QBIT´S Simulations for Develompment of Quantum Computer using the DSP 6711

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ABSTRACT

Quantum computation is a new technology with the potential for providing exponential parallelism, but a quantum computation is a series of quantum logic gates. This work uses the Digital Signal Processor (DSP TI6711) to simulate the different quantum gates for generating a Quantum Bit´s(Qbit´s). This modeling uses the hardware based on Shoeringer equation time-dependent, where the Hamiltonian takes the form of is sufficiently general to capture the salient features of most physical models of the QBit´s. Interactions between Qbit´s that involve different spin components have been left out in because we are not awere of a candidate technology of Quantum Computer where these would be important.

Keywords: Qbits, Gate, Simulator, DSP6711

INTRODUCTION

The fundamental resource and basic unit of quantum information is the Quantum Bit (QBit), which behaves like a classical bit enhanced by the superposition principle. From a physical point of view, a QBit is represented by an ideal two-state quantum system. Examples of such systems include photons (vertical and horizontal polarization), electrons and other spin- 1 2 systems (spin up and down), and systems defined by two energy levels of atoms or ions. From the beginning the two-state system played a central role in studies of quantum mechanics. It is the most simple quantum system, and in principle all other quantum systems can be modeled in the state space of collections of Qbit´s. From the information processing point of view, a QBit's state space contains the two “logical”, or “computational”, states |0> and |1>. The so-called “ket” notation for these states was introduced by P. Dirac, and its variations are widely used in quantum physics. One can think of the pair of symbols “|” and “>” as representing the QBit system. Their content specifies a state for the system. In this context 0 and 1 are system-independent state labels. When, say, 0 is placed within the ket, the resulting expression |0> represents the corresponding state of a specific QBit. The initial state of a QBit is always one of the logical states. Using operations to be introduced later, we can obtain states which are “superpositions” of the logical states. Superpositions can be expressed as sums A|0> + B|1> over the logical states with complex coefficients. The complex numbers A and B are called the “amplitudes” of the superposition. The existence of such superpositions of distinguishable states of quantum systems is one of the basic tenets of quantum theory called the “superposition principle”.

An enormous amount of computing power is required for the complex software used in development for simulate a quantum mechanical systems. Using the current technologies, it´s very difficult to implement gates with many QBits. But it´s very important to simulate quantum systems and numerical methods on existing computers. The purpose of the simulation is:

- to investigate quantum algorithms behavior.
- to analyze performance and robustness of gates based on QBits in the presence of decoherence and operational errors.

The demands of these simulations have led to researchers to develop distributed computing systems harnessing the power of thousands, and in some cases more than a million, processors into clusters. Yet there are limits to this approach. Using many methods like adding more processors to increases the computing capacity of these machines only linearly, yet many problems, particularly in physics and computer science, increase exponentially with the size of their inputs. However, simulations often require more computational power than is usually available on sequential computers.

Therefore, we have developed the simulation method for gates based on QBits on a classical Harvard Architecture. That is, we have developed a general-purpose simulator for quantum algorithms and gates based on QBits systems on the Digital Signal Processor (DSP). The DSP is designed for real-time processing, optimum performance with streaming data, separate program and data memories, special instructions for SIMD (Single Instruction, Multiple Data) operations.

METHODOLOGY

For the devolpment of the project, we use the DSP, like the TI TMS6711, with an architectural optimizations to speed up processing. This DSP can be connected on clasical personal computer for transfer the data between them, and the architectural features is the next:

Program flow:

- Floating-point unit integrated directly into the data-path.
- Pipelined architecture
- Highly parallel accumulator and multiplier
- Special looping hardware. Low-overhead or Zero-overhead looping capability

Memory architecture:

- DSPs often use special memory architectures that are able to fetch multiple data and/or instructions at the same time:
- Harvard architecture
- modified von Neumann architecture
- Use of direct memory access
- Memory-address calculation unit

Data operations:

- arithmetic is often used to speed up arithmetic processing.
- Single-cycle operations to increase the benefits of pipelining.

Instruction sets:

- Multiply-accumulate (MAC) operations, which are good for all kinds of matrix operations, such as convolution for filtering, dot product, or even polynomial evaluation (see Horner scheme, also fused multiply-add).
- Instructions to increase parallelism: SIMD, VLIW, superscalar architecture.
- Specialized instructions for modulo addressing in ring buffers and bit-reversed addressing mode for FFT cross-referencing.
- Digital signal processors sometimes use time-stationary encoding to simplify hardware and increase coding efficiency

This DSP can be programed with a friendly Software (Code Composer Studio) with the with the algorithms of the quantum mechanics systems and numerical methods to solve the states of the QBits.

The system are modeled in term of Quantum Spin (QBits), that envolve in time according to the time-dependant Schrödinger equation (TDSE)

$$i\frac{\partial}{\partial t}\left|\phi(t)\right\rangle = H(t)\left|\phi(t)\right\rangle \qquad (1)$$

in units such that $\hbar = 1$ and where

$$\begin{aligned}|\Phi(t) \; &= a(\downarrow,\downarrow,\ldots,\downarrow;t)|\downarrow,\downarrow,\ldots,\downarrow \; + \\ &+ a(\uparrow,\downarrow,\ldots,\downarrow;t)|\uparrow,\downarrow,\ldots,\downarrow \; + \ldots \\ &+ a(\uparrow,\uparrow,\ldots,\uparrow;t)|\uparrow,\uparrow,\ldots,\uparrow \quad , \end{aligned} \qquad (2)$$

describes the state of the whole QC at the time t. The complex coefficients $a(\downarrow,\downarrow,...,\downarrow;t),...,a(\uparrow,\uparrow,...,\uparrow;t)$ completely specify the state of the quantum system. The time-dependent Hamiltonian H(t) take the form [22]

$$\begin{aligned}H(t) = &-\sum_{j,k=1}^{L}\sum_{\alpha=x,y,z} J_{j,k,\alpha}(t)S_j^\alpha S_k^\alpha \\ &-\sum_{j=1}^{L}\sum_{\alpha=x,y,z}\left(h_{j,\alpha,0}(t) + h_{j,\alpha,1}(t)sen\left(f_{j,\alpha}t + \varphi_{j,\alpha}\right)\right)S_j^\alpha\end{aligned} \qquad (3)$$

Where the first sum runs over all pairs P of QBits, S_j^α denotes the α-th component of the spin ½ operator representing the j-th QuBit, $J_{j,k,\alpha}(t)$ determines the strength of the interaction between the QBits labeled j and k, $h_{j,\alpha,0}(t)$and $h_{j,\alpha,1}(t)$ are static and periodic field acting on the j-th spin respectively. The frequency and the phase of the periodic field are denoted by $f_{j,\alpha}$ and $\varphi_{j,\alpha}$. The number of QBits is L and the dimension of the Hilbert space $D = 2^L$. Hammiltonian (Eq 3) is sufficiently general to capture the salient features of the most physical models of the QC. Interactions between QBits that involve different spin components have been left out in equation 3 because we are no aware of a candidate technology of QC where these would be important. Incorporating these interactions requires some trivial additions to the simulator program.

Procedures to construct unconditionally stable, accurate and efficient algorithms to solve the TDSE of a wide variety of continuum and lattice models have been reviewed elsewhere [3,4]. A detailed account of the applications of this approach to two-dimensional quantum spin models can be found in [5]. According to the equation 2 the time evolution of the QC, the solution of TDSE (Equation 1), is determined by the unitary transformation

$$U(t+\tau,t) \equiv \exp_{+}\left(-i\int_{t}^{t+\tau} H(u)du\right) \qquad (4),$$

where exp denotes the time-ordered exponential function.

Using the semi-group property of U(t+τ,t) we can write

$$U(t+\tau,t)=U(t+m\delta,t+(m-1)\delta)..U(t+2\delta,t+\delta)U(t+\delta,t) \tag{4}$$

where τ=mδ(m≥1). In general the first step is to replace each U(t+(n+1)δ,t+nδ) by simmetrized Suzuki product-formula approximation [5, 6].

The state of a quantum system is described by a wave function which in general depends on the space or momentum coordinates of the particles and on time. In Dirac's representation independent notation, the state of a system is a vector in an abstract Hilbert space | (t) >, which depends on time, but in that form one makes no choice between the coordinate or momentum space representation. The transformation between the space and momentum representation is contained in a transformation bracket, which for each particle is simply.

$$\left\langle r_i \middle| p_i \right\rangle = (2\pi)^{3/2} \equiv \left\langle pi \middle| ri \right\rangle^*$$

The two representations are related by Fourier transformation, which is the way Quantum Mechanics builds localized wave packets. In this way, uncertainty principle limitations on our ability to measure coordinates and momenta simultaneously with arbitrary precision are embedded into Quantum Mechanics (QM). This fact leads to operators, commutators, expectation values and, in the special cases when a physical attribute can be precisely determined, eigenvalue equations with Hermitian operators. That is the content of many quantum texts. Our purpose is now to see how to define a density matrix, in particular a spin density matrix. Spin is missing from the above discussion. In non-relativistic 2 QM, this subtle degree of freedom, whose existence is deduced by analysis of the Stern-Gerlach experiment, is simply tacked on as an additional Hilbert space vector feature. For example, for a single spin 1/2 system the wave function including both space and spin aspects is:

$$\left(r1, t \middle| s m_s \right\rangle)$$

where | s ms > denotes a state that is simultaneously an eigenstate of the particle's total spin operator s2 = s2 x + s2 y + s2 z , and of its spin component operator sz.

RESULTS

The simulation fo QBits was development in DSP 6711 onto personal computer Pentium IV to 2GHz and 512 KB in RAM. For the CNOT, and TOFFOLI Gate, the result is the next:

The multi QBits states, built as tensor products of two QBits ($[00][01][11][10]$), corresponds to $\frac{a}{2}+\frac{b}{2}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and for three QuBits, the tensor product correspond to:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

And the density matrix it`s build like this:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The density matrix operator, acting on the density matrix of the system, produced the Von Neuman entropy, and the next plot (figure 1) show the behavior. The figure 2 show the fidelity vs density matrix.

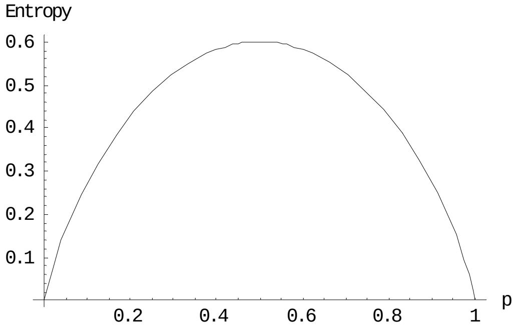

Figure 1: Entropy vs Density function, in GHz, using in multi QuBits states.

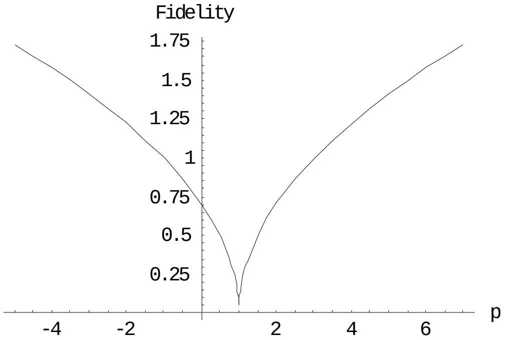

Figure 2: Fidelity vs Density function, using in multi QuBits state

DISCUSION

We can simulate the Von Neuman Model, using different combinations of gate set for the development a Quantum-processor. The next step of this project is to emulate an ALU and then make the different option for logical and arithmetical operations, approaching the QuBits advantage, like multi processing, parallelism, and logical reverse. If we can make the before, the Quantum Fourier Transform, will be the next step, with the advantage of the QuBits

CONCLUSIONS

Hamiltonian (21) is sufficiently generic to represent most models for candidates of physical realizations of quantum computer hardware. The spin-spin term in Eq. (3) is sufficiently general to describe the most common types of interactions such as Ising, anisotropic Heisenberg, and dipolar coupling between the spins. Furthermore, if we also use spin-1/2 degrees of freedom to represent the environment then, on this level of description, the interaction between the quantum computer and its environment is included in model. In other words, the Hamiltonian (3) is sufficiently generic to cover most cases of current interest.

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