Efficient Generation of Reduced-Order Circuit and Device Models for Wide Frequency Applications

S. Mijalković and H.-C. Wu

Delft University of Technology
HiTeC-DIMES, 2628 CD Delft, The Netherlands, S.Mijalkovic@TUDelft.NL

ABSTRACT

A methodology for efficient reduction of large scale circuit and device models have been discussed. The basis for Galerkin subspace projection is created by the singular value decomposition of the system frequency response. The proposed methodology allows efficient creation of reduced-order models for wide frequency applications. It is practically verified in two test examples from circuit and semiconductor device simulation.

Keywords: model order reduction, frequency response, singular value decomposition, electronic circuits, semiconductor devices.

1 INTRODUCTION

The circuits, systems and their components are often represented by highly dimensional models with large computational, accuracy and storage requirements. The aim of model order reduction (MOR) algorithms is to formulate a similar model of considerably smaller dimension but with approximately the same input-output characteristics as the original one [1]. To this end, MOR exploits the correlation among the original model state variables in order to gain control over the model dynamics from a certain lower dimensional subspace having much smaller number of variables. The reduced-order model (ROM) is obtained by projecting the original model governing equations onto such a lower dimensional subspace.

The standard global approximation (GA) MOR techniques, like *balancing transformations* [2], are capable of generating ROMs valid in a wide frequency range. However, the computational cost of GA methods makes them applicable only for the relatively small problems. On the other hand, local approximation reduction techniques, like *moment matching* and *Krylov subspace methods*[3], can handle large-scale problems but provide accurate approximation only in the vicinity of the selected expansion complex frequencies.

A methodology for automated and efficient generation of GA MOR subspaces from the system frequency response is discussed in this paper as an alternative solution. The method is practically demonstrated in the reduction of large LRC electronic circuit and small signal modeling of active semiconductor devices.

2 PROBLEM FORMULATION

A large class of continuous linear time-invariant circuit and device systems with m inputs and p outputs can be modeled by the state and output equations in the form

$$C\frac{dx(t)}{dt} = -Gx(t) + Bu(t) \tag{1}$$

$$y(t) = Lx(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the model state variables at time t, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are vector-valued functions of the model input and output variables while $C \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $L \in \mathbb{R}^{p\times n}$ are system matrices. Using Laplace transformation, the model (1) can be equivalently represented in the frequency domain as

$$y(s) = H(s)u(s) \tag{3}$$

$$H(s) = L\left(G + sC\right)^{-1} B \tag{4}$$

where $H(s)$ is the system transfer function and s is the complex frequency. The model is fully defined by a set of the system matrices

$$M = (C, G, B, L) \tag{5}$$

in both time and frequency domain. The MOR problem can be specified as: find a model M_R such that, for the same input vector u, M and M_R have approximately the same output vector y.

It is widely accepted today to solve the MOR problem in the framework of the subspace projection [4]. The subspace projection is defined as a set of two matrices

$$\Pi = (V, W) \tag{6}$$

where $V \in \mathbb{R}^{n\times q}$ is *the basis matrix*, whose columns are subspace basis vectors and $W \in \mathbb{R}^{n\times q}$ is *the restriction matrix*. For the projection matrices holds $W^TV = I$, where $I \in \mathbb{R}^{q\times q}$ is the identity matrix. The reduced-order model obtained by projection Π is determined as

$$M_R = \left(W^TCV, W^TGV, W^TB, LV\right). \tag{7}$$

The restriction matrix W is often selected as $W = V$ in order to preserves important numerical range properties of the original model [4]. It is often referred to as *Galerkin projection* while for $W \neq V$ we speak of the *oblique* or *Petrov-Galerkin* projection.

Within the Galerkin projection framework the MOR problem may be reformulated as: find the basis matrix V such that for any model state x there is the unique vector of generalized coordinates $x_R \in \mathbb{R}^q$ satisfying

$$x(t) = V x_R(t) \quad \text{or} \quad x(s) = V x_R(s) \tag{8}$$

with sufficient accuracy. In other words, the state vectors x should belong to the subspace formed by columns of V. Linear independence of the subspace basis is achieved by requiring that $V^T V = I$.

The most important for MOR is the system impulse response since it uniquely determines any other input waveform. Let $X(t) \in \mathbb{R}^{n \times m}$ be the solution of the state equation

$$C \frac{dX(t)}{dt} = -GX(t) + B\delta(t)I \tag{9}$$

which amounts to applying an impulse excitation at each input channel in turn and collecting the resulting state vectors as columns of X. The subspace basis V with

$$X(t) = \text{colspan}(V) \quad t \in [0, \infty) \tag{10}$$

would provide GA property to the corresponding reduced-order model. The methods of *balanced truncations* aims at capturing full impulse response $X(t)$ of the system. The price is computationally expensive procedure requiring the order of n^3 operations. Together with accuracy considerations this makes the method of balanced truncation applicable only to systems of modest dimensions.

On the other hand, *the moment matching methods* employ the Tailor expansion of the state vector around the selected complex frequency s_0:

$$x(s_0 + \sigma) = \sum_{i=0}^{\infty} x_i \sigma^i \tag{11}$$

in order to create the subspace V with

$$[x_0, x_1, \ldots, x_r] = \text{colspan}(V) \tag{12}$$

that is, the subspace that spans the Tailor coefficients of the state vector. A variety of the well established Krylov subspace techniques are used to create sets of the Tailor expansion coefficients. The moment matching methods typically require only the order of n^2 operations. However, these methods have only local approximation property in the neighborhood of the expansion point s_0.

3 FREQUENCY RESPONSE SUBSPACES

The state-space system (1) can be also fully characterized in the real frequency domain by the transfer characteristics

$$H(j\omega) = L^T (G + j\omega C)^{-1} B \tag{13}$$

obtained by replacing the complex variable s by $s = j\omega$ where ω is the angular frequency. Notice that the transfer characteristics $H(j\omega) \in \mathbb{C}^{n \times m}$ actually represents the spectrum of the system output $y(t)$ for unit impulse response. This spectrum of the state vector $x(t)$ for unit impulse response applied in turn at different system inputs is the complex matrix $X(j\omega) \in \mathbb{C}^{n \times m}$ being the solution of the steady state frequency response problem

$$(G + j\omega C)\, X(j\omega) = B. \tag{14}$$

The columns of the matrix $X(j\omega_k)$ are state phasors corresponding to different inputs. Separating $X(j\omega)$ into the real and imaginary part as

$$X(j\omega) = P(\omega) + jQ(\omega), \tag{15}$$

it is obvious that an orthonormal projection basis containing the columns of $P(\omega) \in \mathbb{R}^{n \times m}$ and $Q(\omega) \in \mathbb{R}^{n \times m}$ will preserve the spectrum od the system state in the reduced-order model formulation. Namely, the impulse response in time domain $X(t)$ can be recovered from the frequency response as $X(s)$ as [5]

$$X(t) = \frac{1}{\pi} \int_0^{\infty} [P(\omega) cos(\omega t) - Q(\omega) sin(\omega t)]\, d\omega. \tag{16}$$

In other words, it is sufficient to look for the subspace basis matrix V that contain the columns of $P(\omega)$ and $Q(\omega)$, that is,

$$[P(\omega)\, Q(\omega)] \in \text{colspan}(V) \quad \omega \in [0, \infty). \tag{17}$$

in order to capture the system impulse response in ROM. In practice it is reasonable to look only for a subspace that sufficiently accurately spans an ensemble of state phasor's components sampled at finite set of selected discrete frequency points ω_k, $k = 1, 2, \ldots, r$. The single frequency ensemble blocks are collected into the full data ensemble

$$X = [P(\omega_1)\, Q(\omega_1)\, P(\omega_2)\, Q(\omega_2) \ldots P(\omega_r), Q(\omega_k)] \tag{18}$$

where $X \in \mathbb{R}^{n \times 2mr}$.

A subspace basis matrix V that spans data ensemble X with required accuracy, is obtained in efficient way by the singular value decomposition (SVD) of X

$$X = U \Sigma Z^T \tag{19}$$

where Σ is a diagonal matrix

$$\Sigma = \text{diag}\,(\sigma_1, \sigma_2, \ldots, \sigma_{2mr}) \tag{20}$$

with singular values $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{2mr} \geq 0$ while $U \in \mathbb{R}^{n \times 2mr}$ and $Z \in \mathbb{R}^{2mr \times 2mr}$ are orthogonal matrices. The subspace basis of frequency response coherent structures is simply obtained as

$$V = U(:, 1:q) \tag{21}$$

that is, taking the first q ($q < 2mr$) columns of matrix U as the subspace basis matrix V.

The two most important features of the resulting subspace basis are optimality and controllability. Moreover, the minimum value of $e_{\mathcal{V}}$ over q-dimensional subspace is bounded as [1]

$$e_{\mathcal{V}} \geq \sum_{i=q+1}^{2mr} \sigma_i^2 \tag{22}$$

which may be employed as an estimate of the achievable accuracy in approximating a data ensemble X and adaptive selection of probing frequency points within frequency range of interest.

4 EXAMPLES

4.1 LRC Electronic Circuit

As a first practical example, the proposed MOR technique is applied to the generalized state-space system (1) originating from the modified nodal analysis of multi-port linear LRC circuit. It belong to the collection of benchmark examples [6] for testing MOR of linear time invariant systems. The selected problem defines 4-port network ($m = p = 4$ and $L = B$), with $n = 980$ degrees of freedom. Fig. 1 compares y-parameters of the original and corresponding reduced model. The reduced order model has provided sufficient accuracy in the frequency range $10^4 - 10^9$ Hz with only $q = 4$ internal degrees of freedom.

4.2 Active Semiconductor Devices

MOR is so far mainly considered as a tool for reduction of passive networks and devices. Here it is demonstrated how it can be also successfully applied to active semiconductor devices. Fig. 2 shows a typical doping profile and grid structure of the bipolar junction transistor (BJT). The device matrices C, G, B and L in (1) are assembled using open simulation environment SG Framework (SG) [7] while the ROM generation and testing have been performed in Matlab®.

Figs. 3 shows the frequency dependence of the BJT h_{21} parameter obtained by SG and ROMs of different complexity. Notice that the ROM complexity can be automatically adjusted to the required modeling accuracy. While ROM with $q = 4$ (2 basis vectors per each port) is sufficient for the accurate small-signal modeling up to 1 GHz, the ROM with $q = 8$ captures also the non-quasi-static phenomena in the higher frequency range.

This example indicates a possibility to employ MOR for automated generation of efficient CAD models of small signal semiconductor device operation in the wide frequency range. It could be particularly useful in situations when there is no an appropriate compact device model. The ROM complexity is easily adapted to the required modeling accuracy. Extension to nonlinear modeling could be possible via multibias piecewise-linear data-base approaches.

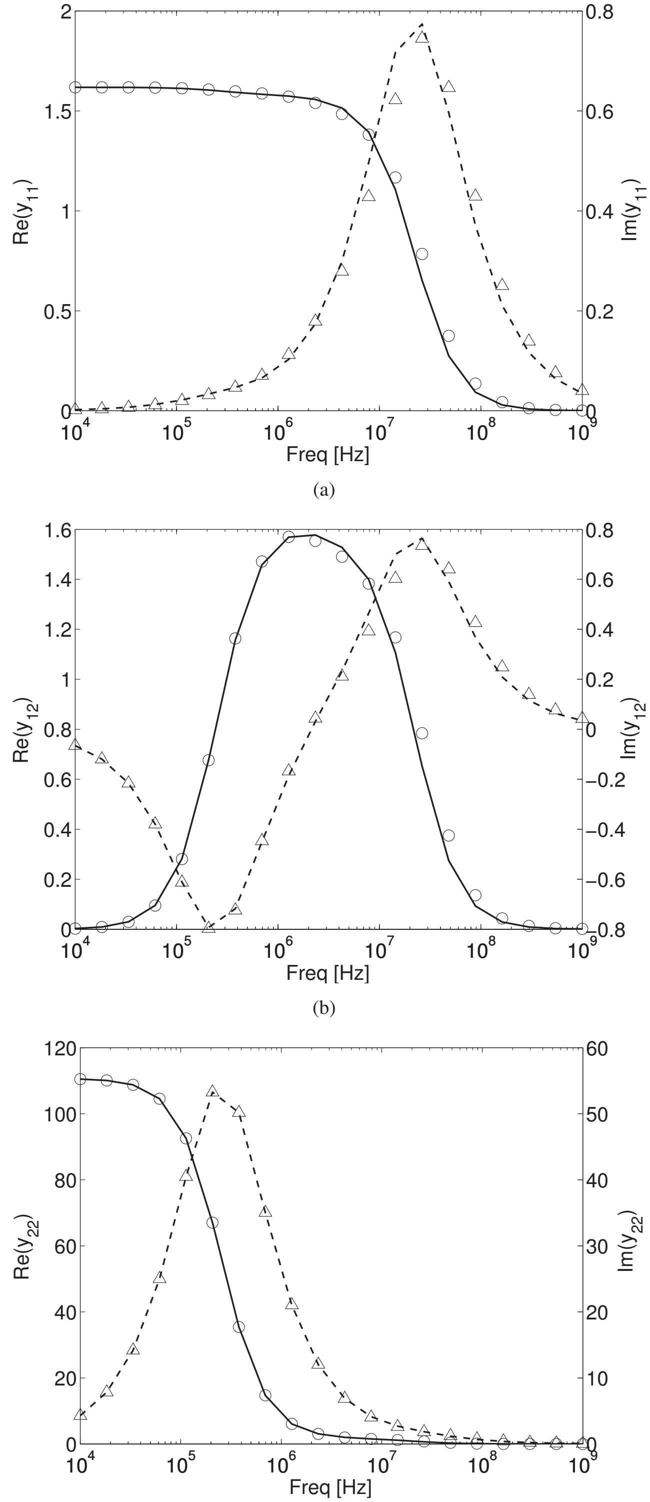

Figure 1: Frequency dependence for the real and imaginary parts of y parameters

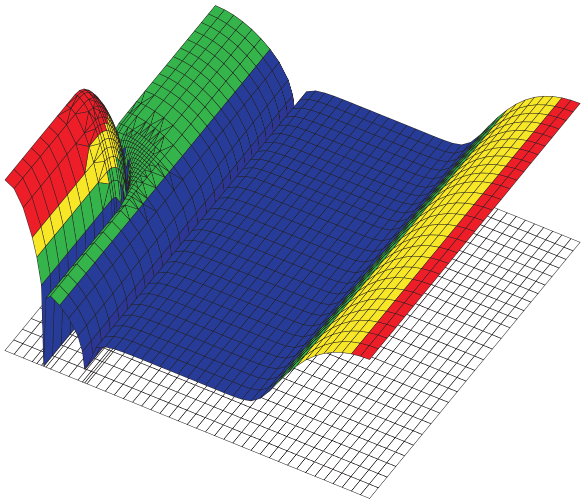

Figure 2: BJT doping profile and grid structure.

5 CONCLUSIONS

An efficient approach to the creation of MOR projection subspaces for stable and accurate model order reduction has been presented. It is based on the SVD of data assembled in a systematic way by sinusoidal steady-state analysis produces frequency responses, subspaces of high fidelity for model order reduction in the wide frequency range.

The proposed model order reduction approach is simple to implement. It requires only a set of state vector phasors from the standard sinusoidal steady-state analysis in the frequency range of interest and standard linear algebra SVD procedure. The computational cost of the required frequency sweep and SVD procedure could be significantly smaller then the cost of full eigensystem analysis. The selection of the test real frequency points is quite straightforward in comparison to the selection of complex frequency points in Krylov subspace methods.

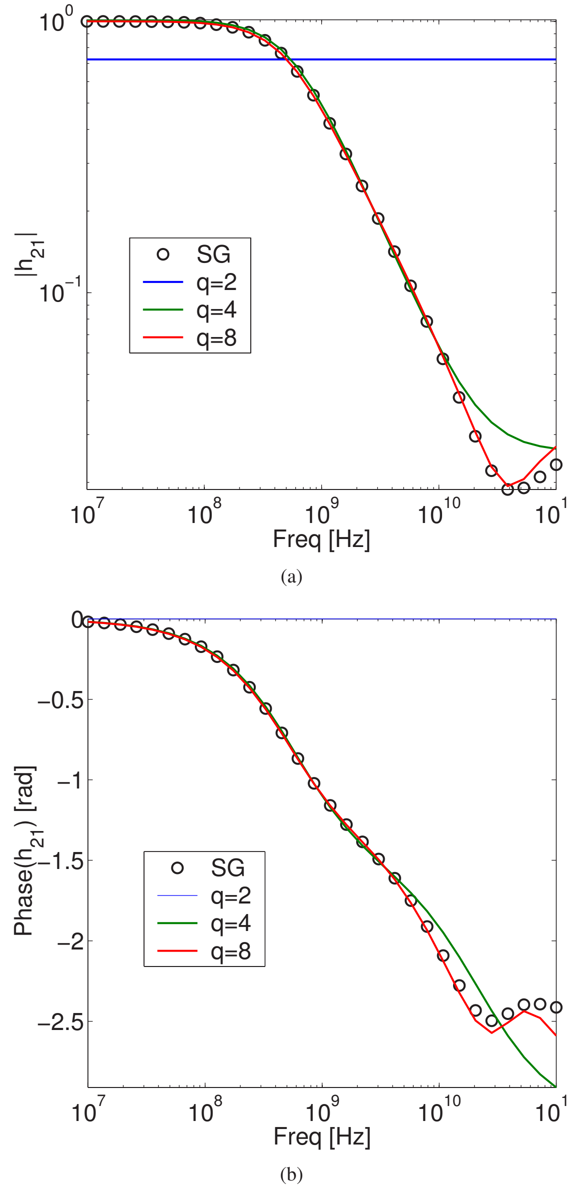

Figure 3: Bipolar transistor example: (a) frequency dependence of the normalized h_{21} parameter (a) magnitude (b) phase (symbol-original model, lines-ROM)

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