Gummel Symmetry with Higher-order Derivatives in MOSFET Compact Models

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ABSTRACT

A new concept for the MOSFET saturation voltages at the drain and source sides referenced to ground is introduced, which are formulated and applied to the popularly-used smoothing functions for the effective drain–source voltage ($V_{ds,eff}$). The proposed model builds in physically source–bulk bias dependence in the surface-potential-based models employing the $V_{ds,eff}$ concept without the need for terminal-voltage swapping. The new model also resolves a key bottleneck in existing models for passing Gummel symmetry test (GST) in higher-order derivatives, which has to be traded off for geometry-dependent $V_{ds,eff}$ smoothing parameter.

Keywords: compact model, effective source–drain voltage, Gummel symmetry, higher-order derivatives, saturation voltage

1 INTRODUCTION

The necessary and sufficient conditions (NSC's) for a MOSFET drain-current, I_{ds} to pass the Gummel symmetry test (GST) [1]–[4] with n^{th}-order derivative are: $I_{ds}(V_{ds}) = -I_{ds}(-V_{ds})$ must be an odd function and the n^{th}-order derivative $d^{(n)}I_{ds}(0) \equiv d^nI_{ds}(V_{ds})/dV_{ds}^n|_{V_{ds}\to 0}$ exists (i.e., no singularity at $V_{ds} = 0$). GST requires a smooth transition in I_{ds} and its derivatives at $V_{ds} = 0$ when source and drain terminals are swapped with respect to (w.r.t.) ground.

It has been recognized [2] that Gummel symmetry was one of the main showstoppers in the industry *de facto* standard model BSIM [5]. Special care has been taken in passing the GST in the next-generation standard model PSP [6]. Gummel symmetry at higher-order derivatives, which is often required for distortion analyses [4,7], still remains a problem due to use of the mathematical smoothing function for the $V_{ds,eff}$, which has to be traded off for the appropriate range of the smoothing parameter for modeling geometry-dependent output (drain current and drain conductance) characteristics.

In this paper, we present a simple modification in the commonly-used smoothing functions for $V_{ds,eff}$, with the essential change from a "source-referenced" to a "ground-referenced" core model. We demonstrate that the proposed new model satisfies GST with any higher-order derivatives while maintaining physical scaling with terminal-voltage variations. In addition, terminal-voltage swapping of V_{ds} is not necessary in circuit implementation for the new model. This allows further extension of the intrinsic model to include source/drain asymmetry behavior of MOSFETs.

2 EXISTING AND PROPOSED MODELS

The MOSFET drain-current model is a function of all terminal voltages. With symmetrically linearized bulk charge approach (e.g., [6]), I_{ds} can be expressed as

$$I_{ds} = f\left(V_d, V_g, V_b, V_s\right)\cdot V_{ds,eff} = \overline{\beta}\left(\overline{V_{gt}} + \overline{A_b}v_{th}\right)\cdot V_{ds,eff}, \qquad (1)$$

where $\overline{V_{gt}}$ and $\overline{A_b}$ are the gate overdrive and bulk-charge factor at the source/drain surface-potential midpoint, respectively, $\overline{\beta} = \overline{\mu_{eff}}C_{ox}W/L$ is the gain factor, and v_{th} is the thermal voltage. $\overline{\mu_{eff}}$ as well as $\overline{V_{gt}}$ and $\overline{A_b}$ must be formulated to satisfy the NSC's for the GST. $V_{ds,eff}$ is then playing a key role in ensuring I_{ds} to pass the GST.

Two popular mathematical smoothing functions for $V_{ds,eff}$ have been used:

$$\begin{aligned}&\vartheta_1\left\{V_{ds}, V_{ds,sat}; \delta_s\right\} \\ &\equiv V_{ds,sat} - \frac{1}{2}\left(V_{ds,sat} - V_{ds} - \delta_s + \sqrt{\left(V_{ds,sat} - V_{ds} - \delta_s\right)^2 + 4\delta_s V_{ds,sat}}\right)\end{aligned} \qquad (2)$$

$$\vartheta_2\left\{V_{ds}, V_{ds,sat}; a_x\right\} \equiv \frac{V_{ds}}{\left[1 + \left(V_{ds}/V_{ds,sat}\right)^{a_x}\right]^{1/a_x}}. \qquad (3)$$

$\vartheta_1(x)$ in (2) is known to have caused asymmetry due to the fact that (2) is not an odd function of V_{ds} whereas $\vartheta_2(x)$ in (3) is strictly an odd function (when considering terminal-voltage swapping for negative V_{ds} as implemented in most circuit simulators). However, when GST at higher-order derivative is important, the $V_{ds,eff}$ function must have its higher-order derivative exist at $V_{ds} = 0$ [4]. It remains a problem for $\vartheta_2(x)$ due to the singularity that exists at $V_{ds} = 0$ for the n^{th}-order derivative when $a_x < n$. It has to be traded off for higher-order Gummel symmetry to cater for geometry-dependent a_x, in which a smaller value of a_x is usually required for short-channel devices, but larger value of a_x is required to satisfy higher-order GST. Therefore, it is highly desirable to have an odd function of $V_{ds,eff}$ that has infinite derivatives with no singularities at $V_{ds} = 0$.

In (2) and (3), $V_{ds,sat}$ is actually referenced to source. We propose a simple modification of (2), based on "ground-reference" for the new $V_{ds,eff}$, by defining $V_{ds,eff}$ as the difference of the effective drain and source voltages:

$$V_{ds,eff} = V_{deff} - V_{seff} = \vartheta_1\left\{V_d, V_{d,sat}; \delta_s\right\} - \vartheta_1\left\{V_s, V_{s,sat}; \delta_s\right\}, \quad (4)$$

in which V_d and V_s are the drain and source terminal voltages, respectively, and $V_{d,sat}$ and $V_{s,sat}$ are the corresponding saturation voltages, all referenced to ground.

We take the conventional threshold-voltage-based approach [8] to show the derivations of the saturation voltages used in (4). The saturation voltage is defined as the channel voltage where I_{ds} reaches the saturation current, I_{dsat} (i.e., reaching saturation velocity, v_{sat}). The saturation current (after Taylor expansion at the source end) can be written as

$$I_{dsat} = v_{sat} W C_{ox}\left(V_{gt,s} - A_{b,s}\Delta\phi\right), \quad (5)$$

where $V_{gt,s} = Q_{i,s}/C_{ox}$ is the inversion charge normalized to the oxide capacitance C_{ox}, evaluated at the source end

$$V_{gt,s} = \Upsilon\sqrt{\phi_s(0) + v_{th}e^{\left[\phi_s(0) - 2\phi_F - V_{sb}\right]/v_{th}}} - \Upsilon\sqrt{\phi_s(0)}\,. \quad (6)$$

$$A_{b,s} = 1 + \Upsilon\Big/\left(2\sqrt{\phi_s(0)}\right) \quad (7)$$

is the bulk-charge factor, and $\Delta\phi = \phi_s(L) - \phi_s(0)$ is the surface potential difference between drain $\phi_s(L)$ and source $\phi_s(0)$. $\Upsilon = (2q\varepsilon_{Si}N_{ch})^{1/2}/C_{ox}$ is the body factor, ϕ_F is the bulk Fermi potential, and v_{th} is the thermal voltage. The drift current and diffusion current (after Taylor expansion at the source end) is given by

$$I_{dd,s} = \mu_{eff0} C_{ox}\frac{W}{L}\left(V_{gt,s} - \frac{A_{b,s}\Delta\phi}{2} + A_{b,s}v_{th}\right)\Delta\phi\,. \quad (8)$$

where μ_{eff0} is the lateral-field mobility based on the piecewise velocity–field relation, which also includes the vertical-field mobility [9]. Based on the "pinned" surface-potential approximations, $\phi_s(y) \approx 2\phi_F + V_{cb}(y)$ where V_{cb} is the channel voltage w.r.t. bulk, $\Delta\phi$ can be approximated as

$$\Delta\phi \approx \left(2\phi_F + V_{db}\right) - \left(2\phi_F + V_{sb}\right) = V_{ds}\,. \quad (9)$$

Solving (5) and (8) at $V_{ds} = V_{ds,sat}$ together with (9), the saturation voltage is obtained as

$$V_{ds,sat} = \frac{V_{gt,s}LE_{sat}}{V_{gt,s} + A_{b,s}LE_{sat} + 2A_{b,s}v_{th}}, \quad (10)$$

in which the $2A_{b,s}v_{th}$ term in the denominator comes from the diffusion current. The above derivation is in fact "source-referenced" as Taylor expansion is done at the source end. The onset of saturation actually happens only at the drain side; therefore, from the convention $V_{ds} = V_d - V_s$, it is possible to define the "saturation drain voltage" w.r.t. ground as:

$$V_{d,sat} = V_{ds,sat} + V_s, \quad (11a)$$

in which $V_{ds,sat}$ is the drain–source saturation voltage (referenced at the source side). Similarly, an equivalent analysis for the drain side leads to

$$V_{s,sat} = V_{sd,sat} + V_d \quad (11b)$$

together with a set of equations similar to (5)–(10) referenced at the drain end. This makes $V_{ds,eff}$ in (4) strictly an odd function of V_{ds}, while retaining no singularity for any higher-order derivatives.

In fact, if the same idea is applied to the smoothing function (3), as below:

$$V_{dse} = V_{de} - V_{se} = \vartheta_2\left\{V_d, V_{d,sat}; a_x\right\} - \vartheta_2\left\{V_s, V_{s,sat}; a_x\right\}, \quad (12)$$

it is also possible to remove the singularity at higher-order derivatives for small values of a_x since the term that causes singularity at higher-order derivative is canceled in the subtraction. However, it is easy to see that the square-root function in (4) is much less computational expensive compared to the power function in (12), which is the preferred model.

The new "ground-referenced" formulation has the correct terminal-voltage scaling built into the new model in (4). V_{deff} and V_{seff} are readily to be used for the evaluation of the effective surface potentials in a surface-potential-based I_{ds} model. In conventional "source-referenced" formulation, the source and drain terminal voltages have to be swapped (as is normally done in circuit implementation) for model evaluation in forward-biased junction simulation when V_{ds} is less than zero. Such implementation is not required with the new model, as V_{deff} and V_{seff} are "ground-referenced" and able to give the right behavior even for forward-biased condition in which V_{ds} is negative.

Other sources of asymmetry may arise if velocity saturation, effective mobility and effective field are not properly formulated for Gummel symmetry. In this paper, the long/short-channel model has been carefully formulated to satisfy the NSC's for the GST, details of which will be presented elsewhere.

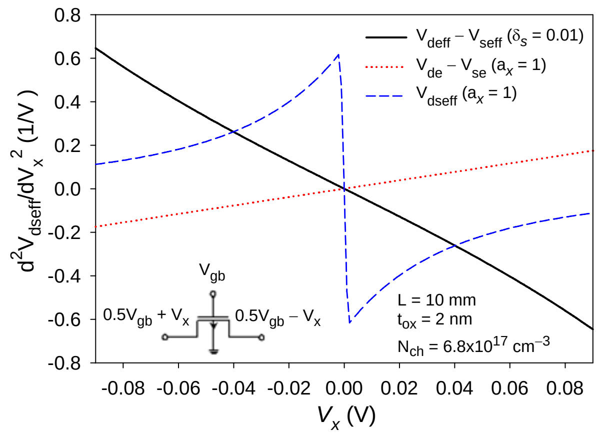

Figure 1: GST for the new function (4) with $\delta_s = 0.01$ (solid line). Singularity of the existing function (3) for $a_x = 1$ (dashed line), which is removed using (12) (dotted line). The inset shows the GST circuit.

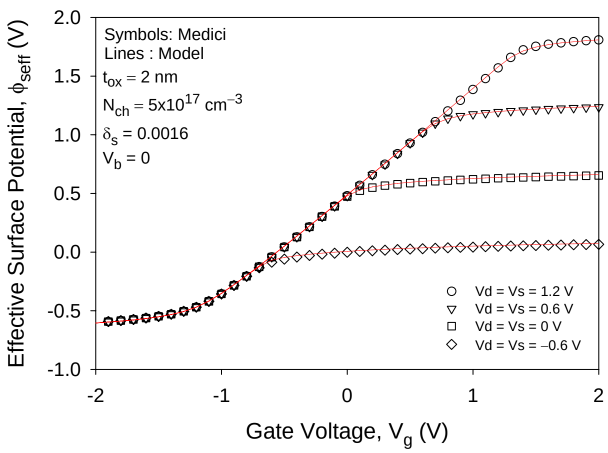

Figure 2: Comparison of effective surface potentials using V_{deff} (and V_{seff}) at drain (and source) side (lines) together with numerical data (symbols), from –0.6 V to 1.2 V.

3 RESULTS AND DISCUSSION

GST on the newly proposed (4) and (12) are shown in Fig. 1, and compared with (3). The inset shows the GST circuit. It can be seen that the new model (4) meets the requirement of GST and the singularity arising from (3) has been removed in (12) even for a small value of $a_x = 1$ in the second-order derivative of the $V_{ds,eff}$ functions. This shows that the new models (4) and (12) are symmetrical when swapping the source/drain terminal without any constraint on the $V_{ds,eff}$ smoothing parameters.

Besides GST requirement, one of the other key concerns is the model dependence on the terminal voltages. The model has to behave physically/correctly in all operating regions. If all the terminal (absolute) voltages are formulated consistently with the common reference (ground), the model will have physical behavior with the correct terminal-voltage dependence. To demonstrate this, we compute the effective surface potentials through Newton–Raphson (NR) by replacing the V_{cb} term with the proposed $V_{deff} - V_b$ and $V_{seff} - V_b$ in the Pao–Sah implicit voltage equation [10] and compare with numerical data (MEDICI), as shown in Fig. 2. The same voltage is applied at both source and drain so as to extract the surface potentials with different quasi-Fermi level from the channel center of the numerical device. It can be seen that the new model matches the numerical data accurately for all biases, including the forward bias (calculated with negative V_{ds} without source/drain terminal swapping), which can show unusual behavior if it is not handled properly.

The effective surface potentials with the new V_{deff} and V_{seff} in (4) are used in a surface-potential-based I_{ds} model. The I_{ds} model with terminal-voltage variations for $V_{d,}$, V_s, and V_b of both subthreshold and linear current are shown in Fig. 3 in comparison with numerical data. The numerical data for V_b variations are depicted in open symbols for $V_b = 0$ to -1.2 V with step of -0.4 V. Numerical data for V_{ds} variations are depicted in solid symbols from $V_d = 1.2$ V ($V_s = 0.3$ V) to $V_d = 1.8$ V ($V_s = 0.6$ V) with step of 0.3 V. The model shows a very good physical behavior following the data accurately.

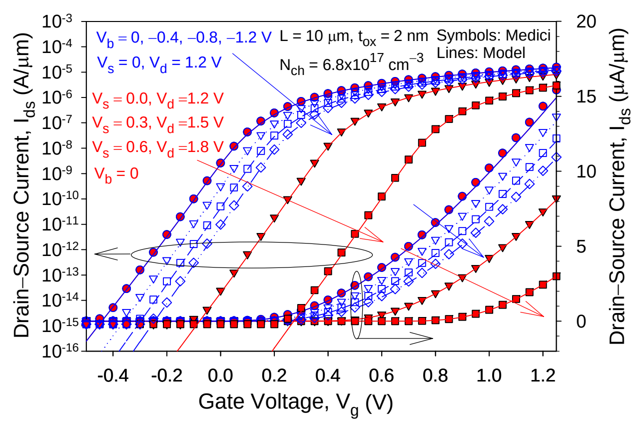

Figure 3: Comparison of the I_{ds} model (lines) with numerical data (symbols) for V_b, V_d, and V_s variations, showing subthreshold (left axis) and strong-inversion (right axis) currents at a fixed $V_d = 1.2$ V ($V_s = 0$) for $V_b = 0, -0.4, -0.8, -1.2$ V (open symbols) and varying $V_d = 1.2$ V ($V_s = 0$ V), $V_d = 1.5$ V ($V_s = 0.3$ V) and $V_d = 1.8$ V ($V_s = 0.6$ V) at a fixed $V_b = 0$ (solid symbols).

Finally, correct physical behavior for GST on course-grid and large-voltage range in saturation is equally important as for fine-grid and small-voltage range in linear operation. The I_{ds} model with the proposed (4) has been verified for GST by sweeping through $V_x = 0$ *without terminal swapping* for both long and short-channel devices from a 90-nm technology (numerical) data, as shown in Fig.

4. The inset shows the symmetry in the 6th-order derivative of the corresponding devices, which is not possible if using the existing $V_{ds,eff}$ due to its singularity at high-order derivatives. This shows that the new $V_{ds,eff}$ model meets the GST requirement in higher-order derivatives for both long- and short-channel devices, without the constraint on the value of the smoothing parameter to trade off for its Gummel symmetry.

In those models in which negative V_{ds} cannot be used in model evaluations, it is usually implemented with source/drain-terminal swapping at the circuit level based on MOSFET source/drain structural symmetry. In our "ground-referenced" core model, negative V_{ds} sweeping is allowed as shown in Fig. 4, which also gives *identical* results if the source/drain terminals are swapped for negative V_{ds}. This feature allows the model to be applied for forward S/D-junction bias analysis including layout-dependent asymmetric source/drain structures, which is not possible with source/drain-terminal swapping.

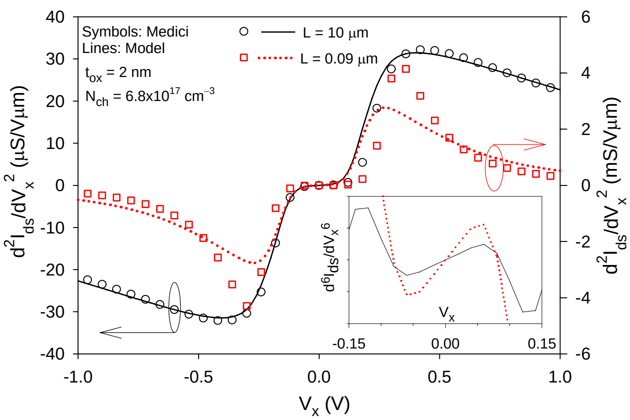

Figure 4: Comparison of the I_{ds} model (lines) for GST with numerical data (symbols) (V_{gb} = 1.2 V, $V_d = V_{gb} + V_x$, $V_s = V_{gb} - V_x$, step size: 0.02 V). Inset: 6th-order derivative of the I_{ds} model.

4 CONCLUSIONS

In conclusion, one of the critical problems in Gummel symmetry at higher-order derivatives that still exists in current and next-generation MOS compact models has been solved with a simple modification of the mathematical smoothing function used for the effective drain–source voltage. The proposed approach of formulating the $V_{ds,eff}$ function builds in terminal-voltage dependency and is always symmetrical regardless of the value of the smoothing parameter. Together with other necessary and sufficient conditions to meet the model symmetry requirements, the proposed model has made a critical milestone contribution to resolve one of the key showstoppers in the current industry-standard MOS model, as well as for next-generation model to trade off symmetry at higher-order derivatives and geometry-dependent effective drain–source voltage smoothing parameter. The ground-referenced V_{deff} and V_{seff} implementation also allows the intrinsic model to be extended to include modeling of asymmetric MOSFET structures for which a model implemented with source/drain-terminal swapping cannot achieve.

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