MOSFET Analytical Inversion Charge Model with Quantum Effects using a Triangular Potential Well Approximation

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ABSTRACT

The eigenfunctions from solutions of the Schrödinger equation for a triangular potential well are the Airy functions. The triangular potential approximation has been shown to be a good approximation for the charge density when the MOS device is in depletion or weak inversion. However, the approach has not had comparable success in approximating the inversion charge density when the device is at strong inversion (see Stern [1] and Moglestue [2]). In this paper we continue to use the triangular potential to estimate the inversion charge, but we use asymptotic solutions of the Poisson equation for the MOS device at strong inversion. The electrostatic potential asymptotic expression is given in [3], which was improved in [4]. Our analytical Schrödinger-Poisson (SP) result is compared with the Bohm potential [5] or Density-Gradient (DG) numerical solutions [6, 7] and Hansch analytical quantum models [8]. Our SP analytical model gives a close approximation to the full numerical inversion charge density simulation results of the DG model (see Figures 1 and 2).

Keywords: Charge density, Device modeling, MOSFET, Quantum effect, SPICE

1 INTRODUCTION

Investigating quantum mechanical effects in the MOS structure requires both the self-consistent numerical and analytical techniques. These techniques are used to solve the Schrödinger-Poisson (SP) equations. The triangular potential well approximation is often used to get analytical solutions from the Schrödinger equation for application in SPICE circuit simulators. However, the triangular potential approach, [1], is valid only in a weak inversion region of the MOS structure. It gives good results if there is little or no charge in the inversion layer but has not had comparable success when the charge density in the inversion layer is comparable to that in the depletion layer. Regardless of this disadvantage, the triangular potential well approach of [1] is widely used for modeling quantum mechanical effects in MOS structures, [9, 10]. The central assumption for the triangular potential approximation is that the change in the electrostatic potential in the direction perpendicular to the transistor channel is constant.

In this paper we continue to use the triangular potential to estimate the inversion charge, but we use asymptotic solutions of the Poisson equation for the MOS device at strong inversion, [3]. We show improvements over the results of [1] that were derived from the Schrödinger equation for strong inversion region application.

2 TRIANGULAR POTENTIAL APPROXIMATION

We estimate the channel electrostatic potential V of the NMOS transistor from the asymptotic solution of the Poisson equation using the triangular potential approximation and solve the Schrödinger equation

$$\frac{d^2\psi_j}{dx_1^2} + \frac{2m^*}{\hbar^2}[E_j - U]\psi_j = 0 \qquad (1)$$

ψ_j is the electron wavefunction with the corresponding energy eigenvalue E_j, m^* is electron effective mass for motion perpendicular to the transistor channel surface, $\hbar$ is Planck's constant divided by 2π, $U = -qV$ is the electron potential energy in the channel. The boundary condition for the wavefunction used in this work is: $\psi_j(0) = \psi_j(\infty) = 0$.

The Poisson equation is

$$\frac{d^2V}{dx_1^{\,2}} = \frac{q}{\varepsilon_s}(n - p + N_a) \tag{2}$$

where $n = n_i e^{(V-\phi)/V_{th}}$ and $p = n_i e^{-(V-\phi)/V_{th}}$ are representations of the classical charge density.

The assumption here is that the doping density $N_a >> N_d$ in the silicon, where N_d and N_a are the donor and acceptor doping densities respectively. The parameter q represents electron charge, ε_s semiconductor permittivity, n electron density, p hole density, n_i intrinsic density, $V_{th} = kT/q$, k Boltzmann constant and T temperature.

The boundary conditions consist of the continuity of electric potential and the electric displacement, $\varepsilon_i \partial V / \partial x$, at the oxide interface $x_1 = 0$, where ε_i is the oxide permittivity.

The scaling used for the co-ordinate perpendicular to the channel, electrostatic potential, quasi-Fermi potential parameters are given respectively:

$$x_1 = xL_d\sqrt{\ln\lambda/\lambda},\ (V,\phi) = (w,\varphi)V_{th}\ln\lambda \tag{3}$$

where $\lambda = \frac{N_a}{n_i}$, N_a is the channel doping density, $L_d = \sqrt{kT\varepsilon_s / n_i q^2}$ is the Debye length. Rewriting equation (2) using the scaling in (3) gives

$$\frac{d^2w}{dx^2} = \frac{1}{\lambda}(e^{(w-\varphi)\ln\lambda} - e^{-(w-\varphi)\ln\lambda}) + 1 \tag{4}$$

In the inversion layer the appropriate scaling is $\tilde{x} = x\ln\lambda$ and the first order asymptotic approximation gives the Poisson equation in a scaled variable as [3]

$$\frac{d^2w}{d\tilde{x}^2} = \frac{1}{\lambda(\ln\lambda)^2}e^{(w-\varphi)\ln\lambda} \qquad w(0) = w_s \tag{5}$$

The solution of (5) is

$$w(\tilde{x}) = 1 + \varphi + \{\ln(\ln\lambda) + 2\ln\alpha_0 - 2\ln(\sinh(\frac{\alpha_0\tilde{x}}{\sqrt{2}} + \gamma))\}/\ln\lambda \tag{6}$$

where

$$\gamma = \sinh^{-1}(\alpha)$$
$$\alpha = \alpha_0\sqrt{\ln\lambda}\,\lambda^{(1-w_s+\varphi)/2}$$

In [4] the scaled surface potential at strong inversion is improved as

$$w_s = 1 + \varphi + \frac{2}{\ln\lambda}[\ln(\ln\lambda) + \ln z_0]$$
$$z_0 = \frac{c}{\sqrt{2}}(\frac{V_g}{V_{th}\ln\lambda} - \varphi - 1), \tag{7}$$
$$c = \varepsilon_i L_d / (t_{ox}\varepsilon_s\sqrt{\lambda})$$

where V_g the effective electrostatic potential at the gate contact and t_{ox} is the silicon oxide thickness. Since the inversion layer thickness is very small, expansion of equation (6) for small $\tilde{x}$ gives

$$w(\tilde{x}) = w_s - f_s\tilde{x} \tag{8}$$

where $f_s = \frac{\sqrt{2}\alpha_0\coth\gamma}{\ln\lambda}$.

The triangular potential well approximation at strong inversion is then becomes

$$U(x_1) = -qV = -qV_s + qF_s x_1 \tag{9}$$

where $F_s = \frac{V_{th}\sqrt{\lambda}(\ln\lambda)^{3/2}f_s}{L_d}$ and $V_s = V_{th}w_s\ln\lambda$

The band bending near the silicon/siliconoxide interface confines the carriers to a narrow surface channel at strong inversion and an electron in the semiconductor conduction band is bounded and its energy is quantized. The model used for electron concentration in j[th] subband is,

$$N_j = 0.38m_e(\frac{kT}{\pi\hbar^2})\ln(1 + \exp(E_f - E_j/kT)) \tag{10}$$

where E_f is the Fermi energy and m_e is the electron mass.

Substituting (9) in to (1) and solving the eigenvalue problem gives the Airy functions as solutions for the Schrödinger wavefunction,

$$\psi_j = aAi\left[\left(\frac{2m^*qF_s}{\hbar^2}\right)^{1/3}\left(x_1 - \frac{(E_j + qV_s)}{qF_s}\right)\right]$$
$$E_j = -qV_s + \left(\frac{\hbar^2}{2m^*}\right)^{1/3}\left((3/2)\pi qF_s(j + \frac{3}{4})\right)^{2/3} \tag{11}$$

where *a* is the normalization constant.

The triangular-potential approximation that was used in [1] is shown to be a good approximation for electron ground

state energy calculation. In this work α_0 is used as a free parameter and its value is determined from the ground state energy value of [1], where

$$E_0 \sim (\hbar^2 / 2m^*)^{1/3} \left(\frac{9}{8} \pi q V_s \sqrt{q N_a / 4 \varepsilon_s V_{th} \ln \lambda} \right)^{2/3}$$

The carrier concentration at x_1 is

$$n(x_1) = \sum_j N_j \psi_j^2(x_1) \qquad (12)$$

In the above equation (12) the eigenfunction is normalized on the half-line: $\int_0^\infty \psi_j^2(x_1) dx_1 = 1$

3 RESULTS

In this section we present results for our final model, equation (12), compared with other commonly used analytical quantum models, namely the DG and Hansch models. Our SP analytical model gives a close approximation to the full numerical inversion charge density simulation results of the DG model (see Figures 1 and 2).

Since more than 90% of the electrons are concentrated in the ground state and the remaining less than 10% are in the first and second excited states, the computation that requires determining the charge density from (12) becomes simple. This model may be pursued further to improve the analytical estimate of the semiconductor energy band gap widening, average charge distance increase from the silicon/siliconoxide interface to the bulk and an increase on the oxide thickness, due to quantization effect [9,10].

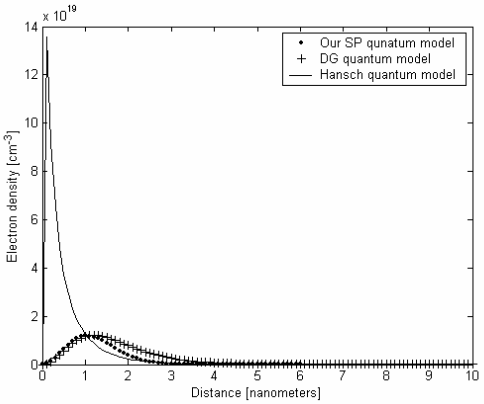

Figure 1: Electron density perpendicular to the inverted channel. Relative effective mass of electron 0.2, $E_f=0$, effective gate voltage 2V, substrate doping 5×10^{17} cm^{-3} and 2nm oxide thickness.

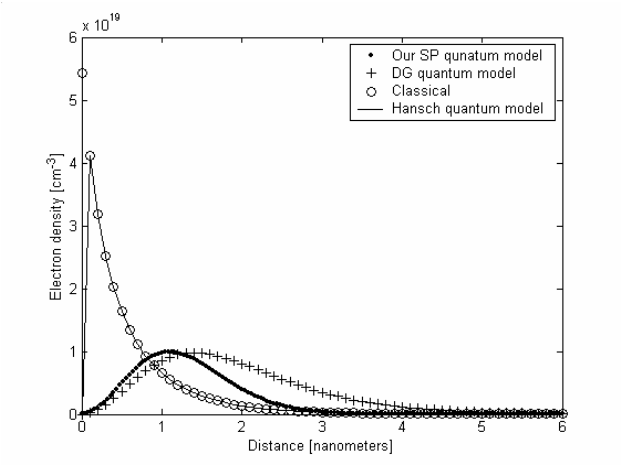

Figure 2: Electron density perpendicular to the inverted channel. Relative effective mass of electron 0.2, $E_f=0$, effective gate voltage 2V, substrate doping 5×10^{17} cm^{-3} and 4nm oxide thickness.

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