Analytic Charge Model for Double-Gate and Surrounding-Gate MOSFETs

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ABSTRACT

An analytic charge model for both double gate and surrounding gate MOSFETs is presented. With only the mobile charge term, Poisson’s equation is rigorously solved and the analytic electrostatic potential is derived. The development of charge model is based on closed-form solution of Poisson’s equation, current continuity equation, and Ward-Dutton linear charge partition. The model can continuously cover all the operation regions, i.e., linear, saturation, and subthreshold, with unique analytic expressions. The physics-based nature makes this model free of fitting parameters as well as the charge sheet approximation. It is shown that the C-V characteristics generated by this model agree with two-dimensional numerical simulation results.

Keywords: MOSFETs, modeling, double gate, surrounding gate.

1 INTRODUCTION

Double gate (DG) and surrounding gate (SG) MOSFETs have been proposed to scale down CMOS technology more aggressively [1], since bulk CMOS scaling [2], [3] is approaching the limit imposed by gate oxide tunneling, body doping, and band-to-band tunneling, etc.. Efforts toward analytic model for these new device structures are presented in this paper, with emphasis on the charge model. Recently, continuous analytic drain current models have been developed for DG [4] and SG [5] MOSFETs, respectively. Without charge sheet approximation [6], these two models are derived directly from the Pao-Sah integral [7] for DG and SG MOSFETs with undoped or lightly doped silicon body. However, drain current model is not enough for circuit simulation [8]. Therefore, terminal charge models for both DG [9] and SG [10] MOSFETs have been proposed to calculate the dynamic behavior of the device and hence to enable AC and transient circuit simulation. The conventional Ward-Dutton charge partition method [11] for bulk MOSFETs can also be applied to define terminal charges for DG and SG MOSFETs. It has been validated by numerical simulations that these models can continuously cover all the three operation regions without the need for non-physical fitting parameters.

Figure 1: Schematic diagram of an SG MOSFET (cut through symmetry axis).

2 ANALYTIC CHARGE MODEL FOR SG MOSFETS

Consider an undoped (or lightly doped) cylindrical SG MOSFET schematically shown in Fig. 1. Under the gradual channel approximation, Poisson’s equation takes the following form with only the mobile charge (electrons) term:

\[ \frac{d^2 \psi}{d \rho^2} + \frac{1}{\rho} \frac{d \psi}{d \rho} = \frac{q}{\epsilon_{si}} n_i e^{\frac{q(\psi - V)}{kT}} \]  

(1)

where \( q \) is the electronic charge, \( \epsilon_{si} \) is the permittivity of silicon, \( n_i \) is the intrinsic carrier density, \( \psi(\rho) \) is the electrostatic potential, and \( V \) is the electron quasi-Fermi potential which is constant in the \( \rho \)-direction. Here we consider an nMOSFET with \( q\psi/kT \gg 1 \) so that the hole density is negligible.

The solution of Eq. (1) has been presented in a previous paper by Jimenez et al. [5]. We reorganize it as

\[ \psi(\rho) = V - \frac{2kT}{q} \ln \left[ \frac{R}{2L_{Di}} \frac{1}{\sqrt{1 - \alpha}} \left( 1 - \frac{(1 - \alpha) \rho^2}{R^2} \right) \right] \]  

(2)

where \( L_{Di} = \sqrt{2e_{si}kT/q^2n_i} \) is the intrinsic Debye length, and \( \alpha \) is to be determined by the boundary condition...
which has final form as

\[
\frac{1}{2} \ln (1 - \alpha) - \ln \alpha + \frac{1 - \alpha}{\alpha} = \frac{q (V_g - \Delta \phi - V)}{2kT} - \ln \left( \frac{2Ld_i}{R} \right). \tag{3}
\]

Here \( s = 2\epsilon_s \ln(1 + \frac{t_{ox}}{R})/\epsilon_{ox} \) is a structural parameter, \( \epsilon_{ox} \) is the permittivity of oxide, \( V_g \) is the voltage applied to the gate, \( R \) is the radius of silicon cylinder, \( t_{ox} \) is the oxide thickness, and \( \Delta \phi \) is the work function of gate electrode with respect to the intrinsic silicon. The range of \( \alpha \) is \( 0 < \alpha < 1 \). Because (3) is an implicit equation of \( \alpha \), the general approach to solve \( \alpha \) is Newton-Raphson numerical method.

From Gauss’s law, the total mobile charge per unit gate area \( Q_i = 2\epsilon_s(2kT/q)(1-\alpha)/\alpha R \). \( dV/d\alpha \) can also be expressed as a function of \( \alpha \) by differentiating (3):

\[
dV/d\alpha = (kT/q)[1/(1-\alpha) + 2/\alpha + 2s/\alpha^2]. \tag{4}
\]

Therefore, the drain current can be calculated as

\[
I_{ds} = \frac{2\pi R}{L} \int_{V_s}^{V_d} Q_i(V) dV
= \frac{2\pi R}{L} \int_{\alpha_s}^{\alpha_d} Q_i(\alpha) \frac{dV}{d\alpha} d\alpha
= \frac{8\pi \epsilon_s}{L} \left( \frac{kT}{q} \right)^2 [f(\alpha_d) - f(\alpha_s)].
\]

where \( \mu \) is the effective mobility, \( L \) is the channel length, \( V_s, V_d \) are the voltages applied to the source and drain, respectively, \( \alpha_s, \alpha_d \) are solutions to (3) corresponding to \( V = V_s \) and \( V = V_d \), respectively, and \( f(\alpha) = (-2/\alpha - \ln \alpha) + s (-1/\alpha^2 + 2/\alpha) \).

Analytic expressions for three terminal charges, i.e., \( Q_g, Q_s, \) and \( Q_d \), associated with gate, source, and drain, respectively, are required for circuit simulation of SG MOSFETs. The gate charge can be computed directly by integrating the charge density along the channel. For the source and drain charges, it is physically sound to adopt Ward-Dutton linear charge partition method, which is widely accepted for bulk MOSFETs [11]. Integrating the current continuity equation

\[
dy = \mu(2\pi R)Q_i dV/I_{ds}
\]

leads to \( y = L[f(\alpha) - f(\alpha_s)]/[f(\alpha_d) - f(\alpha_s)] \). With this result, we can calculate the terminal charges as

\[
Q_g = 2\pi R \int_{0}^{L} Q_i(y) dy
= (2\pi R)^2 \frac{\mu}{I_{ds}} \int_{\alpha_s}^{\alpha_d} Q_i(\alpha) \frac{dV}{d\alpha} d\alpha
= 8\pi \epsilon_s L \frac{kT}{q} g(\alpha_d) - g(\alpha_s)
\]

\[
Q_s = 2\pi R \int_{0}^{L} \left( 1 - \frac{y}{L} \right) Q_i(y) dy = 8\pi \epsilon_s L \frac{kT}{q} W_{sd}
\]

\[
Q_d = -2\pi R \int_{0}^{L} \frac{y}{L} Q_i(y) dy = 8\pi \epsilon_s L \frac{kT}{q} W_{ds}
\]

where

\[
W_{ij} = \frac{f(\alpha_i) [g(\alpha_j) - g(\alpha_i)] + [h(\alpha_j) - h(\alpha_i)]}{[f(\alpha_j) - f(\alpha_i)]^2} \tag{7}
\]

and functions \( g(\alpha) \) and \( h(\alpha) \) are defined as

\[
g(\alpha) = \left( -\frac{1}{\alpha^2} + \frac{3}{\alpha} + \ln \alpha \right) + s \left( -\frac{2}{3\alpha^3} + \frac{2}{\alpha^2} - \frac{2}{\alpha} \right)
\]

\[
h(\alpha) = \left( -\frac{4}{3\alpha^3} + \frac{5}{2\alpha^2} + \frac{1}{\alpha} - \frac{1 - 3\alpha}{\alpha^2} \ln \alpha + \frac{(\ln \alpha)^2}{2} \right)
\]

\[
- s \left( \frac{3}{2\alpha^3} + \frac{43}{9\alpha^2} + \frac{9}{2\alpha^2} + \frac{2 - 6\alpha + 6\alpha^2}{3\alpha^3} \ln \alpha \right)
\]

\[
- s^2 \left( \frac{2}{5\alpha^3} - \frac{2}{\alpha^3} + \frac{10}{3\alpha^3} - \frac{2}{\alpha^2} \right). \tag{8}
\]

The imaginary component of the small-signal behavior of a three-terminal device can be characterized by nine nonreciprocal capacitance coefficients. Here, we shall adopt the convention in which the capacitance coefficient \( C_{mn} \) is defined as \( \delta_{mn} \times \partial Q_{mn}/\partial V_n \), where \( m, n = g, s, d, \) and \( \delta_{mn} \) is equal to 1 if \( m = n \), and -1 if otherwise. For simplicity, we will only list the results of four linearly independent components, from which all
Based on the charge conservation law [8], the nine capacitance coefficients can be obtained directly based on the charge conservation law [8]:

\[
C_{gs} = \frac{L^2 (g_{ds} + g_m)^2}{\mu I_{ds}} - \frac{Q_g (g_{ds} + g_m)}{I_{ds}} \\
= 8\pi \epsilon_s L \frac{1 - \alpha_s}{\alpha_s} \frac{[(1 - \alpha_s)/\alpha_s + W_{sd} + W_{ds}]}{f(\alpha_d) - f(\alpha_s)} \\
\]

\[
C_{gd} = -\frac{L^2 g_{ds}^2}{\mu I_{ds}} + \frac{Q_g g_{ds}}{I_{ds}} \\
= 8\pi \epsilon_s L \frac{1 - \alpha_d}{\alpha_d} \frac{[(1 - \alpha_d)/\alpha_d + W_{sd} + W_{ds}]}{f(\alpha_s) - f(\alpha_d)} \\
\]

\[
C_{dy} = -\frac{L^2 g_{ds}^2}{\mu I_{ds}} + \frac{Q_g g_{ds}}{I_{ds}} - \frac{(Q_s - Q_d) g_m}{I_{ds}} \\
= 8\pi \epsilon_s L \left( \frac{1 - \alpha_d}{\alpha_d} \frac{[(1 - \alpha_d)/\alpha_d + W_{sd} + W_{ds}]}{f(\alpha_s) - f(\alpha_d)} \\
+ (W_{ds} - W_{sd}) \frac{1/\alpha_s - 1/\alpha_d}{f(\alpha_s) - f(\alpha_d)} \right) \\
\]

\[
C_{sd} = \frac{(Q_s - Q_d) g_{ds}}{I_{ds}} \\
= 8\pi \epsilon_s L \frac{1 - \alpha_d}{\alpha_d} \frac{(W_{ds} - W_{sd})}{f(\alpha_s) - f(\alpha_d)}. \tag{9}
\]

Here transconductance \( g_m \) and conductance \( g_{ds} \) are given by

\[
g_m = \frac{\partial I_{ds}}{\partial V_{gs}} V_{ds} = \frac{8\pi \epsilon_s kT}{L} \frac{1}{\alpha_s} - \frac{1}{\alpha_d} \\
\]

\[
g_{ds} = \frac{\partial I_{ds}}{\partial V_{gs}} V_{ds} = \frac{8\pi \epsilon_s kT}{L} \frac{1 - \alpha_d}{\alpha_d} \tag{10}
\]

where \( V_{gs} = V_g - V_s \) and \( V_{ds} = V_d - V_s \). It is worthy to mention that this model is non-source-referenced and automatically preserves the symmetry with respect to source-drain interchange. This property can be obviously observed in the final expressions of the terminal charges as well as the capacitance coefficients.

All these analytic expressions have infinite order of continuity, however, they become the form 0/0, i.e., both the numerator and denominator are 0, when \( V_s = V_d \). Inasmuch as the indeterminacies of type 0/0 will cause problems in numerical implementation, we have to provide expressions for the exact values at \( V_s = V_d \), which are the limits of general expressions as \( V_d \) approaches \( V_s \). These limits can be calculated through L’Hospital’s rule or Taylor expansion. After a lengthy derivation, we finally obtain the results as

\[
Q_g = -2Q_s = -2Q_d = 8\pi \epsilon_s L \frac{kT}{q} \frac{1 - \alpha_s}{\alpha_s} \tag{11}
\]

\[
C_{gg} = 2C_{gs} = 2C_{gd} = 2C_{sy} = 2C_{dy} = -6C_{sd} = -6C_{ds} = 3C_{ss} = 3C_{ds} = 8\pi \epsilon_s L \frac{1 - \alpha_s}{2\alpha_s - \alpha_s^2 + 2s(1 - \alpha_s)}. \tag{12}
\]

Fig. 2 plots terminal charges as functions of drain voltage. In this case, \( \alpha_s \) is small. When \( V_{ds} = 0 \), \( Q_g = -2Q_s = -2Q_d \approx 8\pi \epsilon_s L kT/q\alpha_s \). When the device enters saturation region, \( Q_g \approx -10Q_s/6 \approx -10Q_d/4 \approx 10(8\pi \epsilon_s L kT/q\alpha_s)/15 \) according to (6). These approximate relations are exactly the same with those for bulk MOSFETs [8], and are clearly reflected in Fig. 2.

In Fig. 3, 4, and 5, we have compared our capacitance coefficient model with numerical simulation re-
Figure 5: Normalized capacitance coefficients of an SG MOSFET as functions of $V_{ds}$ obtained from the analytic model (solid lines), in comparison with the 2-D numerical simulation results (open circles). The normalization constant is $4\pi\epsilon_{si}L/s$.

results of a long channel SG MOSFET. Because we use cylindrical coordinate instead of Cartesian coordinate, the numerical simulation is essentially two-dimensional. Fig. 3, where $V_{ds} = 2V$, shows that the general expressions of the model are in complete agreement with numerical simulations. In this case, the device is always in saturation region, and it is reasonable to assume $\alpha_d = 1$. Under this approximation, we can easily obtain the simple relation between capacitance coefficients from (9):

$$C_{gg} = C_{gs},$$
$$C_{sg} = C_{ss},$$
$$C_{dg} = -C_{ds},$$
$$C_{gd} = 0.$$  

If we further consider the well-above threshold region, where $\alpha_s$ is small, we can even derive the simplified approximate expressions for these capacitance coefficients as follow

$$C_{gg} \approx \frac{10}{6} C_{sg},$$
$$C_{sg} \approx \frac{10}{4} C_{dg} \approx \frac{10 \pi \epsilon_{si} L}{15 s} \left(1 - \frac{\alpha_s}{s}\right).$$  

Fig. 4 indicates the validity of (12) for the special case of $V_s = V_d$. In Fig. 5, the capacitance coefficients from both analytic model and numerical simulation are compared as functions of $V_{ds}$. We can clearly observe the transition between two degenerate cases, which have been introduced above: one degeneracy is from source-drain-interchange symmetry at $V_{ds} = 0$; another is induced by saturation at the drain end.

3 ANALYTIC CHARGE MODEL FOR DG MOSFETS

Consider an undoped (or lightly doped) symmetric DG MOSFET schematically shown in Fig. 6. Under the gradual channel approximation, Poisson’s equation takes the following form with only the mobile charge (electrons) term:

$$\frac{d^2\psi}{dx^2} = \frac{q}{\epsilon_{si}} ne^{\psi - \psi_0}$$

where $\psi(x)$ is the electrostatic potential, and $\psi_0$ is the electron quasi-Fermi potential which is constant in the $x$-direction.

The solution of Eq. (14) has been presented in our previous paper [4] as

$$\psi(x) = V - \frac{2kT}{q} \ln \left[ \frac{t_{si}}{2L_{Di} \beta} \cos \left( \frac{2\beta x}{t_{si}} \right) \right]$$

where $\beta$ is to be determined by the boundary condition which has final form as

$$\ln \beta - \ln (\cos \beta) + r\beta \tan \beta = \frac{q(V_g - \Delta\phi - V)}{2kT} - \ln \left( \frac{2L_{Di}}{t_{si}} \right).$$

Here $r = 2\epsilon_{si}t_{ox}/\epsilon_{ox}t_{si}$ is a structural parameter, $t_{si}$ is the silicon thickness. The range of $\beta$ is $0 < \beta < \pi/2$. Because (16) is an implicit equation of $\beta$, the general approach to solve $\beta$ is Newton-Ralphson numerical method.

Similarly, the drain current of DG MOSFET can be calculated as

$$I_{ds} = \frac{8\epsilon_{si} W}{L t_{si} \left( \frac{kT}{q} \right)^2} [p(\beta_d) - p(\beta_s)]$$

where $\beta_s, \beta_d$ are solutions to (16) corresponding to $V = V_s$ and $V = V_d$, respectively, and $p(\beta) = -2\beta \tan \beta + \beta^2 - r\beta^2 \tan^2 \beta$.

Analytic expressions for three terminal charges, i.e., $Q_g, Q_s$, and $Q_d$, associated with gate, source, and drain, respectively, are required for circuit simulation of DG MOSFET.
MOSFETs. Following the Ward-Dutton linear charge partition, we can calculate the terminal charges as

\[ Q_g = 8\varepsilon_s L \frac{W}{t_{si}} \frac{kT}{q} u(\beta_d) - u(\beta_s) \]
\[ Q_d = 8\varepsilon_s L \frac{W}{t_{si}} \frac{kT}{q} U_{sd} \]
\[ Q_s = 8\varepsilon_s L \frac{W}{t_{si}} \frac{kT}{q} U_{ds} \]

where

\[ U_{ij} = p(\beta_i) [u(\beta_j) - u(\beta_i)] + [v(\beta_j) - v(\beta_i)] \frac{[p(\beta_j) - p(\beta_i)]^2}{2} \]

and functions \( u(\beta) \) and \( v(\beta) \) are defined as

\[ u(\beta) = -2 \int_0^\beta \beta^2 \tan^2 \beta \left( \frac{1}{\beta} + \tan \beta \right) d\beta - \frac{2}{3} \beta^3 \tan^3 \beta \]
\[ v(\beta) = 2 \int_0^\beta \beta^2 \tan^2 \beta \left( \frac{1}{\beta} + \tan \beta \right) (\beta^2 - 2\beta \tan \beta) d\beta \]
\[ -2r \int_0^\beta \beta^4 \tan^4 \beta \left( \frac{1}{\beta} + \tan \beta \right) d\beta \]
\[ + 2r \int_0^\beta \beta^2 \tan^2 \beta \left( \frac{\beta}{\cos^2 \beta} + \tan \beta \right) d\beta \]
\[ \times (\beta^2 - 2\beta \tan \beta) d\beta - \frac{2}{5} \beta^5 \tan^5 \beta. \]

Combining above results, we can finally derive the capacitance coefficients as follows

\[ C_{gs} = 8\varepsilon_s L \frac{W}{t_{si}} \beta_s \tan \beta_s \frac{(\beta_s \tan \beta_s + U_{sd} + U_{ds})}{p(\beta_d) - p(\beta_s)} \]
\[ C_{sd} = 8\varepsilon_s L \frac{W}{t_{si}} \beta_d \tan \beta_d \frac{(U_{ds} - U_{sd})}{p(\beta_d) - p(\beta_s)}. \]

Here, we only list the results of two components, because all the other capacitance coefficients can be easily obtained based on source-drain interchange symmetry and charge conservation law.

When \( V_s = V_d \), specific expressions are needed to avoid numerical instability:

\[ Q_g = -2Q_s = -2Q_d = 8\varepsilon_s L \frac{W}{t_{si}} \frac{kT}{q} \beta_s \tan \beta_s \]
\[ C_{gg} = 2C_{gs} = 2C_{gd} = 2C_{sg} = C_{dd} \]
\[ = -6C_{sd} = 3C_{as} = 3C_{dd} \]
\[ = 8\varepsilon_s L \frac{W}{t_{si}} 2 \left( \frac{1}{\beta_s} + \tan \beta_s \right) + 2r \left( \frac{\beta_s}{\cos^2 \beta_s} + \tan \beta_s \right). \]
As follow approximate expressions for these capacitance coefficients β where
If we further consider the well-above threshold region, capacitance coefficients from (22): approximation, we can easily obtain the simple relation region, and it is reasonable to assume relations. In this case, the device is always in saturation model are in complete agreement with numerical simulations that charge and capacitance coefficient be-
has been preserved, it has been verified by numerical simulations that charge and capacitance coefficient be-
haviors in all the operation and transition regions are properly described by continuous functions without ad hoc fitting parameters. For SG MOSFETs, these continuous functions are simply composed of fundamental functions, making it suitable and easy to implement this predictive model in circuit simulation programs. However, for DG MOSFETs, because non-integrable functions are involved, table lookup method has to be employed to calculate terminal charges. Therefore, we are aiming at improving the computational efficiency by making additional approximations. An optimal result is expected in the trade-off between efficiency and accuracy of the model.

4 CONCLUSION

In conclusion, we have presented an analytic charge model for both DG and SG MOSFETs. The model is non-source-referenced and preserves the source-drain symmetry inherently. Inasmuch as the essential physics has been preserved, it has been verified by numerical simulations that charge and capacitance coefficient be-

Figure 9: Normalized capacitance coefficients of an DG MOSFET as functions of $V_{ds}$, obtained from the analytic model (solid lines), in comparison with the 2-D numerical simulation results (open circles). The normalization constant is $(4\epsilon_{si}L/r)(W/t_{si})$.

In Fig. 7, 8, and 9, we have compared our capacitance coefficient model with numerical simulation results of a long channel DG MOSFET. Fig. 7, where $V_{ds} = 2V$, shows that the general expressions of the model are in complete agreement with numerical simulations. In this case, the device is always in saturation region, and it is reasonable to assume $\beta_d = 0$. Under this approximation, we can easily obtain the simple relation between capacitance coefficients from (22): $C_{gg} = C_{gs}$, $C_{sg} = C_{ss}$, $C_{dg} = C_{ds}$, and $C_{gd} = C_{sd} = C_{dd} = 0$. If we further consider the well-above threshold region, where $\beta_s \approx \pi/2$, we can even derive the simplified approximate expressions for these capacitance coefficients as follow

$$C_{gg} \approx \frac{10}{6} C_{sg} \approx \frac{10}{4} C_{dg} \approx \frac{10}{15} \frac{4\epsilon_{si} L W}{r t_{si}} \left( 1 - \frac{1}{r \beta_s \tan \beta_s} \right)$$

Fig. 8 indicates the validity of (24) for the special case of $V_s = V_d$. In Fig. 9, the capacitance coefficients from both analytic model and numerical simulation are compared as functions of $V_{ds}$. We can clearly observe the transition between two degenerate cases.

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