A Novel Topology Optimization Scheme for Micro Electro-Thermo-Mechanical Domains

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ABSTRACT

Developing a systematic approach to design optimum Micro-Electro-Mechanical Systems, MEMS, recently has received a numerous attention among researchers [1-3]. Duo to the lack of a systematic design method, typical MEMS devices design procedures encompass trial and errors and are based on designer intuition. Obviously such structures cannot be optimum and until now no methods have been able to predict better designs for MEMS. In this paper, a new topology optimization method is developed for the systematic design of electro-thermo-mechanical devices with embedded actuation scheme.

Keywords: topology optimization, MEMS, electro-thermal actuators, genetic algorithm

1 INTRODCTION

The topology optimization method distributes finite number of elements in a design domain, such that some output performance is optimized. Topology optimization techniques enable systematic design directly from the behavioral specification. That is, the size, number and connectivity of elements in the structure and its entire shape are obtained as an original new solution such that the performance criterion called objective function is maximized, without relying upon human intuition. The topology optimization method was originally developed for large scale structural design problems with sole mechanical boundaries and is an iterative numerical method.

This paper improves the classical topology optimization of structures with mechanical boundaries to MEMS devices in which a combined mechanical, electrical, and thermal boundaries co-exit. The optimized solution depends on objective function and lower limits on design variables. Here a new objective function is introduced to incorporate mixed boundary effects. The MEMS fabrication process limits are included in optimization constraints as lower and upper bounds on design variables. Also a new algorithmic approach to deal with multi-disciplinary domains is presented. Furthermore, the current method in the optimization of mechanical compliant mechanisms has been improved by the application of beam elements and implementation of the genetic algorithm.

A graphical user interface, GUI, is designed to facilitate the user interface, to control and trace the optimization procedure and to visualize the optimized solution. Several examples of optimized designs are included. The optimum structures are presented and compared with the examples from literature. We also show how the topology-dependent modeling such as heat transfer can be dealt with in a topology design technique where the number and size of elements are varied freely.

Figure 1 (a) shows a general mixed boundary domain for this type of problems. Figure 1(b) is the defined domain and boundaries for an electro-thermal actuator. Applying the voltage at one end, displacement at the shown direction at output port is desired.

Figure 1 (c) and (d) show the reported optimized material distribution and the deformed shape of actuator, respectively [2]. As seen here, the resulted topology consist of several small flexural hinges which are prone to high stress, fatigue and failure and long slender beams which go under buckling at very small forces. Computationally, the homogenization method is very expensive and involves large number of elements and iterations to converge. Moreover, it generates the checker board regions in the solution as shown in Figure 1(d) [3]. Checkerboards are the areas of disconnected gray cubes of material which need an additional filtering to interpolate void or material. The reported optimized structures in literature, using homogenization method need extra filtering schemes to be feasible to fabricate [2] and this adds to the complexity and load of calculations in this method, yet the structures do not seem very efficient.

2 PROBLEM STATEMENT

A quite general problem in micro actuator MEMS design is to convert an electrical input to a mechanical output, e.g. voltage to displacement. In the electro-thermo-mechanical actuation principle an electrical current is converted to heat by Joule's heating and the heat then causes thermal strain, which in turn causes structural deformation. How to design an efficient electro-thermo-mechanical actuator is obviously a complicated task.

The first step in the topology optimization procedure is to develop a model for the evaluation of the mechanical response of the micro actuator subject to applied electric or thermal field. Since the model for uniform heating is a simplification of the electro-thermal model, only the latter will be described here. Assuming that the geometrical

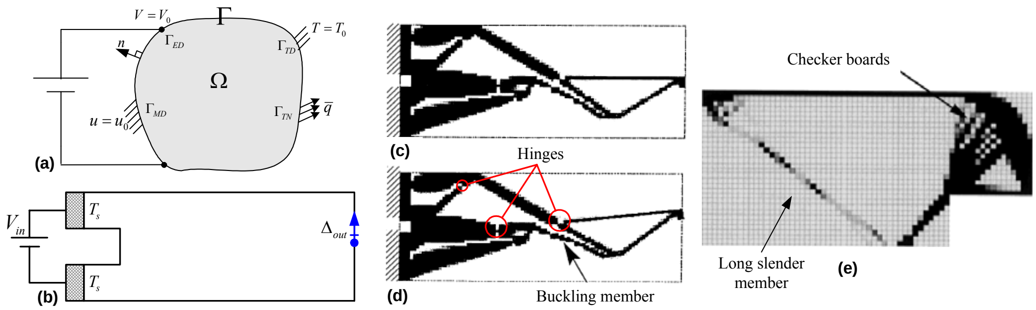

Figure 1: a) General domain with mixed boundaries, b) design domain for a thermal actuator including input voltage, thermal boundaries at substrate contacts and desired output location and direction c) optimized actuator topology using homogenization method [2], (d) deformed actuator and (e) optimal configuration of a clamping mechanism, generated with homogenization method, addressing the checker board problem [3].

changes do not influence the convection or conduction properties, the small strain allows modeling electrical and thermal fields linearly and thus system is only weakly coupled in the sense that the heat equations do not depend on the elasticity equations and that the electric field equations are independent of heat equations. To simplify the equations, it is assumed that the body force and the internal electric current source are zero.

In heat activated compliant mechanisms, the deformation is caused by thermal expansion of material due to change of temperature distribution. The difference with thermal actuation is, instead of elastic strain energy, what system stored is the thermal strain energy. The strain-stress relation with temperature change can be written as

$$\varepsilon = \mathrm{C}^{-1}\sigma + \varepsilon_{\mathrm{th}} \tag{1}$$

where C is the material elasticity coefficient, σ is the stress distribution and ε_{th} is the thermal strain defined as

$$\begin{Bmatrix} \varepsilon_{\mathrm{th}_x} \\ \varepsilon_{\mathrm{th}_y} \end{Bmatrix} = \alpha \begin{Bmatrix} \Delta T \\ \Delta T \end{Bmatrix} \tag{2}$$

In addition, initial and boundary conditions are needed to achieve the solution. The equilibrium equation can be expressed in an integration form according to the principle of virtual work as:

$$\int_\Omega \varepsilon^T \sigma d\Omega = \int_\Omega \varepsilon^T C(\varepsilon - \varepsilon_{th}) d\Omega = 0 \tag{3}$$

Assuming infinitesimal strains and $f_T = \mathrm{C}\alpha\Delta\overline{T}$, Equation (3) is simplified in a general form as:

$$\int_\Omega \varepsilon^T \varepsilon d\Omega = \int_\Omega f_T d\Omega = F_T \tag{4}$$

2.1 Formulation

The topology optimization of compliant mechanisms with voltage and heat actuation shares the similar design criteria as the general compliant mechanism with force actuation. It follows the basic concept of a compliant mechanism, which is supposed to be flexible enough to deform and stiff enough to resist the failure. Two types of design criteria are considered to formulate an optimization problem: flexibility requirement and stiffness requirement.

The flexibility requirement, also called mechanisms requirement means the designed compliant structure must be deformed in a favorable manner to complete its functionality. Mathematically this requirement can be captured by using the concept of mutual strain energy, MSE, based on the reciprocal theorem for linear elasticity.

The structure to be analyzed is a sub domain of the open domain Ω, shown in Figure 2 (1). If displacement field in case (1), U_1, caused by temperature change is used as the virtual displacement for equilibrium equation under the dummy loading case (2), the principal of virtual work can be expressed as the following:

$$\int_\Omega \varepsilon_1^T \sigma_2 d\Omega = \int_\Omega \varepsilon_1^T C \varepsilon_2 d\Omega = F_2^T U_{1A} \tag{5}$$

where ε_i, σ_i, U_i and F_i refer to strain, stress, displacement field and load at loading case i and U_{1A} is the actual displacement at output point A due to actual forces at case (1). If U_2 is used as the virtual displacement:

$$\int_\Omega \varepsilon_2^T \sigma_1 d\Omega = \int_\Omega \varepsilon_2^T C \varepsilon_1 d\Omega = F_1^T U_2 \tag{6}$$

Considering that the loading at case (1) is in fact the thermal load vector F_T, Equation (6) is rewritten as

$$\int_\Omega \varepsilon_2^T \sigma_1 d\Omega = \int_\Omega f_T U_2 d\Omega \tag{7}$$

The bilinear form $\Psi(U_1, U_2) = \int_\Omega \varepsilon_1^T C \varepsilon_2 d\Omega$ defines the mutual strain energy between case (1) and case (2). When the dummy load F_2 is a unit load, this mutual strain energy equals to the displacement at A, in the desired direction due to temperature change, denoted by U_{1A}.

The stiffness requirement can be formulated as the concept of compliance. Consider loading case (3) in Figure 2, if an external force is to be loaded to the mechanism, in the form of resistive force or functioning disturbance force, the stiffness required to sustain the loading is characterized by the displacement at the loading location. Using the same displacement field as the virtual displacement:

$$\Psi(U_3, U_3) = \int_{\Omega} \varepsilon_3^T C \varepsilon_3 d\Omega = F_3^T U_{3B} \quad (8)$$

where U_{3B} is the displacement field at resistant point B due to resistive force F_3. When the resistive dummy force F_3 is a unit load, Equation (8) results in displacement at B. The directions and locations of required flexibility, as well as those of required stiffness need to be determined according to individual design problem. Here the formulation is simplified for two general common cases: design for maximum displacement and design for maximum force.

- To design a mechanism that transfers maximum displacement at point A yet has enough stiffness to resist failure, the required flexibility is in the direction of desired output displacement at point A, while required stiffness is in the direction opposite to that at the same location. This concept is the familiar mutual strain energy and strain energy of the system as a measure for flexibility and stiffness and is shown in Figure 2 (a) where F_d is the unit dummy load in direction of desired output displacement Δ_{out}. This problem is formulated as:

$$\begin{aligned} MSE &= \Psi(U_1, U_2) = U_2^T K_2 U_{1A} \\ SE &= \Psi(U_3, U_3) = U_3^T K_3 U_{3A} \end{aligned} \quad (9)$$

where $K_1U_1=F_T$, $K_2U_2=F_d$ and $K_3U_3=-F_d$.

- To design a mechanism that transfers displacement at point *A* yet has enough stiffness to resists a work piece at point *B*, the required flexibility is in the direction of desired output displacement at the output point A, while required stiffness is in the direction opposite to displacements at resistant force at point B. This concept is shown in Figure 2 (b) where F_d is a unit dummy load. This problem is formulated as:

$$\begin{aligned} MSE &= \Psi(U_1, U_2) = U_2^T K_2 U_{1A} \\ SE &= \Psi(U_3, U_3) = U_3^T K_3 U_{3B} \end{aligned} \quad (10)$$

where k_s is a linear spring that simulates the resistance from a work piece at output point B. This stiffness k_s contributes to the system stiffness matrix K_3 at loading case (3) for required stiffness design.

2.2 Objective Function

The multi objective optimization is formulated as:

$$\begin{aligned} &\textit{Maximize}: \Psi(MSE, \frac{1}{SE}) \\ &\textit{Subject to}: \text{electrical equilibrium equations} \\ &\qquad \text{thermal equilibrium equations} \\ &\qquad \text{elastic equilibrium equations} \\ &\qquad \text{lower and upper bounds on variables} \\ &\qquad \text{buckling constraints} \end{aligned} \quad (11)$$

To solve the optimization problem stated in Equation (11), the design domain is discretized using N beam elements. The cross section of each element is the design variable. Considering the planar micro domain, the width of the elements are the design variable are considered as continuous variables allowing the design variables to take intermediate values in the constrained upper and lower bounds. For computational reasons and avoiding the singularity of the stiffness matrix, the minimum value of the continuous design variables is not zero but takes a small value. The optimization problem in Equation (11) is nonlinear and must be solved by an iterative procedure. The iterative optimization scheme used here is based on a Real Coded Genetic Algorithm with elitism.

3 IMPLEMENTATION

Figure 3 shows the schematic view of implemented scheme and the GUI. It includes an input menu and output window where the results are graphically represented. The optimization is an iterative procedure. It starts with an initial frame like random ground structure as shown in figure 3(a). User then defines the mechanical, thermal and input voltage boundary conditions and the direction of desired output displacement. An innovative finite element code [4] calculates the voltage, temperature, displacement field and the value of objective function in each iteration. A buckling constraint is imposed in the optimization formulation to prevent long slender members in optimized design. Optimization is implemented with a modified real coded genetic algorithm. The optimized solution depends on objective function, constraints and lower and upper limits on design variables. The MEMS fabrication process limits are included in optimization constraints as lower and upper bounds on design variables.

3.1 Numerical Examples

Figure 4 (a) shows the initial frame ground structure and applied boundaries for the electro-thermal actuator. Figure 4(b) is the optimized configuration of the actuator along with displacement field superimposed in dashed lines and Figure 4(c) shows the convergence history of the genetic algorithm. As seen here, the resulted topology is deforming in desired direction, has no flexural hinges and the buckling constraint has prevented generation of long slender beams. Extracting the fabrication layout is straight forward and there is no need for filtering or interpolations of the resulted configuration. This solution is saved as IGES to export to L-edit. It converges with 129 elements and less than 100 iterations. This method is computationally very effective and takes less than 10 minutes on a personal computer to converge.

4 CONCLUSIONS

The topology optimization method was applied to the design of compliant thermal and electro-thermal micro actuators. The performance values for the electro-thermal actuators have been compared with the performance of finite element model and tend to get very close.

Results presented in this chapter indicate that the topology optimization method can produce highly efficient micro actuators. Based on the findings of this research, it is concluded that the topology optimization method is a promising tool for systematic design of the mechanical parts and the electro-thermo-mechanical parts of MEMS.

The presented topology optimization method is a powerful tool to design optimized MEMS structures with on-chip actuations.

REFERENCES

[1] A. Saxena, "Topology optimization of geometrically nonlinear compliant mechanisms for flexibility, stiffness and strength," Ph.D. dissertation, University of Pennsylvania, 2000.

[2] Ole Sigmund, "Design of multiphysics actuators using topology optimization – Part I," Journal of Computer methods in applied mechanics and engineering,, 2001, 190, 6577-6604.

[3] M. I. Frecker, et. al., "topology optimization of compliant mechanisms using the homogenization method," International Journal for Numerical Methods in Engineering, vol. 42, 1998, 535-559.

[4] M. Motiee, et. al, "A New Finite Element Formulation for Micro Electro-Thermo-Mechanical Domains" ASME IMECE, Chicago, IL, November 5-10, 2006, 14992.

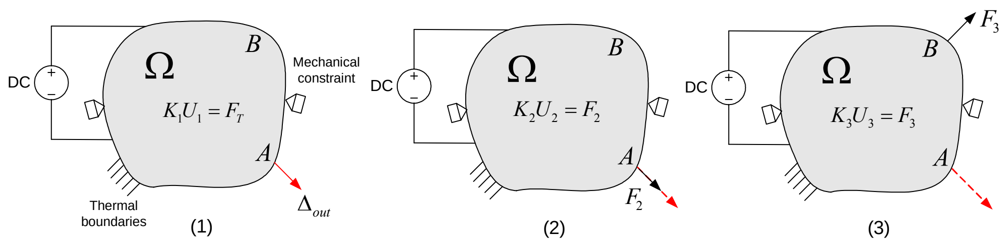

Figure 2: Loading cases for flexibility and stiffness design.

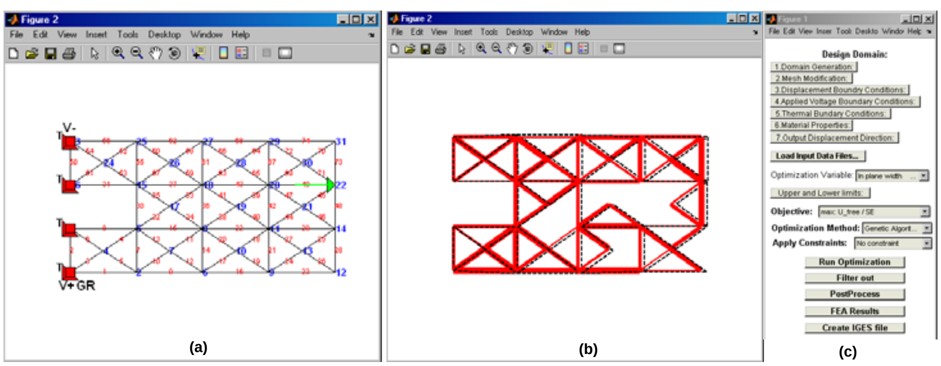

Figure 3: The graphical user interface of the optimization scheme, showing the main result window with a) initial guess ground structure, b) changing topology towards optimal design and FEA results superimposed as dashed lines and c) input menu.

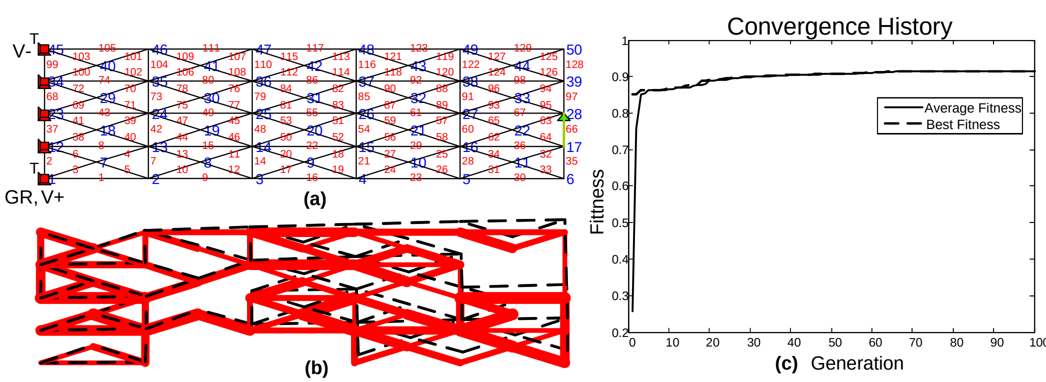

Figure 4: a) Initial ground structure for thermal actuator, material is nickel; design domain is 200 µm by 50 µm and is initially meshed with 50 nodes and 129 random elements. Width of elements is the design variable and lower and upper bounds are 3 and 8 µm, respectively. The thickness of structure is 3 µm b) optimized compliant actuator topology has 46 nodes and 82 elements, dashed line shows the deformed shape of actuator with 5 V input c) convergence history of average and best fitness value for each population.