Electromechanical Model for a Single Port Nano-Mechancial Beam Resonator

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ABSTRACT

A simple electromechanical model for a FIB fabricated nano-mechanical, magneto-motively driven beam resonator is derived. The model is developed in order to determine the feasibility of a single port device and enable potential applications of these nano-resonators to be considered. The model is based on a lumped parameter approach using an equivalent electrical circuit.

Keywords: FIB fabrication, nano-resonator, single port, electrical equivalent approach.

1 INTRODUCTION

Focused ion beam (FIB) technology has the capability to fabricate structures down to tens of nanometers [1]. It has been established [2] that the most viable method of excitation and detection is by using Lorentz force. It is proposed that due to the small scale of these devices, it would not be practical to use two independent drive and pick-off. The parasitic capacitance between the two tracks would mean the cross talk would mask the induced signal in the pick-off track. Therefore, it is proposed that a single port device would be most viable since it would require just a single drive track. Here we analyse the electrical properties of such a device.

The full electromechanical behavior of beam resonators can be described by differential equations of motion of the mechanical elements, and characteristic equations of the transducer elements. Solving these equations can often only be accomplished through finite element modelling and hence can be time consuming. Frequently, the full behaviour of the resonator is not required since it will operate at the fundamental mode [3]. In this situation it is possible to use the equivalent circuit approach [4, 5], where both the mechanical and electrical features of the resonator are represented by electrical components. Using this technique, the complex differential equations are replaced by lumped electrical circuit components that are representative of the system's mass, stiffness etc. The lumped element circuit can then be easily solved using electric network analysis software such as SPICE.

In this work, the equivalent circuit approach is used to model the electrical properties of a magneto-motively exited [2] nano-mechanical beam resonator. This resonator is typical of that which would be found in a resonant sensor or filter.

2 MODEL FORMULATION

The nano-mechanical beam resonator to be modelled is a single port actuation-detection device; that is, it has a single conduction track performing both drive and pick off function. A single port configuration is chosen since it is most compatible with nano-scale devices. The fabricated device is illustrated in figure 1. The only inputs to the model are the quality factor of the resonance and the geometrical dimensions of the beam. The validity of the model is based on: homogeneous mass distribution, the negligible affect of drive tracks, and simple beam theory.

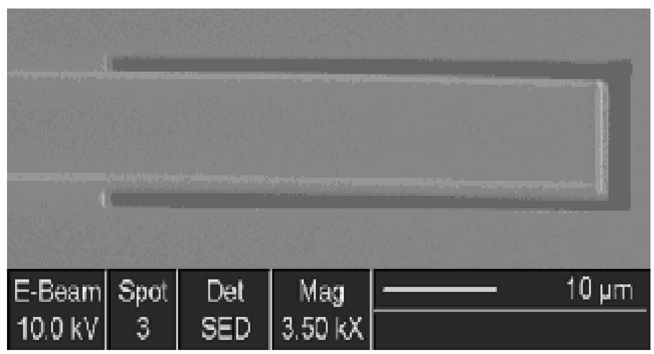

Figure 1: A 50x15x0.5μm Si_3N_4 cantilever fabricated by FIB technology [1], featuring a single FIB-deposited platinum drive track (0.25x0.25μm cross section) around the perimeter. An external magnetic field is applied in plane and parallel to the longitudinal-axis of the resonator.

The method of applying the equivalent electrical approach illustrated here derives the equations of motion for the system, and characteristic equations for the transducer elements and then makes the analogy with the equations for an electric circuit to derive the lumped electrical parameters.

We begin by determining the resonant frequency and stiffness of the cantilever using standard beam theory [6]. The equation of motion of the system is given by:

$$m_{eff}\frac{d^2y(t)}{dt^2}+c\frac{dy(t)}{dt}+ky(t)=L_aBI_d(t) \quad (1)$$

where m_{eff}, c, and k are the lumped, or effective, mass, damping and stiffness terms. L_a is the active length of the drive track, B the external magnetic field, and finally $I_d(t)$ the driving current. Simplifying and transforming to the frequency domain yields the system transfer function:

$$y(s) = \frac{L_a B / m_{eff}}{s^2 + 2\xi\omega_n s + \omega_n^2} I_d(s) \tag{2}$$

where ω_n is the resonant frequency of the system and ξ is the damping coefficient. The motion of the cantilever in the magnetic field generates an electromotive force in the drive track. The magnitude of this emf is given by [7]:

$$V_{emf}(t) = L_a B \dot{y}(t) \tag{3}$$

Thus, the back emf generated in the drive track is given by:

$$V_{emf}(t) = \frac{s L_a^2 B^2 / m_{eff}}{s^2 + 2\xi\omega_n s + \omega_n^2} I_d(s) \tag{4}$$

Modelling the magnetically coupled oscillator as a parallel combination of a resistor R_m, an inductor L_m, and a capacitor C_m, figure 2, the voltage drop $V(s)$ across the combination is:

$$V(s) = Z_{total}(s) I(s) = \frac{s/C_m}{\omega_{LC}^2 + s^2 + s/R_m C_m} I(s) \tag{5}$$

or

$$V(j\omega) = \frac{j\omega/C_m}{\omega_{LC}^2 - \omega^2 + j\omega/R_m C_m} I(j\omega) \tag{6}$$

where $I(s)$ is the drive current, and $\omega_{LC} = (L_m C_m)^{-0.5}$. It can be seen that the two transfer functions have the same form, and as such the electrical equivalent is appropriate. By comparing the two expressions it is possible to identify the circuit parameters in terms of the mechanical properties of the resonator.

$$R_m = \frac{L_a^2 B^2}{\omega_n m_{eff}} Q \quad L_m = \frac{L_a^2 B^2}{\omega_n^2 m_{eff}} \quad C_m = \frac{m_{eff}}{L_a^2 B^2} \tag{7-9}$$

Having established these electrical equivalents, it is then possible to construct a model and analyse the response using SPICE modelling software.

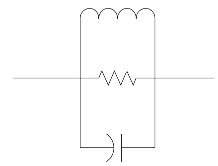

Figure 2: Electrical equivalent circuit for a magnetically coupled resonator.

3 RESULTS

For the device illustrated in figure 1, these electrical equivalent components were then determined and used to create a SPICE model. The resonant frequency of the cantilever is [6]:

$$\omega_n = \frac{1.875^2}{2\sqrt{3}} \sqrt{\frac{E}{\rho}} \frac{t}{l} \tag{10}$$

where E, ρ, t and l are the Young's modulus, density, thickness and length of the cantilever respectively. For the fabricated cantilever:

$$\rho = 3100 kgm^{-3} \tag{11}$$

$$E = 385 GPa \tag{12}$$

and

$$\omega_n = 7.972 \times 10^6 rads^{-1} = 1.269 MHz \tag{13}$$

The effective mass of the system can be determined using the general equation for the resonant frequency of a system [6]:

$$\omega_n = \sqrt{\frac{k}{m_{eff}}} \tag{14}$$

where the stiffness is given by [8]:

$$k = \frac{Ewt^3}{4l^3} \tag{15}$$

where w is the width of the cantilever. Hence for the fabricated device:

$$k = 1.444 Nm^{-1} \tag{16}$$

$$m_{eff} = \frac{k}{\omega_n^2} = 22.717 \times 10^{-15} kg \tag{17}$$

Using these properties, the electrical equivalent properties of the device can be determined (a Q of 10000 is assumed):

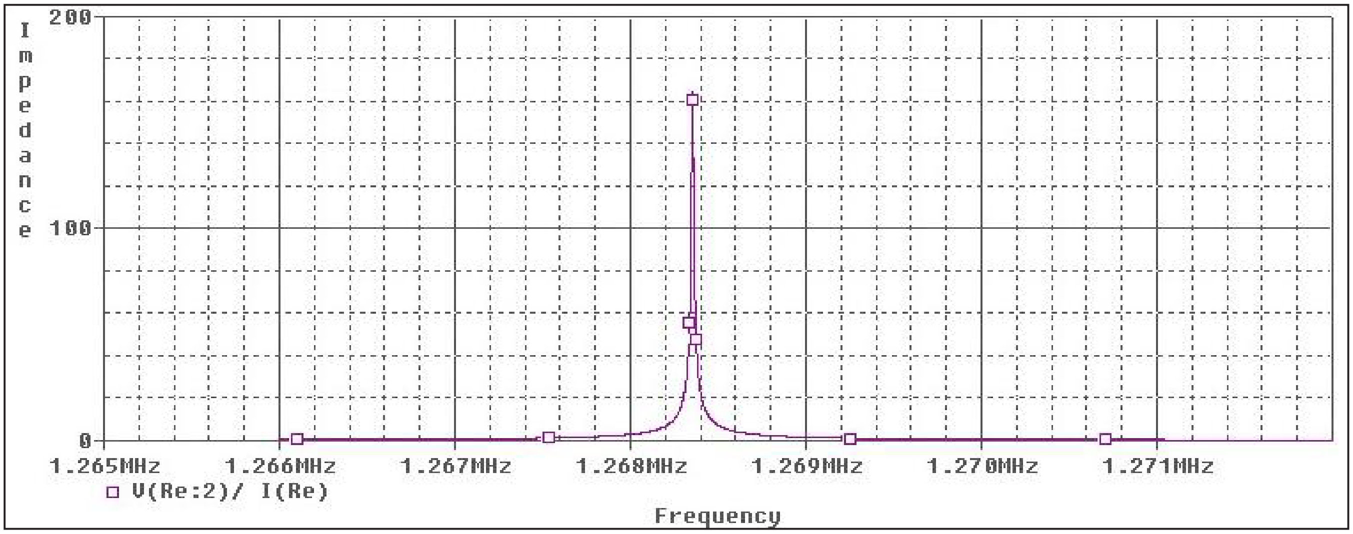

Figure 3: The electrical response spectrum of the fabricated cantilever (figure 1) yielded using the equivalent electrical approach.

$$R_m = 124.2\Omega \quad (18)$$

$$L_m = 155.8 \times 10^{-12} H \quad (19)$$

$$C_m = 101.0 \mu F \quad (20)$$

Using SPICE software, the impedance of an electrical circuit with these properties can be determined for range of frequencies (figure 3). As can be seen, there is a considerable change in the impedance at the resonant frequency of the device. This demonstrates the feasibility of such a single port device. Work is continuing to test the response of the device experimentally.

4 CONCLUSION

An electromechanical model has demonstrated the feasibility of a single port magneto-motive nano-resonator. The model was developed using the electrical equivalent approach. The electric circuit would also help in the design of the signal processing electronics. Finally, the model could equally be applied to other magneto-motive driven structures with consideration to the initial lumped equation of motion.

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