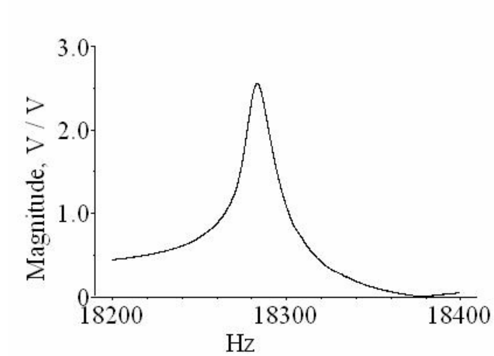

Figure 4: Frequency response of the MEMS ring resonator

The electrostatic force between the ring and the electrode is proportional to the square of the applied periodic voltage and may be shown to contain a term which results in periodic modulation in the ring stiffness. Parametric resonance can be achieved in such a system by ensuring that the frequency of oscillation ω of the excitation voltage is related to the natural frequency ω_α of the system by the condition

$$\omega \approx \frac{2\omega_\alpha}{l} \quad \text{where } l=1, 2, 3... \tag{1}$$

In this work, only *l*=1, i.e. first order parametric resonance has been considered and experimentally demonstrated. However, higher order parametric resonances can be realized experimentally and has been demonstrated in [7], but they would need higher excitation voltages and are not considered in the present experiment.

Capacitive sensing of the response vibration with the sense electrodes necessitates a fixed DC bias V_{dc} on the ring. The periodic voltage function for parametric excitation was chosen to be a sine waveform. A DC bias of 25 V was used in the experiment. The excitation voltage across the capacitive gap is of the form $V_{dc} + \eta \sin(\omega t)$.

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The electrostatic force being proportional to the square of the applied periodic voltage, would then be proportional to

$$V_{dc}{}^2+\frac{\eta^2}{2}\left(1+\cos(2\omega t)\right)+2V_{dc}\eta\sin(\omega t)$$

Frequency sweeps centered on approximately twice the measured resonant frequency were performed at various values of voltage amplitudes of the sine wave. This resulted in responses at half the excitation frequency. These were mapped to obtain the 'boundary curve'. It can be shown that this 'boundary curve' demarcates regions of stability and instability. The experimentally determined 'boundary curve' is shown in Figure 5. The circles in the figure represent the experimentally determined 'boundary curve '.

4 MODELLING AND ANALYSIS

The equation of motion of the micro-ring resonator may be shown to be of the form

$$m\ddot{q}+c\dot{q}+[k - K(t)]q=F(t). \tag{2}$$

The terms *m, k* and *c* are modal mass, stiffness and viscous damping terms of the ring. The terms *K(t)* and *F(t)* are the electrostatic stiffness and the forcing terms associated with electrostatic actuation and *K(t)q+ F(t)* constitutes the total electrostatic force due to the electrodes.

$$m=\rho Aa\pi\left(1+\frac{1}{n^2}\right)$$

$$c=2m\zeta\omega_\alpha$$

$$k=\frac{EI_z\pi}{a^3}\left(\left(1-n^2\right)^2\right)$$

$$K(t)=\frac{\varepsilon_0 ad}{h_0^3}\sum_{k=1}^{8}V(t)_k^2\left(\alpha+\frac{\cos(2n\delta_k)\sin(2\alpha)}{2n}\right)$$

$$F(t)=\frac{\varepsilon_0 ad}{nh_0^2}\sum_{k=1}^{8}V(t)_k^2\left(\frac{\cos(n\delta_k)\sin(n\alpha)}{n}\right)$$

Equation (2) represents the equation of motion for an actuated mode of vibration of the micro-ring resonator and is of the form of an inhomogeneous Hill's equation.

The periodic voltage excitation is of the form used in experiment

$$V(t)=V_{dc}+\eta\sin(\omega t) \tag{3}$$

There are various approximate methods to obtain the boundary of instability in a parametrically excited system. The method of harmonic balance is one such approximate method which has been used in the present analysis to predict the 'boundary curve'. This method is simple and particularly suitable for the case of simple parametric resonance dealt with here [8].

The method of harmonic balance involves assumption of a periodic solution, usually in the form of a Fourier series. Here, we assume the solution to be of the form

$$q(t)=A\cos(\frac{\omega}{2}t)+B\sin(\frac{\omega}{2}t) \tag{4}$$

Substitution of equations (3) and (4) into equation (2) and equating the coefficients of the harmonic components having frequency $\frac{\omega}{2}$ to zero yields

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{pmatrix}\begin{pmatrix} A \\ B\end{pmatrix}=0 \tag{5}$$

where

$$m_{11}=-\frac{\omega^2}{4}+\omega_\alpha^2-DV_{dc}{}^2\frac{\pi}{2}-DF\frac{\eta^2}{2}$$

$$m_{12}=\zeta\omega\omega_\alpha+DV_{dc}\eta F$$
$$m_{21}=-\zeta\omega\omega_\alpha+DV_{dc}\eta F$$

$$m_{22}=-\frac{\omega^2}{4}+\omega_\alpha^2-DV_{dc}{}^2\frac{\pi}{2}-DF\frac{\eta^2}{2}$$

D and *F* are geometrical constants. The condition for obtaining a non-trivial solution is

$$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{vmatrix}=0 \tag{6}$$

Equation (6) yields the relationship between the excitation frequency (ω) and excitation voltage (η). This relationship is plotted as the 'boundary curve' and is shown in Figure 5. The dots in Figure 5 indicate the analytically obtained boundary curve.

To obtain the analytical 'boundary curve' in Figure 5, a Q-factor ≈ 1140 and a natural frequency ≈ 18.3kHz was used. From Figure 3, it can be seen that the capacitive gap size is not constant and it was estimated to vary from 6.6-8.4 μm. The boundary curve shown in Figure 5 corresponds to the average capacitive gap size of 7.5 μm . It can be seen that the experimentally measured and predicted boundary curves are similar. As the amplitude of the parametric excitation described by $K(t)$ is inversely proportional to the cube of the capacitive gap, uncertainty in capacitive gap h_o must be minimised in order to improve the comparison. This requires optimisation of the etch process used to form the gap.

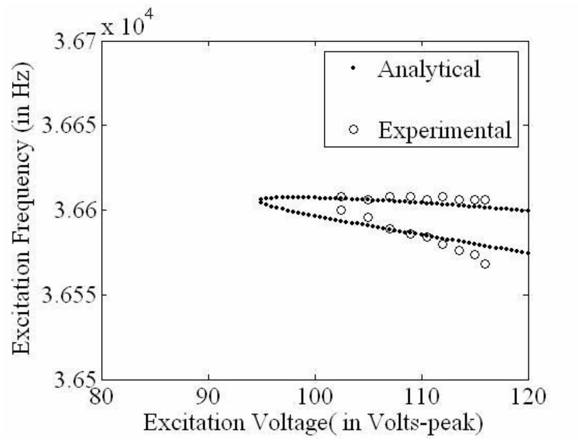

Figure 5: The 'boundary curve'

5 CONCLUSION

First order parametric resonance in a micro-ring resonator was realized experimentally and the 'boundary curve' was plotted. The equation of motion was analyzed using the method of harmonic balance, which proved to be a simple method to analyze first order parametric resonances. The boundary curve was obtained analytically. Owing to a largely varying capacitive gap between the ring and the electrode, it was shown that the experimental and the theoretical boundary curves agree to satisfactory level for the capacitive gap having an average value 7.5 μm. The analytical model for parametric-only excitation is therefore concluded to agree with experiment. Further work is underway to combine the parametric excitation with harmonic forcing in an attempt to controllably amplify the Q-factor of any mode of vibration of the ring resonator as shown through simulation in [5]. Fabrication of new devices with an optimized etch is also underway.

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