Analytical and Computational Solutions to Piezoelectric Bending: A Comparative Study

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ABSTRACT

Computational and analytical studies are conducted on piezoelectric beams of various materials in bending mode investigating the effect of variations in length and thickness on tip deflection that might be useful in MEMS actuators. Specifically, the materials considered are zinc oxide, poly vinylidene flouride, lead zirconate titanate, barium titanate, and piezoelectric sheet element. The computational solutions were obtained with the finite element analysis integrated into IntelliSuite (IntelliSense Corporation) software, and the analytical solutions were obtained with MATLAB. Results show that deflections are in closer agreement for beams of decreasing lengths and increasing thickness. It is noteworthy for the design of actuators that a large piezoelectric constant results in increased deflection.

Keywords: piezoelectric, beam, bending, analysis, computation, comparison.

1 INTRODUCTION

Microsystems are ubiquitous in main-stream engineering. In recent years, micro-dimensional beam-like structures have found useful applications in a variety of microactuators and microsensors. Analysis of micro-beams with different actuation methods such as electrostatic, magnetic and piezoelectric has been reported by Dufour and Sarraute [1]. Analysis of bimetallic beam by Timoshenko [2], modeling of piezoelectric microactuators by DeVoe and Pisano [3], derivation of equations for piezoelectric actuators by Weinberg [4] and Senturia [5] have laid the foundation for the present work.. In this work an attempt has been made to compare analytical and computational solutions of beam type piezoelectric actuators for various parameters such as length, thickness, Young's modulus, and piezoelectric constant. The variation in Young's modulus and piezoelectric constant is effected by a choice of different piezoelectric materials.

2 METHODS

The objective of the present work was to quantify the deflection of a piezoelectric beam for various beam lengths, thicknesses, and (piezoelectric) materials. The choice of materials has naturally offered a variety of Young's moduli and piezoelectric constants. The solutions for beam tip deflection by computational and analytical methods were compared for acceptance in design applications. Computational solutions were obtained by using the finite element analysis integrated into the IntelliSuite software and the analytical solutions were worked out using MATLAB for equations of piezoelectric actuators presented by Weinberg [4].

2.1 Piezoelectric Material Effect

A piezoelectric beam that measures 50 _m wide and 500 _m long was created. The beam was composed of a bilayer: 2 _m of silicon, and 2um of a piezoelectric material. While usually there is a thin metal lead to bring the electricity to the piezoelectric layer, this was assumed to be very thin, and omitted in the modeling. A voltage of +30volts was applied to the face of the piezoelectric material, and the silicon face was grounded.

The beam was meshed into 10 _m x 10 _m elements, and static analysis was performed using IntelliSuite's integrated Finite Element Analysis (FEA) software. The deformed structures were printed, and the maximum tip deflections were noted.

The aforementioned procedure was executed for each of the five materials presented in Table 1.

2.2 Effect of Beam Length Variation

The piezoelectric layer, layer thickness, beam width, and voltage were held constant while the length of the beam was increased. The width of the beam was 50 _m, and the length was varied from 200 _m to 800 _m. The bilayer was composed of 2 _m polysilicon and 2 _m of PZT (note the piezoelectric constant in Table 1), and the system was simulated at 30 volts.

Once again, the Thermo-Electro-Mechanical analysis component of IntelliSuite was used to perform FEA strain analysis on the beam that had been meshed into 10 um x 10um elements. Diagrams of the deflected beams were taken and the tip deflection was recorded for later analysis. A typical FEA output of the beam is shown in Figure 1.

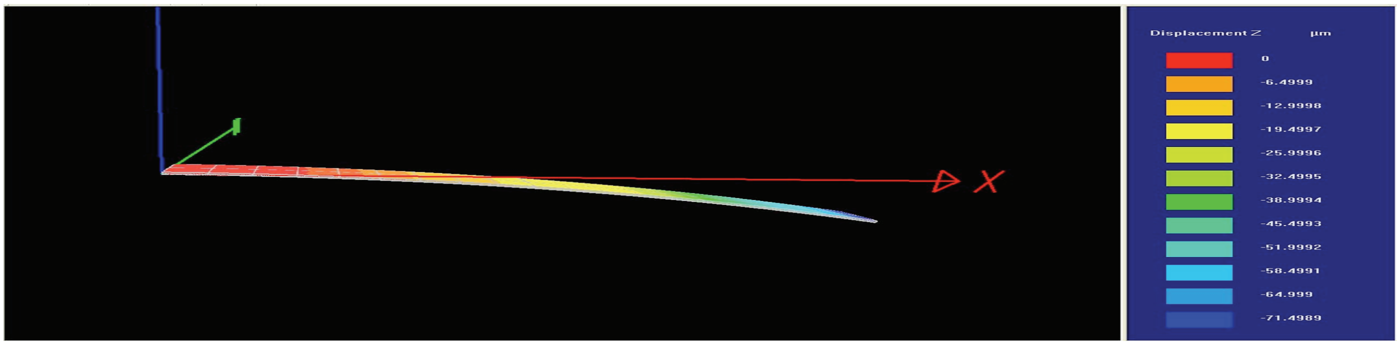

Figure 1: Typical output from FEA deformation prediction.

Table 1: Piezoelectric Materials and Associated Properties

Material	Young's Modulus (GPa)	Piezoelectric Constant (pm/V)	Reference Source
Zinc Oxide(ZnO)	12	12.4	IEEE Micro Electro Mechanical Systems Workshop,Jan-Feb 1991,Nara, Japan, p.118
Poly Vinylidene Flouride (PVDF)	2	23	IEEE 5th International Symposium on Micro Machine and Human Science Proceedings,Nagoya, Oct 1994, p.75
Lead Zirconate Titanate (PZT)	4.8	110	J.MEMS,DEC 1995,Vol.4,NO.4, p.234
Barium Titanate (BaTiO3),	67	78	IEEE 5th International Symposium on Micro Machine and Human Science Proceedings,Nagoya, Oct 1994, p.75
Piezoelectric sheet element	10	195	IEEE 5th International Symposium on Micro Machine and Human Science Proceedings,Nagoya,Oct 1994, p.48

2.3 Effect of Beam Thickness Variation

The thickness of the beam was varied and the beam deflection was determined. The piezoelectric PZT layer was kept at 2 _m (see Table 1 for piezoelectric constant), and the silicon base layer was varied from 0.5 um to 3.5 um by increments of 0.5 _m. The beams were meshed with 10 _m x 10 _m elements, and F E A analysis was performed by the Thermo-Electro-Mechanical module in IntelliSuite. The length and width of the beam were maintained at 500 _m and 50 _m, respectively.

2.4 Analytical solution

An analytical solution for a piezoelectric beam deflection was discussed in Refs. [1] and [5]. In these works are presented the mathematical models for piezoelectric actuators. The equation for vertical deflection is a second order differential equation which is solved here by using MATLAB. For the purpose of comparison, the solutions of the analytical equations were obtained and tip deflections were determined. Solutions were plotted in the MATLAB environment for the same parametric values considered in the finite element analysis of piezoelectric beams using the IntelliSuite software.

3 RESULTS AND DISCUSSION

Results for the beam material, length, and thickness variations are presented in Tables 1 through 3.

Comparison with the analytical solution is presented in Figures 2 through 7.

Both the piezoelectric constant and the Young's modulus of the piezoelectric layer impact beam deflection. If the piezoelectric constant is large, the material will deform more with the voltage input (30 V). If the Young's modulus of the piezoelectric layer is small compared to the silicon layer, the piezoelectric layer will compress instead of bending the beam. To achieve maximum bending, there must be a large piezoelectric constant and a large Young's modulus. The beam tip deflection vs. length for PZT on silicon is shown in Table 2. The beam deflections for various materials considered in this study are presented in Figure 2. Note that while the PZT had a relatively large piezoelectric constant, it compressed substantially due to its relatively low Young's modulus (4.8 GPa), and deflected the beam 24.64 _m as shown in Figure 2. The barium titanate had a lower piezoelectric constant (78pm/V vs. 110 pm/V for PZT), but was able to deflect the beam 51.1 _m (Fig. 2) due to its increased stiffness.

The longer the beam bilayer, the larger the tip deflection (Table 2). The analytical solution suggests that the deflection is proportional to the square of the length.

Table 2: Maximum deflection vs. beam length: finite element Method (Material: PZT on silicon)

Length(_m)	Tip deflection (_m)
200	11.37
300	24.815
400	43.26
500	65.16
600	92.3
700	121.84
800	154.28

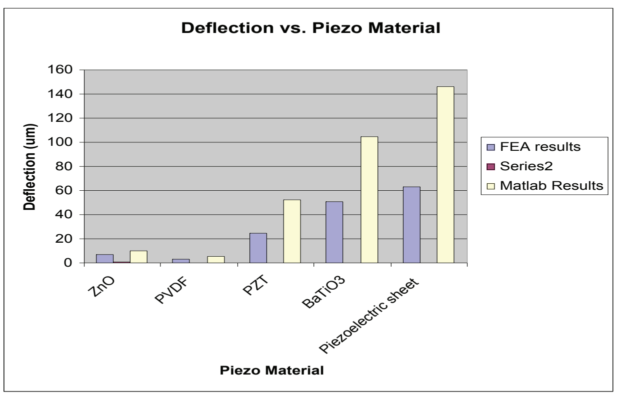

Figure 2: Deflection of beam vs. material type as modeled by FEA and MATLAB.

Table 3: Maximum deflection vs. beam thickness by finite element method (Material; PZT)

Thickness of silicon substrate layer(_m)	Tip deflection (_m)
0.5	54.78
1	71.49
1.5	70.6
2	65.16
2.5	58.64
3	49.41
3.5	46.9156

In Figure 3 is depicted the percentage error of MATLAB model compared to FEA for different piezoelectric materials. The deflection versus beam length for computational and analytical models is presented in Figure 4. These results show that there is an overestimation of the deflection as indicated by the diverging curves. A fitted power trend line with R^2 value of 0.9998 suggests that the deflection is more closely proportional to the length raised

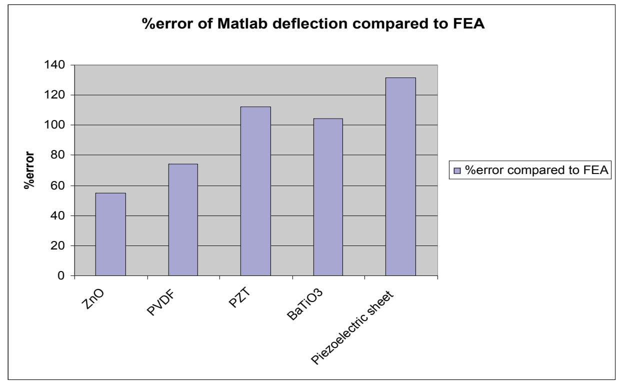

Figure 3: Percent error of Matlab model compared to FEA model for different piezoelectric materials

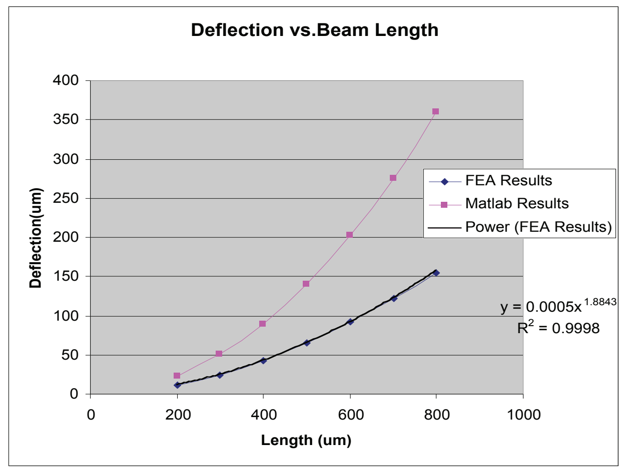

Figure 4: Deflection vs. beam length for computational and analytic model.

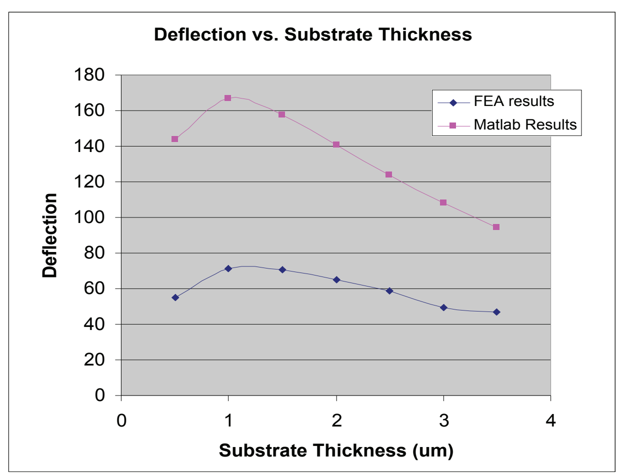

Figure 5: Deflection vs. beam thickness for computational and analytic model.

to the power of 1.884. In Table 3 is shown beam tip deflection as a function of the thickness of silicon substrate layer. For this study, 2 _m thick PZT was laminated with silicon layers ranging from 0.5 _m to 3.5

_m thickness. It can be observed in Figure 5 that as the thickness of the silicon layer increases, there is a rise in deflection followed by a gradual fall. The analytical (MATLAB) solution also shows this trend. The analytical and computational solutions are compared in Figure 5. It is noteworthy in Figures 4 and 5 that while the error between the analytical and computational solutions for deflection versus beam length increases (Figure 4) with increasing beam length, the error decreases for increasing beam thicknesses.

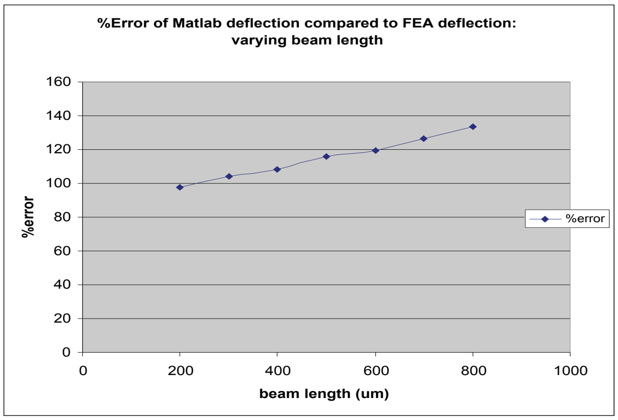

Figure 6: Percent error of analytic model over computational model varying length

The data from Figures 4 and 5 can be more concisely interpreted when percent error of analytic solution over computational one is expressed for varying length and varying thickness situations. The percent errors for the two cases are plotted and presented in Figures 6 and 7, respectively. In Figure 6, the error increases rather monotonically with respect to increasing beam length, while, as shown in Figure 7, the error decreases with respect to increasing thickness.

4 CONCLUSIONS

In this study, computational and analytical solutions for the end deflection for a piezoelectric beam were investigated using FEA integrated IntelliSuite software and analytical models in MATLAB environment. The analytical and

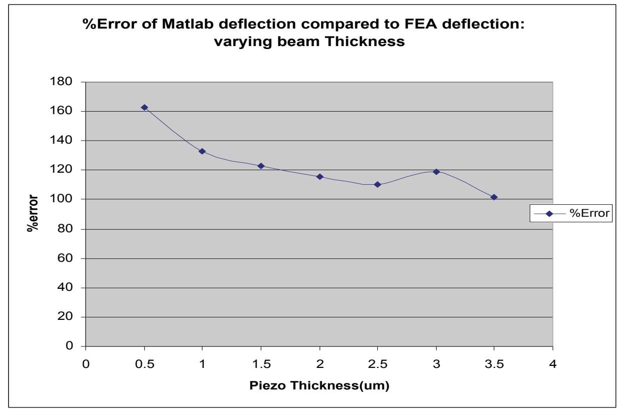

Figure 7: Percent error of analytic model over computational model varying thickness

computational solutions are most similar for short, thick beams. The impact of piezo- material properties (piezo-electric coefficient, stiffness or Young's modulus) on beam deflection were also investigated. Materials having large piezoelectric coefficients and large Young's moduli produced the largest end deflection that can be noted in the design of MEMS actuators and sensors.

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