Numerical Analysis of Nonlinear Deformation and Breakup of Slender Microjets with Application to Continuous Inkjet Printing

E. P. Furlani and K. C. Ng

Eastman Kodak Research Laboratories
1999 Lake Avenue, Rochester, New York 14650-2216
edward.furlani@kodak.com, kam.ng@kodak.com

ABSTRACT

We present a model for predicting the instability and breakup of modulated microjets, and we discuss applications of the model to high-speed inkjet printing. The model is based on a one-dimensional slender-jet analysis and takes into account the coupled thermal/fluidic behavior of the jet. We solve the nonlinear slender jet equations using the method of lines to predict the free-surface, velocity and temperature along the jet. We apply the model to study high-speed continuous inkjet printing.

Keywords: thermo-capillary instability, continuous inkjet printing, slender jet analysis, microjet instability, Marangoni instability

1 INTRODUCTION

The instability of slender liquid cylinders and jets has been subject of countless research papers dating back to the initial studies of Lord Rayleigh [1]. In addition to the academic interest in this phenomenon, there is considerable industrial interest, especially in the field of high-speed inkjet printing. In printing applications, liquid microjets are modulated in a controlled fashion to create steady streams of picoliter-sized droplets at frequency rates that can exceed 500 kHz. In our lab, integrated microfluidic inkjet devices have been developed that utilize thermally modulated jets to enable color printing with unprecedented speed and versatility [2-5]. These devices consist of a pressurized reservoir that feeds a microfluidic nozzle manifold with hundreds of active orifices, each of which produces a continuous microjet of fluid. Controlled thermal modulation of each jet is achieved using CMOS/MEMS technology wherein a resistive heater element is integrated into the nozzle surrounding each orifice. To modulate a jet, a periodic voltage is applied to the heater, which causes a periodic diffusion of thermal energy from the heater into the fluid near the orifice (Fig. 1). Thus, the temperature of fluid, and hence the temperature dependent fluid properties, density, viscosity and surface tension, are modulated near the orifice. The dominant cause of jet instability is the modulation of surface tension. To first order, the temperature dependence of σ is given by $\sigma(T) = \sigma_0 - \beta(T - T_0)$, where $\sigma(T)$ and σ_0 are the surface tension at temperatures T and T_0, respectively. The pulsed heating modulates σ at a wavelength $\lambda = v_0\tau$, where v_0 is the jet velocity and τ is the period of the heat pulse as shown in Fig. 1. The down-stream advection of thermal energy gives rise to a spatial variation (gradient) of surface tension along the jet. This produces a shear stress at the free-surface, which is balanced by inertial forces in the fluid, thereby inducing a Marangoni flow towards regions higher surface tension (from warmer regions towards cooler regions). This causes a deformation of the free-surface (slight necking in the warmer regions and ballooning in the cooler regions) that ultimately leads to instability and drop formation [3]. The drop volume can be adjusted on demand by varying τ, i.e., $V_{drop} = \pi r_0^2 v_o \tau$. Thus, longer pulses produce larger drops, shorter pulses produce smaller drops, and different sized drops can be produced from each orifice as desired.

The design of continuous inkjet printing devices typically involves time consuming CFD analysis [4,5].

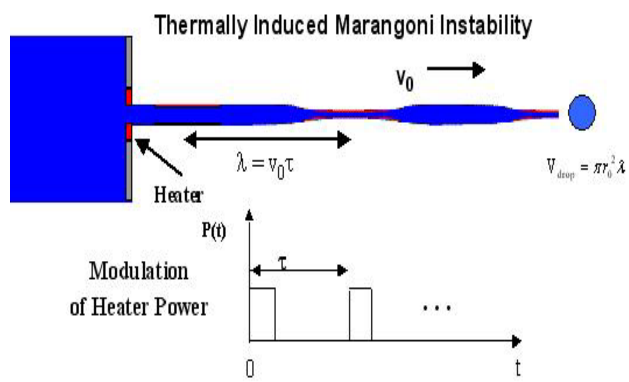

Figure 1: Illustration showing a single nozzle with an integrated heater at the orifice, and a thermal modulation pulse used to induce Marangoni instability and drop formation.

NSTI-Nanotech 2007, www.nsti.org, ISBN 1420061844 Vol. 3, 2007

In this paper, we present a model for performing rapid parametric analysis of jet instability and droplet generation that is useful for the design of inkjet printing devices. The model predicts the nonlinear deformation and breakup of modulated slender liquid cylinders and microjets. We adopt a one-dimensional slender-jet approximation, and model the behavior of the jet using a system of coupled thermal/fluidic PDEs. We use the method of lines to reduce the PDEs to a system of ODEs, We solve for the free-surface, velocity, and temperature along the jet. A typical analysis takes only a few minutes to run on a workstation.

2 EQUATIONS OF MOTION

The equations governing the behavior of a non-isothermal viscous microjet of an incompressible Newtonian fluid with surface tension σ, viscosity μ, density ρ, specific heat c_p, and thermal conductivity k, are as follows:

Navier-Stokes:

$$\rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 v, \tag{1}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + v \bullet \nabla$.

Thermal:

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T. \tag{2}$$

Continuity:

$$\nabla \bullet v = 0. \tag{3}$$

Boundary Conditions:

Thermal:

$$-k \hat{n} \bullet \nabla T = h_c (T - T_\infty). \tag{4}$$

Normal Stress:

$$(\mathbb{T} \bullet \hat{n}) \bullet \hat{n} = -2H\sigma . \tag{5}$$

Tangential Stress:

$$(\mathbb{T} \bullet \hat{n}) \bullet \hat{t} = \hat{t} \bullet \nabla_s \sigma . \tag{6}$$

Kinematic (at jet surface):

$$\frac{D}{Dt}\left(r_s - h(z,t)\right) = 0 . \tag{7}$$

On axis (r = 0):

$$v_r = \frac{\partial v_z}{\partial r} = \frac{\partial T}{\partial r} = 0 , \tag{8}$$

where v, p, and T are the velocity, pressure and temperature distributions along the microjet, v_r and v_z are the radial and axial velocity components, $h(z,t)$ defines the free-surface, $\hat{n}$ and $\hat{t}$ are unit vectors normal and tangential to the free-surface, ∇_s is the gradient operator along the free-surface, $\mathbb{T}$ is the stress tensor, and h_c is the coefficient for thermal convection off the free-surface. The function H is given by

$$H = \frac{1}{2}\left(\frac{1}{h(1+h'^2)^{1/2}} - \frac{h''}{(1+h'^2)^{3/2}} \right), \tag{9}$$

where $h' = \partial_z h$. Equations (1) - (8) need to be solved subject to appropriate boundary conditions.

3 SLENDER JET ANALYSIS

For slender microjets, equations (1)-(8) can be simplified using a perturbation expansion in r for the unknown variables h, T and v, and retaining the lowest order terms [3]. This leads to the following 1-D slender jet equations:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial(2\sigma H)}{\partial z} + \frac{3\mu}{\rho h^2}\frac{\partial}{\partial z}\left(h^2 \frac{\partial v}{\partial z}\right) + \frac{2}{\rho h}\frac{\partial \sigma}{\partial z}, \tag{10}$$

$$\frac{\partial h^2}{\partial t} = -\frac{\partial\left(h^2 v\right)}{\partial z}, \tag{11}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial z} = \alpha \frac{1}{h^2}\frac{\partial}{\partial z}\left(h^2 \frac{\partial T}{\partial z}\right) - \frac{2\alpha h_c}{hk}\left(T - T_\infty\right). \tag{12}$$

In this approximation, there is thermal diffusion along the axis of the microjet and convection from its surface, but no radial thermal diffusion (1-D approximation).

We solve the slender jet equations (10) - (12) (subject to appropriate boundary conditions) using the method of lines (MOL). The MOL is typically implemented using finite differences for the spatial derivatives and ordinary differential equations for the time derivatives. We use a uniform staggered computational grid for the finite differences where h, p, and T are evaluated on one set of nodes, and the velocity v is computed on interlaced nodes midway between the first set (Fig. 2). Thus, in the MOL approach, Eq. (11) reduces to a system of N ODEs of the form

$$\frac{\partial h_i}{\partial t} = -\frac{h_{i+1/2}v_i - h_{i-1/2}v_{i-1}}{2h_i \Delta z} \qquad \left(1 \le i \le N\right) \quad (13)$$

where N is the number of nodes, and $h_{i+1/2} = \frac{1}{2}\left(h_i + h_{i+1}\right)$, $h_{i-1/2} = \frac{1}{2}\left(h_i + h_{i-1}\right)$. A similar system of ODEs is obtained for both Eqs. (10) and (12), and therefore a total of number of approximately 3N ODEs need to be solved for each simulation. Furthermore, one needs to apply a numerical upwind differencing scheme for the accurate solution of Eqs. (10) and (12).

We have implemented the MOL in MATLAB using the ODE solver routines for our numerical studies. We

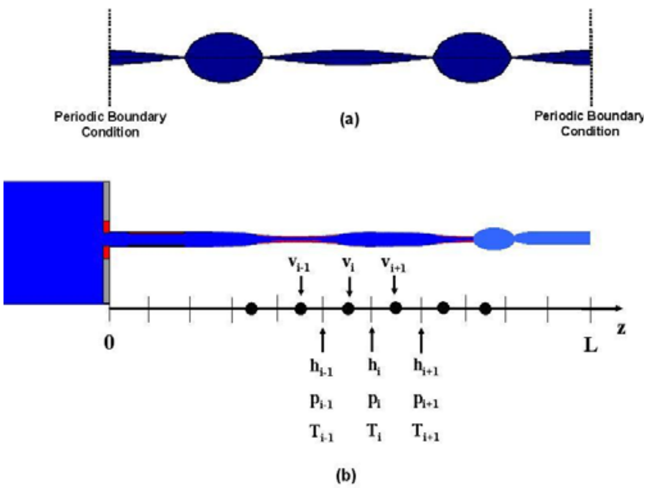

Figure 2: Staggered computational grid: (a) infinite cylinder at pinch-off with periodic boundary conditions, (b) schematic of nozzle driven microjet.

developed models to study both infinite cylinders of fluid with periodic modulation, and nozzle driven microjets wherein the modulation is applied in a time-wise fashion at the orifice, and then convected downstream. Our models take into account temperature dependent fluid properties such as surface tension.

We applied our model to an infinite cylinder (thread) of fluid that has the properties of water and an unperturbed radius of r_0 = 5 μm. We imposed a periodic thermal gradient along the length of the cylinder via an initial temperature distribution of the form,

$$T(z) = T_0 + \Delta T \cos(2\pi z / \lambda). \quad (14)$$

There was no initial deformation to the free-surface, and so the capillary instability was induced purely by the Marangoni effect. The velocity, temperature and free-surface of the jet at pinch-off are shown in Fig. 3 for λ = $9r_0$. It is instructive to note that the cylinder balloons at the cool regions, which is consistent with Marangoni instability as described above. A similar analysis with λ = $18r_0$ is shown in Fig 4. Notice that at longer wavelengths the volume of fluid in the main lobes decreases as the volume in the connecting filaments increases. Parametric analysis of these systems was completed in a few minutes.

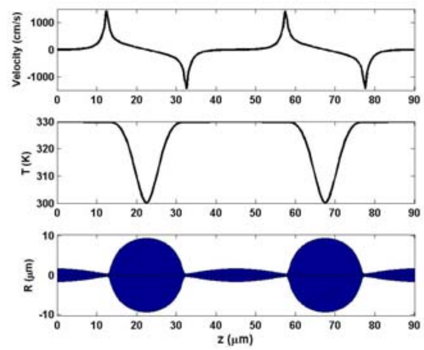

Figure 3: Modulated fluid cylinder at pinch-off, λ = $9r_0$.

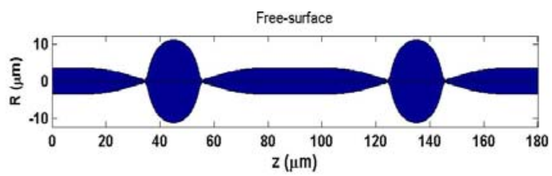

Figure 4: Modulated fluid cylinder at pinch-off, λ = $18r_0$.

We performed a similar analysis for nozzle driven microjets. In this case, the modulation was in the form of a sequence of thermal pulses applied at the orifice. These induced Marangoni instability and pinch-off. We were able to compare the predicted jet profile at pinch-off with measured data in the form of a high-speed CCD image of a strobbed microjet. The comparison is shown in Fig. 5.

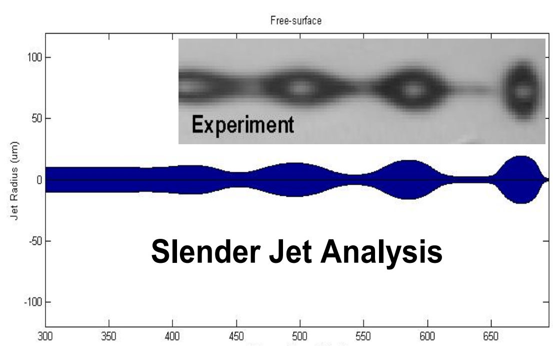

Figure 5: Comparison of predicted and observed pinch-off.

NSTI-Nanotech 2007, www.nsti.org, ISBN 1420061844 Vol. 3, 2007

4 APPLICATION TO INKJET PRINTING

The model presented above has been used in the design of an integrated microfluidic nozzle manifold for continuous inkjet printing applications. This device, which has been fabricated and characterized in our lab, consists of hundreds of active micro-orifices (Fig. 6), each of which can be individually thermally modulated to produce either large or small drops, depending on the image content. The drop size is controlled by increasing or decreasing τ, the period of the thermal pulsing The large drops are used for printing, while the small drops are deflected away from the print media using air flow and then recycled (Figs. 7 and 8).

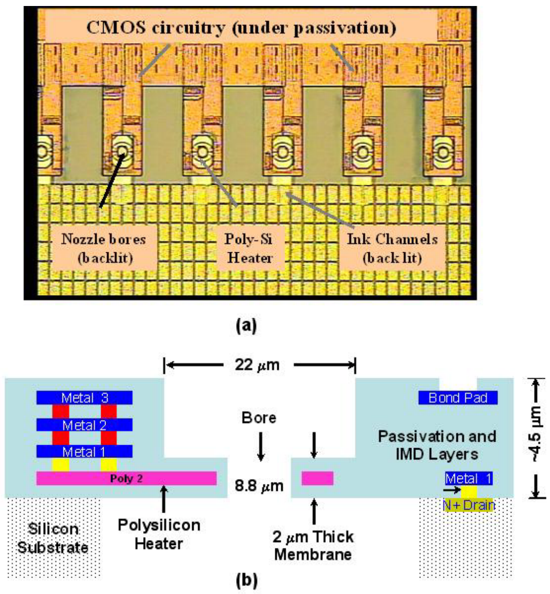

Figure 6: Fabricated inkjet printhead: (a) top surface of linear array of nozzles, (b) cross-section of nozzle/heater structure showing CMOS circuitry.

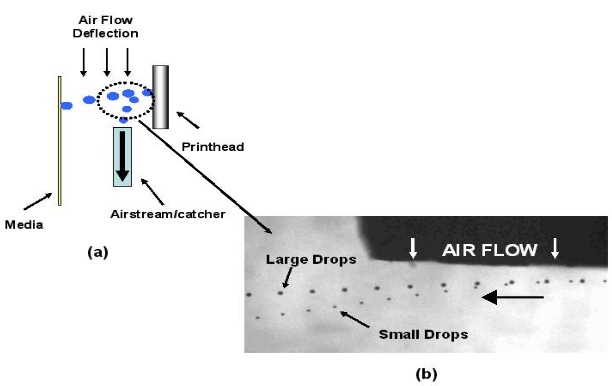

Figure 7: Continuous inkjet printing: (a) schematic showing drop size selective printing (large drops are used for printing, small drops are deflected using controlled air flow), (b) experimental picture showing airflow induced separation of large and small drops.

Figure 8: Color Picture printed using microfluidic printhead with the print media moving at 1m/s.

5 CONCLUSIONS

We have presented a model for predicting the nonlinear deformation and pinch-off of slender fluid cylinders and microjets. The model is easily programmed and well suited for parametric analysis. It has proven to be useful for the design and analysis of high-speed continuous inkjet printing devices.

REFERENCES

[1] Lord Rayleigh, “On the instability of jets,” Proc. Lond. Math. Soc. **10**, 4-13, 1878.

[2] J. M. Chwalek, D. P. Trauernicht, C. N. Delametter, R. Sharma, D. L. Jeanmaire, C. N. Anagnostopoulos, G. A. Hawkins, B. Ambravaneswaran, J. C. Panditaratne, and O. A. Basaran, “A new method for deflecting liquid microjets,” Phys. of Fluids **14**, 6, 37-40, 2002.

[3] E. P. Furlani, “Temporal instability of viscous liquid microjets with spatially varying surface tension,” J. Phys. A: Math. and Gen. **38**, 263-276, 2005.

[4] E. P. Furlani, "Thermal Modulation and Instability of Viscous Microjets", proc. NSTI Nanotechnology Conference 2005.

[5] E. P. Furlani, B. G. Price, G. Hawkins, and A. G. Lopez, "Thermally Induced Marangoni Instability of Liquid Microjets with Application to Continuous Inkjet Printing", Proc. NSTI Nanotechnology Conference, 2006.