Finite Element Modeling of Electro-Mechanical Coupling in Capacitive Micro-Systems

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ABSTRACT

In this paper advanced multi-physics simulations of micro-electro-mechanical systems (MEMS) are used to investigate their dynamic behaviour. The strong coupled electro-mechanical Finite Element (FE) formulation is used to model the electro-mechanical interactions and to perform modal and transient analysis taking into account large deformation effects. The application examples simulate two micro-resonators consisting in a clamped-clamped beam suspended over a substrate (the lower electrode). When a voltage is applied between the beam and the substrate, electrostatic forces appear which force the beam to bend. When the applied voltage is increased up to the pull-in limit, the electrostatic force becomes dominant and the plates stick together. The pull-in voltage is an essential design parameters in capacitive micro-systems. Here we also define a new design parameter describing the limit dynamic behaviour, namely the dynamic pull-in voltage.

Keywords: Electro-mechanical coupling, Finite Element, Nonlinearity, Micro-Systems, Pull-in.

1 INTRODUCTION

MEMS are micro-metric devices in which electric, mechanical and fluid phenomena appear and interact. Because of their microscopic scale, strong coupling effects arise between the different physical fields, and some forces, which were negligible at macroscopic scale, have to be taken into account. In order to design micro-electro-mechanical systems, it is of primary importance to be able to accurately predict the static and dynamic behaviour as a strongly coupled interplay between electric and mechanical forces.

In the paper we will apply advanced modeling techniques based on Finite Element discretization to evaluate static and dynamic behaviour of two capacitive micro-devices. In particular we will introduce a new concept in order to characterize the dynamic behavior of MEMs i.e. the dynamic pull-in.

2 PROBLEM DEFINITION

The structures considered in this paper are beam-type resonators composed of two electrodes. One side of the electrode is a clamped-clamped suspended beam while the other electrode is fixed on the substrate (figure 1). The beam is made out of silicon with density and Young modulus of 2648.38 kg/m^3 and 77GPa respectively . Its length is 300 μm and its thickness is 0.5 μm. The distance d_0 between the electrodes when the beam is undeformed is 6 μm. In the first structure, the lower electrode is located at the center of the substrate and has a length of 60μm. For the second case, the lower electrode is composed of two 60μm parts separated by a distance of 60 μm.

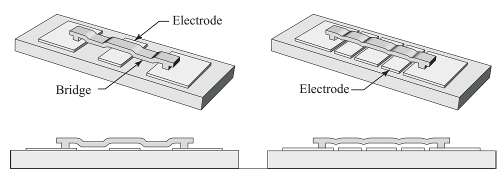

Figure 1: Micro-resonators.

The finite element model of these micro-resonators is simplified as shown in figure 2. The fixation at the extremities of the beam may be considered as clamped and the corrugations along the beam are small enough to be negligible. First a static study is performed to evaluate the *static pull-in voltage*. Then the dynamic behaviour of both structures is studied when a voltage is suddenly applied between the two electrodes. A new concept is then defined: the *dynamic pull-in voltage* representing the step voltage for which the beam, initially at rest, is trapped in an unstable trajectory of the phase diagram and stick to the lower electrode. The static and dynamic pull-in thresholds will be evaluated using the assumptions of small and large deformation respectively as outlined in [2].

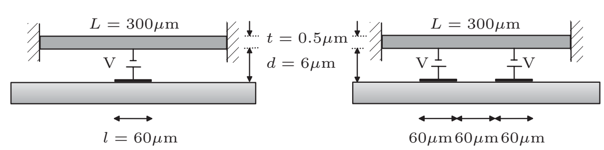

Figure 2: Simplified models of the micro-resonators.

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3 NUMERICAL SIMULATION

Classical methods used to simulate the coupling between electric and mechanical fields are usually based on staggered procedures where successive quasi-static solutions for the electric and mechanical problem are found separately [1].

In this paper, a fully coupled electro-mechanical Finite Element formulation is used. The consistent way of deriving a finite element discretization to model electro-mechanical coupling consists in using the variational calculus [3]. Starting from the energy of the coupled problem, nodal forces are obtained for an element by derivation of the energy. The tangent stiffness matrix of the coupled problem is then obtained by linearization of the equilibrium equations in the vicinity of an equilibrium position (see reference [3]):

$$\begin{pmatrix} \mathbf{K}_{uu}(\phi) & \mathbf{K}_{u\phi}(\phi) \\ \mathbf{K}_{\phi u}(\phi) & \mathbf{K}_{\Phi\Phi} \end{pmatrix} \begin{pmatrix} \Delta\mathbf{U} \\ \Delta\mathbf{\Phi} \end{pmatrix} = \begin{pmatrix} \Delta\mathbf{f_m} \\ \Delta\mathbf{Q} \end{pmatrix} \quad (1)$$

where u and ϕ refer respectively to the mechanical and electric domains; $\Delta\mathbf{U}$ is the displacement vector and $\Delta\mathbf{f_m}$ is the mechanical forces vector; $\Delta\mathbf{\Phi}$ is the electric potential vector and $\Delta\mathbf{Q}$ is the variation of electric charges. The tangent stiffness matrix is symmetric and coupling terms $\mathbf{K}_{u\phi}$ and $\mathbf{K}_{\phi u}$ appear between mechanical and electric degrees of freedom. The tangent stiffness matrix, unlike in the common approach, is not obtained by finite difference, but derives naturally from the variational approach and can be easily assembled using the finite element model. Details about the expression of the coupling matrix are described in reference [3].

This technique allows to compute static equilibrium positions in a non-staggered way, and provides fully consistent tangent stiffness matrices that can be used for transient analysis.

3.1 Static Pull-in voltage

To evaluate the static pull-in voltage using a staggered method, a trial voltage is first applied. If the algorithm converges, a static solution exists and thus the pull-in voltage has not yet been reached. Conversely if the algorithm does not converge, it is likely that the applied voltage is higher than the pull-in threshold (see figure 3a). Hence in a staggered scheme the pull-in voltage is found by a trial and error iteration.

With the method proposed here, continuation algorithms such as Riks-Crisfield, are used. In that case they are able to pass over the pull-in voltage and to enter in the unstable area (Figure 3b). This methodology provides more reliable results than a staggered method.

3.2 Dynamic Pull-in voltage

One of the advantages of the proposed methodology is that the coupled tangent stiffness matrix that arises

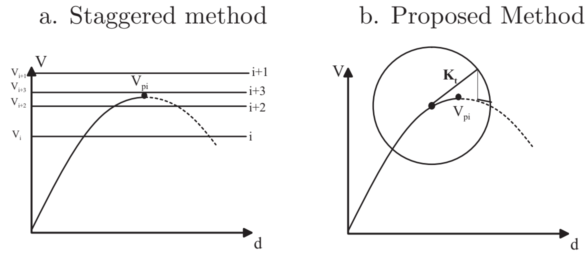

Figure 3: Usual and proposed method to evaluate the static pull-in voltage.

from the formulation allows to predict the dynamic behaviour of the system when the voltage is suddenly applied between the electrodes (figure 4).

With this type of excitation, the overshoot of the response due to dynamic effects may reduce the distance between the plates so that the electrodes stick together even if the static pull-in voltage is not reached. The threshold voltage value for which this instability occurs is defined here as the dynamic pull-in.

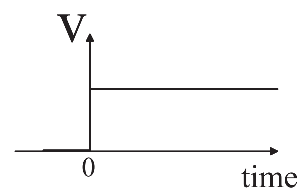

Figure 4: Step in time.

4 RESULTS

Now the behaviour of these two resonators shown in figure 1 is studied. First a static computation allows us to determine the static pull-in voltage and then a time-integration procedure is used to compute the dynamic pull-in voltage. The problem is discretized using 600 electric elements to model the vacuum between the electrodes and 150 mechanical elements to model the beam. Electro-mechanical coupling elements are placed between the beam and the vacuum to generate the proper electric forces.

4.1 Small Displacement Hypothesis

In this section, the displacements are assumed to remain small so that a linear strain theory is applied. This first analysis allows to identify the effect of the electro-mechanical coupling without adding the nonlinearity produced by the large displacement of the beam.

Figure 5: Electric potential at the static pull-in voltage for the case of one centered electrode.

Figure 6: Electric potential at the static pull-in voltage for the case of two electrodes.

4.1.1 Pull-in Voltage

The static pull-in for the central electrode resonator is found to be 26.96 V. When the resonator is composed of two electrodes, the static pull-in voltage is 26.4 V. Figure 7 shows the displacement of the center of the beam when the voltage increases. The displacement necessary to reach the static pull-in voltage is larger in the case of two electrodes than when there is only one. Since the pull-in voltages are nearly the same although the electrode area in the second design is doubled, the first design is more efficient to reaching the pull-in threshold.

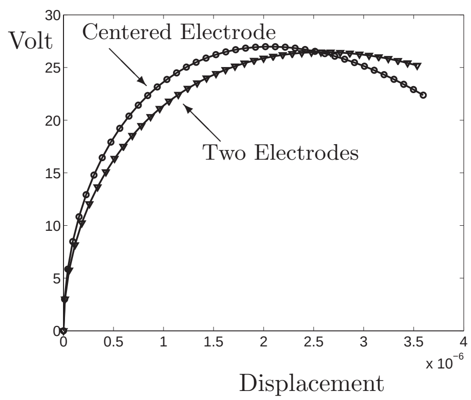

Figure 7: Evolution of the displacement of the center of the beam with the voltage.

In figure 5 and 6 the electric potentials are plotted when the static pull-in voltage is reached for the two resonators.

4.1.2 Dynamic behaviour

The dynamic motion of the beam when a step voltage is applied can be visualized in the phase diagrams of Figures 8 and 9. It may be observed in Figure 8 that when the voltage amplitude is equal to 24.81 V the beam passes a threshold such that the beam slams on the lower electrode. This corresponds to the dynamic pull-in limit for the first design. The dynamic pull-in voltage is found to be 24.24 V in the case of two electrodes (Figure 9). The difference between the static and the dynamic pull-in voltages is 7.97% for the first test case and 8.18 % for the second one.

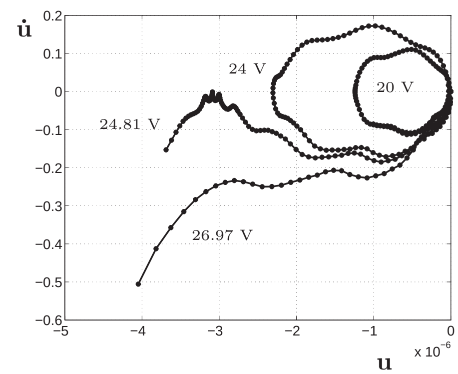

Figure 8: Phase Diagram for one central electrode resonator with small displacement hypothesis.

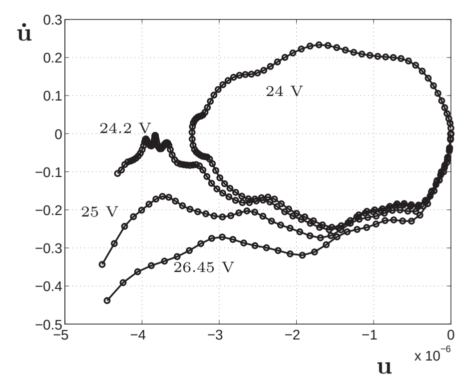

Figure 9: Phase diagram for two electrodes resonator with small displacement hypothesis.

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4.2 Large Displacement Hypothesis

As the beam has a very large aspect ratio (the length thickness ratio is equal to 300/0.5) and as the maximum transverse displacement (equal to the gap) is 6 μm, large strain have to be accounted for.

4.2.1 Pull-in Voltage

With the large displacement hypothesis, the static pull-in voltage increases from 26.96 V to 129.3 V for the centered electrode resonator and from 26.4 V to 141.6 V for the two electrodes resonator. The voltage has been multiplied by 4.79 for the first problem and by 5.36 for the second one. This can be explained by the fact that the two-electrodes resonator induces more deformation in the beam and thus is more sensitive to large displacements effects.

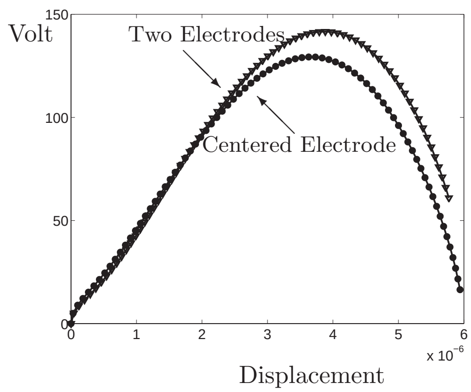

Figure 10: Evaluation of the static pull-in voltage with large displacement hypothesis.

4.2.2 Dynamic behaviour

The dynamic behaviour of the structures is now performed with the large displacement model. The dynamic pull-in voltage in the case of a single centered electrode is about 114.8 V, 10.5% lower than the static pull-in voltage with the same hypothesis. For the second problem, the dynamic pull-in voltage is about 125.1 V, i.e. 11.6% lower than the static one.

Figures 11 and 12 show phase diagrams in the two test cases. The large displacement hypothesis introduces more non-linearity in the behaviour of the resonators.

5 Conclusions

In this paper, strong electro-mechanical coupling elements are used to simulate the static and the transient behaviour of two types of micro-resonators. The first one is composed of one central electrode and the second one of a couple of electrodes. The static pull-in voltage is evaluated and the concept of dynamic pull-in is introduced. We observe that in terms of pull-in threshold, both designs (single centered and double uncentered electrodes) are very similar. From the transient dynamic

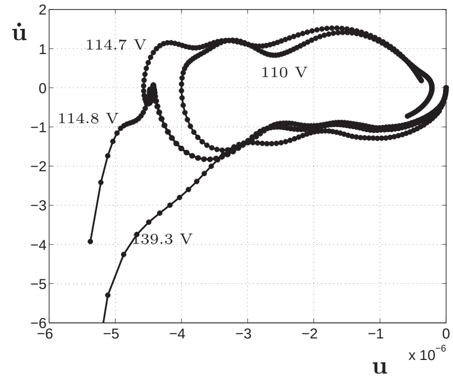

Figure 11: Phase diagram for one central electrode resonator with large displacement hypothesis.

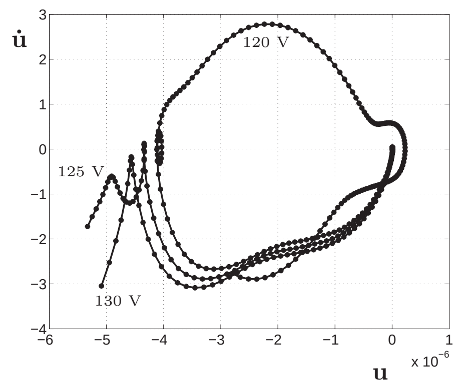

Figure 12: Phase Diagram for two electrodes resonator with large displacement hypothesis.

simulations, it was found that the difference between the static and the dynamic pull-in voltage is about 8 %. The large deformation effects have a significant influence in the behaviour of the structures: they increase the pull-in voltage by a factor 5 and the difference between the static and the dynamic pull-in voltage can be as high as 11 %.

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