Stress optimization of a micromechanical torsional spring

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ABSTRACT

In this paper we present a method for the design of a stress-optimized and reliable micromechanical torsional spring. Our optimization approach is based on a software tool for simulation-based optimization – *MOSCITO* [1], [2], in combination with the FEM tool *ANSYS*®. For a specific spring geometry we successfully increase the maximal possible deflection by 30 percent, keeping a given limit for the stress.

Keywords: MEMS, *ANSYS*®, *MOSCITO*, optimization, torsional spring

1 INTRODUCTION

Microsystems in SOI technology take advantage of the excellent mechanical properties of monocrystalline silicon which are superior to those of steel [3]. As a brittle material silicon does not exhibit plastic deformation at room temperature. Monocrystalline silicon is also free of fatigue unless local stress peaks lead to crack initiation. In this way it is ideal for the fabrication of springs.

Micromechanical torsional springs are important and frequently applied components of MEMS design. They are easy to dimension and show nearly linear behavior concerning the torsional spring constant. Simultaneously they are often the most stressed components and therefore crucial for reliability. For this reason it is important to investigate and understand the mechanisms of stress and failure within the springs. So it becomes possible to find a spring geometry with optimized distribution of mechanical stress, increased reliability and applicability.

Unfortunately universally valid values for material properties like Woehler curves are not available. This is due to the complex and usually unknown properties of the specific technology used to microstructure the silicon [5]. Another point is the orthotropy of monocrystalline silicon in terms of its material properties. It must not be neglected and increases the modeling effort. In order to determine maximal acceptable mechanical stress within the springs some experiments with special testing structures were accomplished at *Fraunhofer IPMS*. The results can be used to verify the two provided physical models introduced later in this paper. The basis of the experiments were micromirrors with comparatively short torsional springs (*figure 1*). They were driven in resonant mode in a vacuum chamber, increasing the deflection gradually. The spring breaks at a specific deflection angle which is measured [4]. Conclusions regarding the stress can be drawn, since the geometry of the torsional spring and the angle of fraction are known. *Table 1* summarizes the results of the experiments.

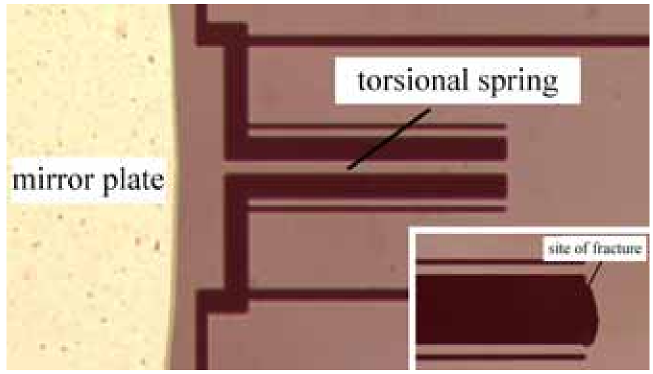

Figure 1: Light microscope photograph of a micromechanical torsional spring before and after failure [4]

Another important finding from the experiments is the preferred location of failure. The test structures always broke at the end of a torsional spring (*figure 1*).

Table 1: Material properties of monocrystalline silicon

Property	Value: Typical†	Value: *IPMS*‡
critical shear stress	$> 3\ GPa$	$1.4\ GPa$
critical tensile stress	2 to 3 GPa	$0.9\ GPa$

†unstructured Si wafer at room temperature; static load [5]
‡experimental results; dynamic load ($N > 10^9$) [4]

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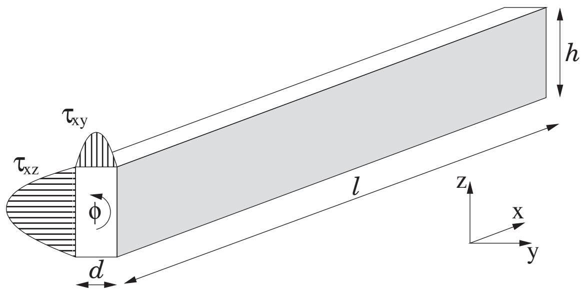

Figure 2: Torsional spring as ideal beam

2 ANALYTICAL MODEL

In order to verify and understand a numerical model, we need an analytical model for the distribution and the strength of the mechanical stresses within the spring. The following assumptions are made:

A1 The torsional spring can be treated as a beam according to solid state mechanics. (linear, ideally elastic materials; straight edges; $l \gg d$; $l \gg h$)

A2 The spring's cross section can be treated as a rectangle.

A3 Clamping effects can be neglected.

A4 Shear stress is responsible for failure.

Considering the orthotropic material properties of monocrystalline silicon, the torsional function Ψ can be described by the following partial differential equation [5].

$$\frac{1}{G_{xz}}\frac{\partial^2\Psi}{\partial y^2} + \frac{1}{G_{xy}}\frac{\partial^2\Psi}{\partial z^2} = -2\frac{\phi}{l} \qquad \Psi_{\mathrm{BC}} = 0 \tag{1}$$

Where G is the shear modulus, ϕ the torsional angle, l the length of the beam and $\Psi_{\mathrm{BC}}=0$ is the boundary condition for the cross section. Then the components of shear stress within the beam's cross section result in

$$\tau_{xz} = -\frac{\partial\Psi}{\partial y} \qquad \tau_{xy} = \frac{\partial\Psi}{\partial z} \tag{2}$$

Introducing a mean shear modulus

$$G_{\mathrm{m}} = \frac{l}{\phi\, I_{\mathrm{t}}}\int \Psi dA \tag{3}$$

[5] with I_{t} as the torsional moment of inertia helps to simplify the problem. Since G_{m} depends on geometry and orientation of the beam within the crystal plane, the orthotropy of monocrystalline silicon is still considered. For an orientation of the beam in $\langle 110\rangle$-direction one can find the following equation between quantities as an approximated result of (3) [5], [6].

$$G_{\mathrm{m}}/_{GPa} = \frac{85.8}{4\left(\frac{d}{h}+0.08517\right)^2+3} + 51.2 \tag{4}$$

The maximum shear stress, which is suspected to be responsible for failures, is located in the center of the longer edges of the spring's cross section. It results in

$$\tau_{\max} = \frac{G_{\mathrm{m}}\, I_{\mathrm{t}}}{W_{\mathrm{t}}\, l}\phi \tag{5}$$

[7] with W_{t} as torsional section modulus. This easy to manage equation can be used to determine the maximal shear stress for every beam with $l \gg d$ and $l \gg h$ (fulfilling assumption **A1**). *Table 2* shows some results of (5) for common spring geometries.

Table 2: Maximum torsional stresses within beams in $\langle 110\rangle$-direction; torsional angle $\phi = 24°$

Beam geometry $(l \times d \times h)/_{\mu m^3}$	I_{t} [7] $\cdot 10^{21}/_{m^4}$	W_{t} [7] $\cdot 10^{15}/_{m^3}$	G_{m} $/_{GPa}$	$\tau_{\max}$ $/_{GPa}$
$150 \times 6.6 \times 30$*	2.48	0.38	76.64	1.40
$200 \times 6.6 \times 30$	2.48	0.38	76.64	1.05
$240 \times 4.0 \times 30$	0.58	0.15	78.09	0.54

* *Fraunhofer IPMS* testing structure, introduced in [4]

3 FEM MODEL

In general FEM models fit reality better than simplified analytic models. It is possible to consider geometrical details like roundings. Furthermore, non-linear effects can be included without additional modeling effort. On the other hand, FEM models also have disadvantages. They generally require large computation efforts. Furthermore, it is hardly possible to generalize single results. Thus it is necessary to repeat model generation and calculations for every single geometry. Another issue is the interpretation of the results. They often strongly depend on parameters like choice of FEM elements or meshing grid.

With our FEM model of the micromechanical torsional spring, we try to reduce some problems described above. Geometry generation and meshing is completely parameterized, where the contour of the spring is defined with a set of spline functions. This approach allows the simple implementation of complex geometries like springs with tapered ends. As a first approach, cubic splines with seven equidistant grid points are used to provide sufficient freedom of geometry. Since the springs are always symmetrical, the splines describe only one quarter contour. The result is a parameterized FEM model with the nine parameters l, h and the half spring width at seven locations.

The FEM model allows to verify the accuracy of the four assumptions which were set up to create the analytical model. Comparing the two models leads to the following results:

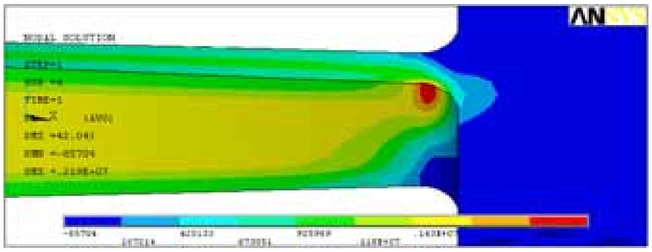

Figure 3: Critical tensile stress at testing structure, introduced in [4]; torsional angle $\phi = 24°$

1. For small torsional angles ($\phi/l < 0.1°/\mu\text{m}$) and geometries fulfilling **A1-A4** the analytical results according to (5) agree with the results of the FEM model. The shear stress is the most important stress component and the maximum shear stress differs by less than 15 percent even for unfavorable spring geometries with $\frac{l}{d} \approx 20$.

2. For large torsional angles ($\phi/l > 0.1°/\mu\text{m}$) and/or spring geometries with $l \ggg d$ new effects arise. The clamping can not be neglected anymore. Tensile and compressive stress components at the suspension of the spring increase faster than linearly (*figure 3*). This matches with the observed failures at the testing structures (*figure 1*).

In conclusion the analytical model can only be used for dimensioning torsional springs under the conditions $l \gg d,\ l \gg h,\ \phi/l < 0.1°/\mu\text{m}$. Other spring geometries require further analysis using FEM techniques. The reason for the additional stress components is the warping of the spring's cross section. This effect arises with all non-circular and non-elliptical cross sections. Because of the insensitivity of silicon to compressive stress only the tensile stress has to be respected in further analysis.

4 OPTIMIZATION APPROACH

To allow for larger torsional angles at maximum stress, defined by reliability objectives, the geometry of a certain torsional spring has to be optimized. Therefore it is necessary to reduce stress peaks. Since the height of the spring is specified by MEMS process, the spring's width/contour and length are the only degrees of freedom. In case of a spring, which can be sufficiently described by the analytical model, shear stress generally decreases with increasing length. In order to retain a given torsional spring constant it is necessary to increase width (increasing $\frac{I_t}{W_t}$), nevertheless, the maximal shear stress will always be at its minimum for maximal possible length.

The case of a spring with local tensile stress as critical stress is even more interesting. With modifications of the contour it is possible to modify the distribution

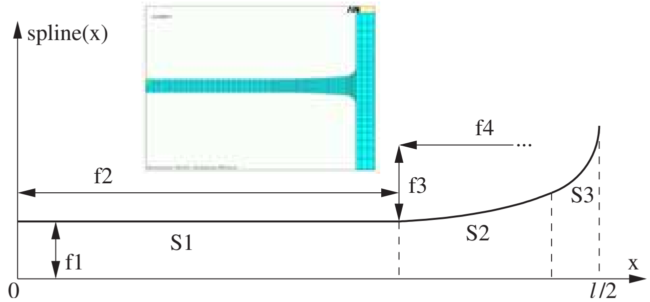

Figure 4: Parameterizing of one quarter of the spring's contour, using straight line, ellipse and circle equations

as well as the magnitude. The maximum tensile stress can be reduced, for example, by widening the spring at its ends. On the other hand, such a change in geometry leads to increasing shear stress and changes of the torsional spring constant. Considering all characteristics of the given problem, a nonlinear optimization is necessary to find an improved spring geometry, providing a certain spring constant with reduced maximum stress.

For solving this typical design problem the optimization tool *MOSCITO* [1], [2] is used. It was designed by the Branch Lab Design Automation of *Fraunhofer IIS* Dresden for simulator-based design of microsystems and may be combined with almost any modeling tool. It contains modules for model parametrization, simulation, calculation of error functions and optimization. Since there is an interface between *MOSCITO* and the FEM tool *ANSYS®* available it is useful to combine it with our parameterized FEM model. Reducing the number of parameters (the dimension of the optimization problem) and considering constraints from our MEMS process, a new set of spline functions is introduced. It consists of three "splines" described by a straight line ($S1$), an ellipse equation ($S2$) and a circle equation ($S3$) with constant radius, specified by the MEMS process. This new spline set describes the contour with just four parameters $f1 .. f4$ (see *figure 4*), where $f3$ and $f4$ are the semimajor and semiminor axes of the ellipse. The location of the transition between $S2$ and $S3$ is determined by the condition

$$\frac{d}{dx}S2 = \frac{d}{dx}S3$$

and is calculated during model generation, using Newton's approximation of roots. To rate a specific set of parameters, for every optimization cycle a static analysis at a given torsional angle, calculating stresses and reaction forces has to be done. Since *MOSCITO* needs a scalar as return value of an optimization cycle the results have to be combined. Therefore an objective function is needed. A suitable objective function for the given

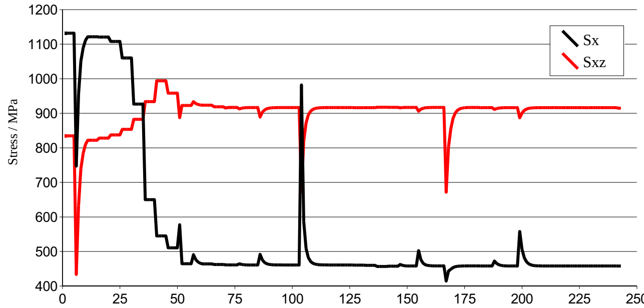

Figure 5: Maximum tensile stress Sx and maximum shear stress Sxy vs. optimization cycles

problem is for example

$$L(f1,..,f4) = \max\{w_1\,\sigma_{\max}, w_2\,\tau_{\max}\} + (w_3\,(k_t - k_{t0}))^2 \tag{6}$$

with $\sigma_{\max}$ as maximum tensile stress, $\tau_{\max}$ as maximum shear stress, k_t as current torsional spring constant and k_{t0} as objective torsional spring constant. w_1, w_2, w_3 are weighting factors. The current torsional spring constant k_t is calculated from the reaction forces and the given torsional angle.

This setup enables us to optimize any desired torsional spring geometry in terms of the maximal stress for a given spring constant and spring length. The importance of several stress components is considered by choosing the weighting factors.

5 OPTIMIZATION EXAMPLE

To illustrate our optimization method an example for a particular spring geometry was chosen and optimized: For a current design at *Fraunhofer IPMS* a torsional spring with a length $l \leq 240\mu m$, a height $h = 30\mu m$, a torsional spring constant $k_{t0} = 205 \cdot 10^{-9} Nm/_{rad}$ and a torsional angle $\phi \geq 40°$ is needed. If such a spring is realized using standard geometry – with constant width and maximum length – the spring's width would result in $d = 4\mu m$, length in $l = 240\mu m$ and the critical tensile stress at an angle of $\phi = 40°$ would be exceeded by 70 percent (see *figure 6*a; *table 1*). Thus an optimization is necessary in order to fulfill the requirements.

The progression of an optimization, using the optimization algorithm FSQP [1], is shown in *figure 5*. Every cycle has a duration time of approximately 10 minutes at an up-to-date *Sun UltraSPARC*® workstation. After 243 cycles, the optimization stops. The result is a geometry providing a similar maximum shear stress and decreased tensile stress, fulfilling all requirements. Compared with the standard geometry, the optimized spring is able to perform about 30 percent larger torsional angles without exceeding the critical torsional and tensile stress respectively. The deviation of the spring constant from the objective spring constant is less than 2 percent. *Figure 6* shows the tensile stress of the example spring before and after optimization. The changes in geometry are quite small but it is well recognizable that the peak is reduced and the stress is distributed more homogeneously.

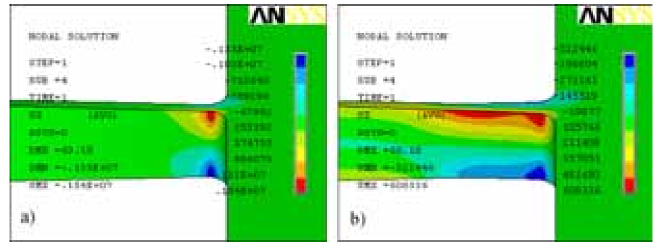

Figure 6: Tensile stress of the example geometry a) before and b) after optimization; torsional angle $\phi = 40°$; The maximum stress is reduced by 60 percent.

6 SUMMARY AND OUTLOOK

The investigations of stress-related failure of micromechanical torsional springs unveiled some interesting phenomenons. Spring geometries with straight contours are not the optimum for applications requiring large torsional angles. In order to improve reliability and applicability of torsional springs an optimization tool based on the *MOSCITO* optimization framework was developed at *Fraunhofer IPMS* in Dresden. Contour-optimized testing structures with tapered ends are fabricated. Thus it will soon be possible to verify the theoretical results by measurements.

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