Stability of micro and nano devices actuated by Casimir forces.

J. Hernández and R. Esquivel-Sirvent

Instituto de Fisica, UNAM
Apdo. Postal 20-364, México D.F. 01000 México, raul@fisica.unam.mx

ABSTRACT

The role of Casimir forces in the stability of micro and nano electromechanical devices is studied taking into account the dielectric properties of the materials using Lifshitz formula. The calculations are done for a one-degree of freedom simple-lumped system for different materials. The results of the stability are derived from the bifurcation diagram of the system.

Keywords: casimir, stability, mems, nems, bifurcation

1 INTRODUCTION

Nano and Micro electromechanical systems (NEMS and MEMS) have reach a length scale at which forces of quantum origin, such as Casimir forces, play an important role [1]–[3]. A limiting factor in the operation of nems/mems is the jump-to-contact or snap-down due to the instability produced when the actuating force equals an elastic restitution force between movable parts. If the elastic force is overcome, moving parts of the device adhere to each other. The phenomena of snap-down has been studied extensively when the actuating force is of electrostatic origin [4]–[7] for different systems such as membranes, cantilevers, fixed-fixed beams, among others. In these situations, the actuating force arises from a voltage difference between movable parts of the device. For a toy-model consisting of two parallel plates with an initial separation $z = L_0$, the force between the plates if they are held at a potential difference V goes as $\sim 1/z^2$, the snap down happens when the plates are at a separation of $z_c = L_0/3$ [6]. In general, it can be shown that if the force varies as $\sim 1/z^n$ the jump to contact occurs at $z_c = L_0/(n+1)$.

Even if the plates are held at a zero potential there is always an attractive Casimir force. This force is due to the fluctuating vacuum electromagnetic field, and can be regarded as a macroscopic manifestation of retarded van der Waals forces [8], [9]. As will be discussed in the next section, this force depends strongly on the dielectric properties of the materials attaining a maximum value when ideal perfect conductors are considered. Although this is a minute force, several authors have studied its role in the operation of mems and nems. For example, Chan showed that a micro torsional balance could be driven into its nonlinear modes when actuated only by a Casimir force [1]. Serry and Maclay [10]calculated the deflection of a micro membrane strip due to the Casimir force taking into account the finite conductivity of the plates in an approximate way. Other authors have studied the problems of stability assuming only perfect conductors [11]–[13] as in the original derivation of this force [14]. The role of finite conductivity was fully taken into account by Lifshitz [15] and the stability analysis for real materials was recently presented by the authors [16]. In this work we give a brief summary of Lifshitz formalism and how to calculate the Casimir force in Section 2. Then, we present the general expressions for the stability analysis and show how it can be modified for different materials.

2 CASIMIR FORCES

The Casimir force can be understood as a macroscopic manifestation of quantum vacuum fluctuations. Casimir original calculation, considered two neutral parallel plates separated a distance L made of perfect conductors. The attractive force per unit area between them will depend only on the separation and fundamental constants as

$$F = -\frac{\hbar c \pi^2}{240 L^4}, \tag{1}$$

where $\hbar$ is Planck's constant and c is the speed of light in vacuum.

If the two plates have a finite conductivity, the attractive force between them is given by Lifshitz formula that considers the retarded Van der Waals force between macroscopic bodies. The Casimir force can be derived considering the fluctuating electromagnetic field between the plates. Let this electromagnetic field be characterized by its wave vector $q = \omega/c = \sqrt{Q^2 + k^2}$, where ω is the angular frequency, Q and k are the wave vector components parallel and perpendicular to the plates respectively. We assume isotropic plate. In terms of the reflection coefficients for p and s polarized electromagnetic waves, r_p and r_s, the Lifshitz formula is

$$F = -\frac{\hbar c}{2\pi^2} \int_0^\infty Q dQ \int_0^\infty k dq (G^s + G^p), \tag{2}$$

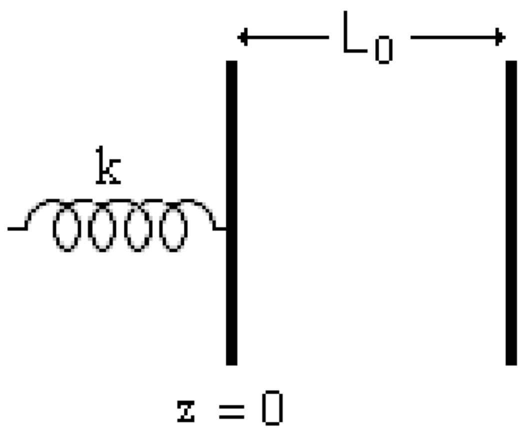

Figure 1: Model and coordinate system used in the calculation

where $G_s = (r_s^{-2}\exp(2ikL) - 1)^{-1}$ and $G_p = (r_p^{-2}\exp(2ikL) - 1)^{-1}$. Eq. (1) for ideal conductors is obtained when $|r_p| = |r_s| = 1$ for both plates.

Lifshitz formula allows the calculation of the Casimir force is a straightforward manner. Given the materials of the plates, calculate the reflectivities for both polarizations for which the dielectric function of the plates $\epsilon(\omega)$ is needed. Then, the integrals have to be calculated. For ease of calculation it is convenient to evaluate the integrals along he imaginary frequency axis changing ω by $i\omega$. Typically Eq. (2) is written in the rotated frequency axis [9].

3 The problem of stability

Many mems and nems can be modeled by a one-degree of system simple lumped system as the one shown in Fig.1.

The moving plate of mass m obeys the equation of motion

$$m\frac{d^z}{dt^2} = -kz + F, \tag{3}$$

where F is the actuating force. As an illustrative example, let us assume that the actuating force is given by Eq. (1) for perfect conductors. The steady state solution (left hand side of Eq. (3) yields

$$0 = -\kappa z + \frac{\hbar c\pi^2 A}{240(L_0 - z)^4}, \tag{4}$$

where A is the area of the plates. Introducing the variable $v = z/L_0$, the previous equation becomes

$$0 = -(\kappa L_0)v + \frac{\hbar c\pi^2 A}{240L_0^4}\frac{1}{(1-v)^4}. \tag{5}$$

Equivalently, we have

$$\lambda = v(1-v)^4, \tag{6}$$

where $\lambda = \frac{A\hbar c\pi^2}{240L_0^4}\frac{1}{\kappa L_0}$.

Physically, λ gives the ratio of the Casimir force when the plates at a distance L_0 to the elastic force. When $v = 1$ the plates are at the initial separation L_0. The set of all points (v, λ) defines the bifurcation diagram. At the maximum point the jump-to-contact occurs. Let this maximum be (v_c, λ_c).

The previous analysis can be use for the more general Lifshitz formula Eq.(2) and in the steady state the resulting equation, equivalent to Eq. (6) is

$$0 = -v + \frac{120}{\pi^4}\lambda\int_0^\infty \tilde{Q}d\tilde{Q}\int_0^\infty d\tilde{q}\tilde{k}(G^s + G^p) \tag{7}$$

The quantities with tilde are normalized to L_0, for example $\tilde{Q} = QL_0$. This is done to introduce again the bifurcation parameter λ.

In Figure 2, we present the bifurcation diagram for the parallel plate system when we have ideal conductors and when the plates are made of Si or SiN. The value of v at which the maximum occurs depends only on the functional dependence of the actuating force with plate separation, while the value of λ depends on the dielectric properties of the plates [16]. From Lifshitz formula Eq. (2) we see that if the reflectivities go to zero, so does the Casimir force. Thus, if mems/nems were made of a low reflectivity material in a wide frequency range, the value of λ_c will go to infinity and no snap-down will occur. Another alternative to inhibit the snap-down is to use repulsive Casimir forces using high μ magnetic materials [17], [18].

Being able to change the value of λ_c is important. For example, if we keep the area and initial separation L_0 constant, from the definition of lambda we see that $\kappa \sim 1/\lambda_c$. Thus, increasing the value of λ_c allows us to decrease the elastic constant of the system keeping the same stability conditions. The same argument can be used if we keep κ constant and want to reduce the initial separation L_0.

4 CONCLUSIONS

The role of Casimir forces in micro and nano technology has just been explored recently. This force depends on the dielectric function of the materials, being stronger for conductors. In this work we have presented a basic stability analysis of a one-degree of freedom simple lumped system using Lifshitz formula for the Casimir force. The critical separation at which the jump-to-contact occurs is the same regardless of the material. It depends only on how the force varies with distance. The maximum value of the bifurcation parameter does

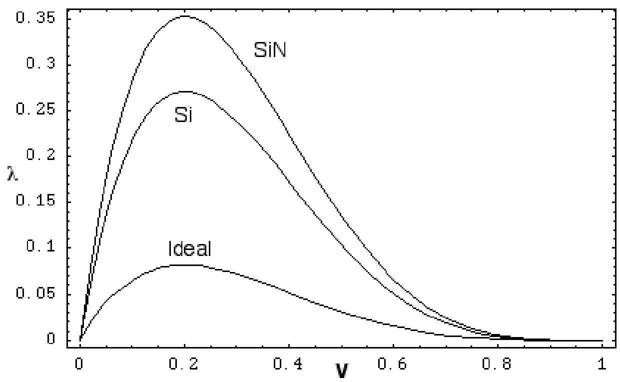

Figure 2: Bifurcation diagram for an ideal metal, Si and SiN. The maximum occurs at $v = 1/5$ due to the dependence of the force with plate separation. The value of λ_c depends on the dielectric functions of the materials.

depend on the dielectric properties of the materials. The jump-to-contact becomes less important if low reflecting materials are used in the fabrication of the devices. In this work we assumed constant values for the dielectric functions of Si and SiN. However, frequency dependent or even non-local dielectric functions can be used [19]–[21]

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