The Computational Abilities of Random Magnetic Structures

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ABSTRACT

Quantum-dot cellular automata (QCA) are proposed to replace conventional CMOS techniques for the construction of logic gates. Previous research has shown that QCA are likely to be sensitive to errors in placement of the individual dots within a QCA cell. An approach using random quantum-dot structures claims to partially alleviate the problem of dot placement in a QCA cell. Magnetic versions of QCA are promising candidates for realizing low-power computing devices. In this paper, we investigated the computational abilities of random magnetic structures and compared their behavior with random electronic quantum-dot structures.

Keywords: QCA, Magnetic QCA, random structures, placement errors

1 INTRODUCTION

Quantum-dot cellular automata (QCA) are proposed to replace conventional CMOS techniques for the construction of logic gates. QCA show great promise for fast computation with low heat generation in the realm of nanometer feature sizes [1]. Previous research has shown that QCA are likely to be sensitive to errors in placement of the individual dots within a QCA cell [2]. An alternative approach using random quantum-dot structures, which claims to partially alleviate the problem of dot placement has been discussed [3]. In random structures, quantum dots are placed to build a device in hopes that the computational ability of the device is assuredly present. Surprisingly small structures almost always compute basic logic functions.

In an offshoot of QCA, magnetic versions have been explored and are promising candidates for realizing low-power computing devices [4].Magnetic QCA have the following advantages. They seem simpler to fabricate and inputs may be easier to control during simulations. Inputs are held at required constant value during computations.
However, it is believed that magnetic QCA like electron based QCA are also sensitive to placement errors. The goal of our work is to simulate magnetic QCA using random structures and to compare and contrast their behavior with electronic random structures.

2 BACKGROUND

In random magnetic structures, each dot can exist at least in two polarities, North-South and South-North. Let North-south and South-North be designated as High and Low respectively. The polarity of a dot is influenced by the polarity of neighboring dots.

Consider a structure where magnetic dots are placed randomly. Let any two dots in the structure, say dot A and dot B are selected as inputs to the structure. Inputs dots can be in one of the two states, say LOW and HIGH and they are held constant throughout the relaxation process.
If one cycles all combinations of input and one records the ground states of the remaining dots, one can generate truth table. If one is lucky, the truth table may reveal functions of interest such as OR and AND.

To show computation with magnetic dots, consider a magnetic structure in which 6 dots are randomly generated. Suppose if the polarity of the two dots say dot A and dot B are kept as input and the polarity of dot E is fixed at 0, which is covered by rectangular box, then dot C will compute the output.

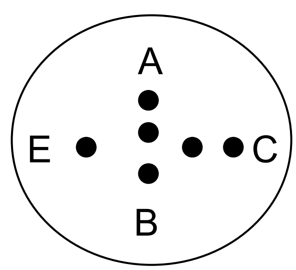


Figure 1

The four possible combinations of inputs A and B are drawn below in Figure 2

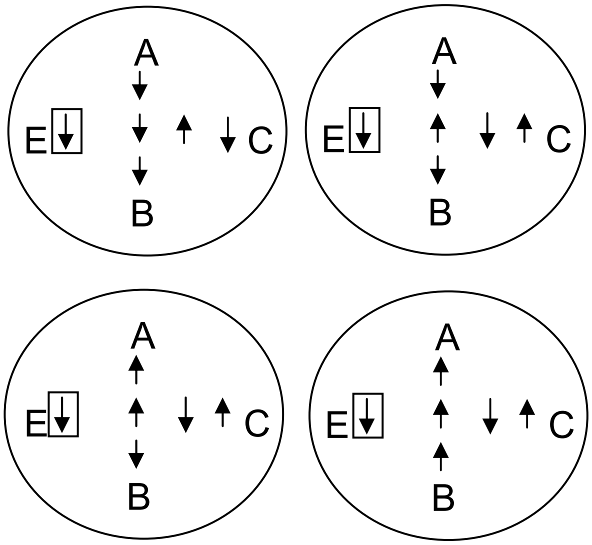


Figure 2

The output dot C depends on the inputs A and B. Here, dot C computes an OR function of A and B. The dots will orient in such a way that total energy of the magnetic system is the least

3 MAGNETIC SYSTEM

We model such a system with classical magnetic dipoles. Energy (E) of magnetic dipole in an external magnetic field is given by the equation [5]

$$E = - \mu B \cos Ø \quad (1)$$

μ is the dipole moment of the magnetic dipole expressed in Weber meter.
B is the extern magnetic field magnetic field expressed in Tesla
Ø is the angle between the dipole moment (μ) and the field (B).

The energy of interaction between two magnetic dipoles having magnetic moments m_1 and m_2 respectively and separated by a distance R is given by equations [5]

$$E = -m_1 \cdot B_2 \quad (2)$$

or

$$E = -B_1 . m_2 \quad (3)$$

Here B_1 is the field generated at the position of dipole #2 by dipole #1, and B_2 is the field at dipole #1 generated by dipole #2.

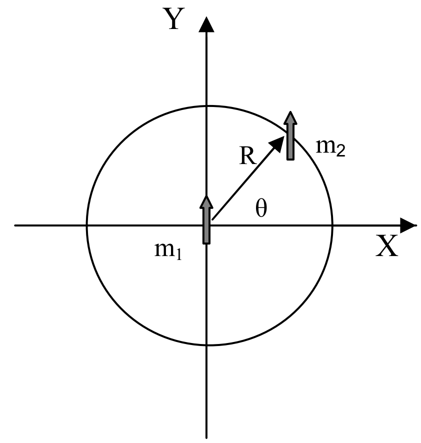


Figure 3

The field B_1 has both x and y components. The value of B_{1x} and B_{1y} are given by the equation [5]

$$B_{1x} = [\mu_0/4\pi]\,[m_1/R^3]\,[3\sin\theta\cos\theta] \quad (4)$$

$$B_{1y} = [\mu_0/4\pi]\,[m_1/R^3]\,[3\sin^2\theta - 1] \quad (5)$$

Therefore, Interaction Energy (E) is given by equation [5]

$$E = -B_1.m_2 = -[\mu_0/4\pi]\,[m_1 m_2/R^3]\,[3\sin^2\theta - 1] \quad (6)$$

From this expression, we say interaction energy between two magnetic dipoles is the minimum when the angle (θ) between them is ($\pi/2$) radians and the minimum energy is given by the equation [5]

$$E_{min} == -2\,[\mu_0/4\pi]\,[m1.m2/R^3] \quad (7)$$

4 SIMULATIONS

Simulations were performed in following three steps.

1) Random Structure generation
2) Exhaustive search for lowest energy computation
3) Search for Boolean functions.

4.1 Random Structure generation

It generates the dimensions for any number of given dots. The dimensions associated with each dot are magnetic length, magnetic moment, x-coordinate value, y-coordinate value, orientation, fixed and possible, where fixed and possible are used in Boolean search. rand () and srand () functions in the C library were used to generate random values.

4.2 Exhaustive Search

Exhaustive search is performed to find the ground state (lowest energy) of the system. In the exhaustive search, all possible polarities of the magnets are considered. For n dots, there are 2^n possibilities. The energy of the structure for each 2^n possible combination is computed. It is guaranteed that exhaustive search will find the ground state of the system. The lowest energy and the corresponding polarities are saved. However, as the number of magnets increases, the time to search increases dramatically.

4.3 Search for Boolean functions

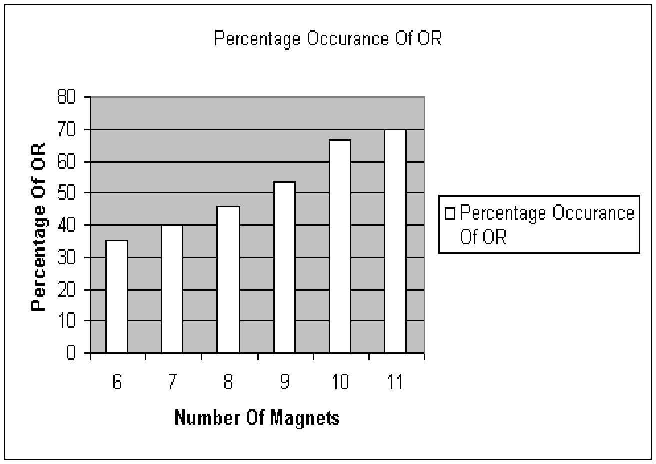
In the Boolean search two inputs are chosen. For each of the four combinations, the ground state is found and the corresponding polarities of the dots are recorded. If an output dot has a polarity opposite of that expected, then that dot is no longer considered a possible output. The pseudo code used to search for OR logic function is given as

```
function orsearch ()
   {
   Var i, j, k, z;

   for (i = 0; i < nMagnets; ++i)
      {
         for (j = i + 1; j < nMagnets; ++j)
            {
            setPossibleToTrue (Magnets);
            Magnets[i]->fixed = 1;
            Magnets[j]->fixed = 1;
            for (k = 0; k < 4; ++k)
               {
                  result = (k != 0);
                  setPloarityToRandom (Magnets);
                  Magnets[i]->polarity = k % 2;
                  Magnets[j]->polarity = (k / 2) % 2;

                  findLowestEnergy (Magnets); // Exhaustive search
                  for (z = 0; z < nMagnets; ++z)
                     {
                     if (z == i || z == j) continue;
                     if (Magnets[z].polarity! = result)
                              {
                              Magnets[z].Possible = False;
                              }
                     }
                  }
            for (z = 0; z < nMagnets; ++z)
                  {
                 if (z == i || z == j) continue;
                  if (Magnets[z].Possible == 1) return 1; //found an
OR
                  }
            }
         }

   return 0; // did not find OR
   }
```

The presence of a Boolean function depends on the polarity and location of the dots.

5 RESULTS

Simulations show that relatively small magnetic structures compute the logical functions such as OR. The graph below summarizes the percentage of OR seen for the corresponding number of dots in the random structure.



We confined our simulations to eleven magnetic dots, due to rapidly increasing search times. However, by extrapolating from the graph 16 or more number of dots will effectively always compute an OR function.

6 DISCUSSION

In contrast to prior results [3], although larger magnetic structures seem to require as compared to random quantum dot structures, the apparent advantages of magnetic structures warrants further investigation.

7 REFERENCES

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